MURA-LWJ-8 MURA #58

A Special Case of Mark V Parallel Sloped Bars

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I. Introduction

If a Mark V F.F.A.G. accelerator is constructed of parallel, sloped bars of magnets such that the field along an equilibrium orbit is alternately zero and a constant value, H, the cosine σ expressions can be written explicitly. The solution may then be compared with an equivalent Mathiew equation solution neglecting scallops and with the smooth approximation solution first neglecting scallops, then including scallops of the equilibrium orbit. This is not intended as a serious accelerator design as much as an illustration of Mark V which may aid in gaining insight into some of the general characteristics of Mark V.

II. Exact Formulation.

If the normals to the edges of the bars make an angle α_0 with a circle of radius r about the center of the machine, the particle enters the region of field H from the straight section at an angle $\alpha_0 - E = \alpha_1$, and leaves the field at $\alpha_0 - E = \alpha_2$, where $E = \beta/4$. Here β is S/ρ, the path length in the magnet divided by the radius of curvature. Also $\ell = S$ and C = 2, where C is the circumference factor and ℓ is the length of straight sections.

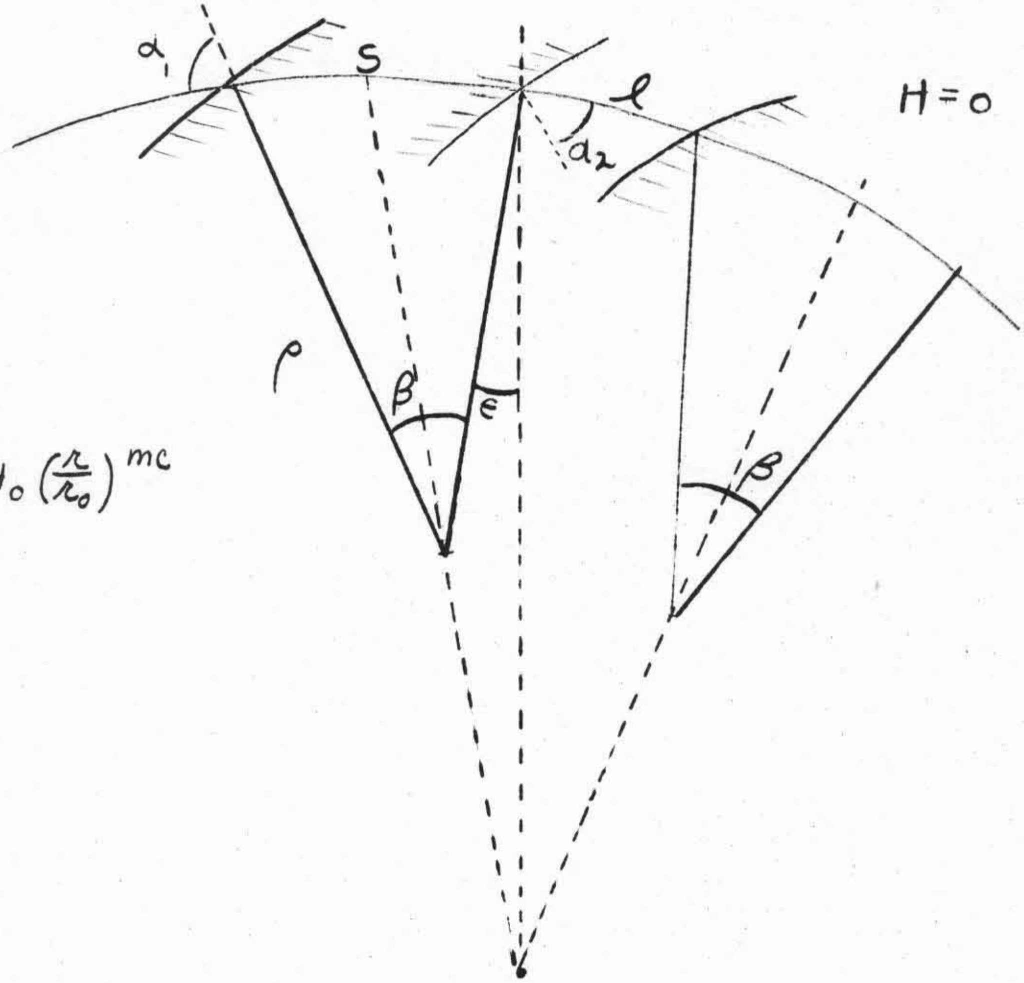
We may define the following:

$$\beta = S/\rho \qquad C = \frac{r}{\rho} \cong \frac{\beta + \delta}{\beta}$$

$$\delta = \ell/\rho \qquad C = 2$$

$$\beta = \delta \qquad H = H_0 \left(\frac{r}{r_0}\right)^{mc}$$

$$n = \left|\frac{\rho}{H}\frac{dH}{d\rho}\right|$$



$$H = H_0 \left(\frac{r}{r_0}\right)^{mc}$$

$\psi = K\beta$ $\qquad$ $k = nc$

$\phi = K_1\beta$ $\qquad$ $K = \sqrt{n'}$,

$t_1 = \tan\alpha_1$ $\qquad$ $K_1 = \sqrt{n+1}$

$t_2 = \tan\alpha_2$

Following the method used in MURA-LWJ-KMT-2 for transformation matrices for Mark Ib with edges and straight sections, the matrix for transformation of displacement and slope for radial oscillation through one magnet and straight section is given by:

$$M_x = \begin{pmatrix} \cos\phi & 1/K_1 \sin\phi \\ -K_1\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -t_2 & 1 \end{pmatrix}\begin{pmatrix} 1 & \delta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ t_1 & 1 \end{pmatrix}.$$

For vertical oscillations, the matrix is:

$$M_z = \begin{pmatrix} \cosh\psi & \frac{1}{K}\sinh\psi \\ K\sinh\psi & \cosh\psi \end{pmatrix}\begin{pmatrix} 1 & 0 \\ t_2 & 1 \end{pmatrix}\begin{pmatrix} 1 & \delta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -t_1 & 1 \end{pmatrix}$$

These matrices refer to a positive momentum compaction design where the gradient within magnets is radially focusing.

The cosine σ expression resulting are:

$$\cos\sigma_x = \cos\phi\left[1 + \frac{(t_1-t_2)}{2}\delta\right] + \sin\phi\left[\frac{t_1-t_2-t_1t_2\delta}{2K_1} - \frac{K_1\delta}{2}\right]$$

$$\cos\sigma_z = \cosh\psi\left[1 + \frac{(t_2-t_1)}{2}\delta\right] + \sinh\psi\left[\frac{t_2-t_1-t_1t_2\delta}{2K} + \frac{K\delta}{2}\right]$$

The following numerical example illustrates the strong effects of scalloping of the equilibrium orbit.

$N = 64$

$k = 205$

$n = 102.5$

$\alpha_0 = 86.49°$

$\beta = 5.45°$

$\alpha_1 = 87.85°$ $\qquad$ $\tan\alpha_1 = 26.45$

$\alpha_2 = 85.13°$ $\qquad$ $\tan\alpha_2 = 11.78$

$\cos\sigma_x = -0.08$ $\qquad$ $\sigma_x = 94°$ $\qquad$ $\nu_x = 16.7$

$\cos\sigma_z = 0.09$ $\qquad$ $\sigma_z = 85°$ $\qquad$ $\nu_z = 15.2$

If the scalloping is neglected, tan α_0 is substituted for tan α_1 and for tan α_2 in the above expressions. Here one assumes that the equilibrium orbit is a perfect circle. The results are:

$$\cos \sigma_x = -0.91 \qquad \sigma_x = 155^\circ \qquad \nu_x = 27.6$$
$$\cos \sigma_z = 0.60 \qquad \sigma_z = 53^\circ \qquad \nu_z = 9.4$$

The scalloping thus clearly improves the design of Mark V with positive momentum compaction.

III. Mathieu Equation Formulation

From the original MURA report on Mark V.[1] it was shown that neglecting forcing terms (scallops) the linear equations of motion of a particle is:

$$\frac{d^2 z}{d\tau^2} + \left[+ a_z + 16q \cos 2\tau \right] z = 0$$
$$\frac{d^2 x}{d\tau^2} + \left[a_x + 16q \cos 2\tau \right] x = 0$$

where:

$$H = H_0 \left(\frac{r}{r_0}\right)^k \left[1 + f \sin\left(\frac{x}{\lambda} - N\theta\right) \right]$$

$$a_x = \frac{4}{N^2}(1+k)$$

$$a_z = -\frac{4}{N^2} K$$

$$q = \frac{f}{4N^2} \frac{\sqrt{k^2 \lambda^2/r^2 + 1}}{\lambda/r}$$

If the sinusoidal flutter factor f is equal to $\sqrt{2}$, the above field approximates the parallel bar field of the preceeding section. Ther ore:

$f = 1.414$

$N = 64$

1 MURA -DWK/KMT/LWJ/KRS -3

$k = 205$

$\theta = 90° - \alpha_0 = 3.51° = .0613$ radians

$\lambda\!\!\!\!- = \lambda/2\pi = \frac{r\theta}{N}$

$\frac{\lambda\!\!\!\!-}{r} = \frac{\theta}{N} = 0.00096$

$a_x = 0.201$

$a_y = -0.199$

$q = 0.091$

From the graph in MURA-DWK-7, these values of a and q lie near the limits of stability.

IV. Smooth Approximation

From later Mark V MURA reports [2,3,4] the smooth approximation solution to the unscalloped Mark V is given as follows:

$$\nu_x^2 = 1 + k + G$$
$$\nu_z^2 = -k + G$$
$$G = \left(\frac{f\,n}{\sqrt{2}\,\lambda\!\!\!\!-\,N}\right)^2$$

For the above values,

$$G = \left(\frac{1}{.00096 \times 64}\right)^2 = 266$$
$$\nu_x = \sqrt{205 + 1 + 266} = 21.7$$
$$\nu_z = \sqrt{266 - 205} = 7.8$$

Recent work by Kerst and Laslett has shown that scallops of the equilibrium orbit modify the smooth approximation solutions as follows:

$$\nu_x^2 = 1 + k$$
$$\nu_z^2 = -k + 2G.$$

2 MURA KRS 7
3 MURA DWK 7
4 MURA DWK 9

This gives

$$\nu_x = \sqrt{205} = 14.35$$
$$\nu_z = \sqrt{2x\ 266 - 205} = 18.1$$

V. Conclusions.

The matrix solutions and the smooth approximation solutions may be compard in both cases: neglecting scallops and including scallops.

	Neglecting Scallops		Including Scallops	
	ν_x	ν_z	ν_x	ν_z
Matrix Solution	27.6	9.4	16.7	15.2
Smooth Approximation	21.7	7.8	14.35	18.1

This comparison must not be taken too seriously since a flutter factor of $\sqrt{2}$ of the sinusoidally varying field is probably not equivalent to the step-wise field of the parallel bars. Also an error in ν of the order of 10% is expected with the smooth approximation when σ reaches $\pi/2$. However both treatments are in general agreement in illustrating the very great effects of scallops of the equilibrium orbit.