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INJECTION INTO FFAG ACCELERATORS

INJECTION INTO F.F.A.G. ACCELERATORS

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I. Introduction

The problem of injection into F.F.A.G. accelerators in the multi-Bev range is greatly simplified due to the elimination of the eddy current problem and great reduction of the field problem. Low energy injection therefore only presents a problem from the standpoints of a) gas scattering, b) space charge repulsion, and c) r.f. range of frequency modulation. We will treat a) and b) below, considering the entire problem of the injection aperture of the accelerator. We will concern ourselves particularly with two numerical designs, a Mark Ib 10 Bev design and a Mark V 20 Bev design.

II. Space Charge:

Laslett[1,2] has treated space charge in the conventional strong focusing accelerator. He assumes a tube of beam of constant cross section and uniform charge density to calculate the effective change in the focusing force of the magnet gradients. It has been pointed out that the effect of space charge is to weaken the focusing and strengthen the defocusing effects both radially and vertically, hence the effect can not be represented on the simple necktie diagram. Symon has suggested that space charge might be compensated for by

pairs of programmed quadrupole lenses which effectively increases the lens strengths to preserve the same σ s. Laslett's expression for space charge is

$$q_0 = \frac{10^9}{60}\left(\frac{2\pi R_0}{c}\right)\left(\frac{\Delta}{R_0}\right)^2 \beta^2 \left|\delta n\right| \quad \text{coulombs}$$

Where R_0 = the machine radius (cm

$c = 3 \times 10^{10}$ cm/sec

β = v/c of the injected particle

Δ = the radius of the beam "tube" of charge.

$|\delta n|$ = the allowable change in n.

In the strong focussing case, $\nu_n \propto \sqrt{k}$ where k = nC for either the Mark V or the Mark I in the smooth approximation. first estimate would be to ask what change in k would move ν_k one eighth the distance between integral radial resonances. If the operating point is midway between an integral and a half-integral resonance, this corresponds to moving half the remaining distance to the closest resonance. In the above, δn is replaced directly by δ k. For 5 Mev injection the above can be rewritten as

$$q_0 = 2.2 \times 10^{14} \frac{\Delta^2 |\delta k|}{R_0} \quad \text{protons.}$$

A lens analogy[3] indicates that the σ_s (or ν_s) for interspersed lenses of different strengths result in an effective σ (or ν) for the combination equal to the sum

of the squares of the components σ_s .

In general, $\sigma = \frac{2\pi\nu}{N}$. If ν_o is the number of betatron wavelengths per revolution neglecting space charge, ν the number including space charge, and i ν_q the effective ν from the defocussing effects of space charge, then

$$\nu_x^2 = \nu_{ox}^2 - \nu_q^2 \ , \quad \delta\nu_x = \nu_x - \nu_{ox} \cong -\frac{\nu_q^2}{2\nu_{ox}}$$

for $|\nu_q| << \nu_o$;

$$\nu_z^2 = \nu_{oz}^2 - \nu_q^2 \ , \quad \delta\nu_z = \nu_z - \nu_{oz} \cong -\frac{\nu_q^2}{2\nu_{oz}}$$

Since ν_{oz} is, for usual F.F.A.G. designs, about $1/2\ \nu_{ox}$, shift toward vertical resonances would probably be more severe by a factor of two than the shift toward radial resonance. We must evaluate δk for a change $\delta\nu_x = 1/8$.

$$\frac{\nu_x + \delta\nu_x}{\nu_x} = \frac{\sqrt{k + \delta k}}{\sqrt{k}}$$

$$1 + \frac{\delta\nu_x}{\nu_x} = 1 + \frac{\delta k}{2k}$$

$$\delta k = 2k\,\frac{\delta\nu_x}{\nu_x} \ , \quad \delta k = \frac{k}{4\nu_x} .$$

For $\delta\nu_z = 1/8$, this should in general be divided by two.

$$\delta k = \frac{k}{8\nu_x}$$

III. Space Charge-Numerical examples.

10 Bev Mark Ib.

$R_o = 10^4$ cm

$k = 200$

$\nu_x = 22$

$\Delta = 5.0$ cm

$\delta k = 200/8x22 = 1.1$

$q_o = 2.2 \times 10^{14} \times 25x1.1/10^4 = 6 \times 10^{11}$ protons per pulse.

For a one turn injector, this would require a current, i

$i = q_o \times 1.6 \times 10^{-19} \times t = 6 \times 10^{11} \times 1.6 \times 10^{-19}/2 \times 10^{-5} =$ 5 milliamperes.

20 Bev Mark V

$R_o = 5 \times 10^3$ cm

$k = 200$

$\nu_x = 14$

$\delta k = 1.8$

Δ cm	q_o protons	i (one turn) milliamperes
1.0	8×10^{10}	1.3
1.5	1.8×10^{11}	2.9
2.0	3.2×10^{11}	5.1

IV. Gas Scattering.

Hammermesh[4] has applied the gas scattering formulation of Courant and Blackman[5] to the F.F.A.G. The probability of survival of a particle in a given accelerator is P (η) where the argument η is

$$\eta_0 = \frac{\pi}{8} Z^2 e^4 N \frac{C \mu^2}{T_i e V} \left[2 \ln \left(\frac{1}{\mu \chi} \right) - 1 \right], \text{ or}$$

$$\eta_0 = 2.82 \times 10^{-13} p \frac{C \mu^2}{T_i e V} \left[2 \ln \left(\frac{1}{\mu \chi} \right) - 1 \right], \text{ and}$$

$$\mu = \frac{2C}{2 \pi A \nu_z}$$

$$\chi = 1.396 \times 10^{-4} T_i^{-1/2} \left[1 + 2.04 T_i^{-1} \right]^{1/2}.$$

C is the circumference in centimeters

A is the vertical semiaperture in centimeters

T_i is the injection energy in Mev

e_V is the energy gain per turn of the particle in Mev.

p is the pressure in the vacuum chamber in units of 10^{-5} mm H_g of air or nitrogen.

ν_z is the number of vertical betatron wavelengths per revolution.

For 5 Mev injection,

$$\chi = 7.4 \times 10^{-5}$$

The function P (η) is plotted in Reference 5. If the beam has an initial amplitude, Δ, as from gas scattering and the phase space from injected particles, the argument, must be increased. From Reference 6,

$$\eta = (\eta_0 + \Delta^2/16A^2)^2/\eta_0 .$$

The aperture, A, should probably be chosen less than the total vertical aperture due to r.m.s. misalignments of the magnets. In the numerical examples considered, A is taken as about 2/3 the semiaperture available physically.

V. Gas Scattering, Numerical Examples.

Mark I. $\nu_z = 5.5$, other parameters as in III.

A = 10 cm

eV = 25 kev/turn

Δ cm	p x 10^{-5} mmHg	η_0	η	P(η)
5.0	1.0	.093	.126	.80
5.0	0.5	.047	.085	.90

Mark V $\nu_z = 6.7$ other parameters as in III

A = 2 cm

eV = 25 kev/turn

Δ cm	p x 10^{-5} mm Hg	η_0	η	P(η)
1.0	1.0	.164	.196	.50
1.0	0.5	.082	.114	.80
1.5	1.0	.164	.31	.28
2.0	0.5	.082	.256	.35

VI. Injector

A 5 Mev electrostatic generator can be expected to give an admittance of 0.5×10^{-3} cm-radians of injected beam[7]. The amplitude of oscillation resulting from such an admittance will be, where δ is the half-angle of divergence and a is the radius of the injected beam,

$$\Delta = \sqrt{a^2 + \left(\frac{2R_o\,\delta}{\nu_z}\right)^2}$$ A minimum amplitude results

when $a = 2R_o\delta/\nu_z$. $a^2 = \frac{2R}{\nu_z}\, a\delta$

Therefore,

minimum $\Delta = a\sqrt{2}$

if $\Upsilon = a\delta$

$$\Delta = \sqrt{4\frac{R_o}{\nu_z}\, a\delta} = 2\sqrt{\frac{R}{\nu_z}\Upsilon}$$

in Mark I $R_o = 10^4$ cm

$\nu_z = 5.5$

$\Upsilon = 0.5 \times 10^{-3}$

$\Delta = 1.9$ cm

in Mark V $R_o = 5 \times 10^3$ cm

$\nu_z = 6.7$

$\Upsilon = 0.5 \times 10^{-3}$

$\Delta = 1.2$ cm

We conclude that values of Δ used in the space charge expressions are realistic.

VII. Conclusions, Numerical Results.

Combining the effects of space charge and gas scattering, one may arrive at a final figure for the protons accepted per pulse. As a first guess, 50% of the protons might be lost due to R.F. phase acceptance, and 50% might be lost in Mark V in crossing the transition energy. The following table summarizes all results.

	Mark I	Mark V
E_{max}	10 Bev	20 Bev
E_i	5 Mev	5 Mev
R_o	100 m	50 m
A	10 cm	2 cm
ν_z	5.5	6.7
ν_x	22	14
eV	25 kev	25 kev
$\lvert\delta k\rvert$	1.1	1.8

Machine	Δ cm	q_o protons	i ma	ρ 10^{-5}	η_o	η	$P(\eta)$	q protons	q after 50% R.F. injection loss	q after 50% Xitron E loss
Mark I	5.0	6×10^{11}	5	1.0	.093	.126	.80	4.8×10^{11}	2.4×10^{11}	
				0.5	.047	.085	.90	5.4×10^{11}	2.7×10^{11}	
Mark V	1.0	$.8\times10^{11}$	1.3	1.0	.164	.196	.50	4×10^{10}	2.0×10^{10}	1.0×10^{10}
				0.5	.082	.114	.80	6.4×10^{10}	3.2×10^{10}	1.6×10^{10}
"	1.5	1.8×10^{11}	2.9	1.0	.164	.24	.40	7.2×10^{10}	3.6×10^{10}	1.8×10^{10}
				0.5	.082	.167	.65	12×10^{10}	6.0×10^{10}	3.0×10^{10}
"	2.0	3.2×10^{11}	5.1	1.0	.164	.31	.28	9×10^{10}	4.5×10^{10}	2.3×10^{10}
				0.5	.082	.256	.35	11×10^{10}	5.5×10^{10}	2.7×10^{10}

Appendix: Justification of substituting δk in the F.F.A.G. case for δn in the case treated by Laslett[1]. From reference 1,

$$F_{s.c.} = \frac{q e (1-\beta^2)}{4\pi^2 \epsilon \Delta^2 R_0} \quad y \quad \text{, the space charge force.}$$

From the magnetic field of the accelerator

$$F_z = \delta B e v_1 = \frac{m e v B_0}{R_0} y$$

where $\delta B = n \frac{B_0}{R_0} y$

since $n = \frac{-R_0}{B_0} \frac{dB}{dR} = \frac{R_0}{B_0} \frac{\delta B}{y}$

In the F.F.A.G. case, (e.g. Mark Ib)

$$n = \frac{\rho}{B} \frac{dB}{dR} = \frac{R}{CB} \frac{dB}{dR} = \frac{R}{C\bar{B}} \frac{\delta \bar{B}}{\delta R}$$

$$k = mc = \frac{R}{\bar{B}} \frac{\delta \bar{B}}{\delta R} = \frac{R}{B} \frac{\delta B}{\delta R}$$

where $\bar{B}$ is the magnetic field averaged over one sector pair,

C is the circumference factor.

$$F_r = \delta B e v = \frac{k B e v}{R} \delta R$$

So for x motion, δk in the F.F.A.G. case plays the role of δn in the y motion of a conventional machine.

In Mark V, k is shown by the smooth approximation to uniquely determine radial focusing.

$$F_r = \overline{\delta B} e v = k \frac{\bar{B} e v}{R} \delta R$$

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1. L. Jackson Laslett, LJL(MAC)-2.

2. L. Jackson Laslett, MURA Notes August 14, 1954.

3. L. W. Jones, MURA-LWJ-7.
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4. Hammermesh, Argonne Accelerator Group Progress Report #3.

5. Blachman and Courant, Phys. Rev. 74, 144 (1948).

6. Blachman and Courant, Phys. Rev. 75, 315 (1949).

7. M. Stanley Livingston, Design Study for a 15-Bev Accelerator.