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ESTIMATE OF POSSIBLE SPATIAL VARIATION OF n

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1. It is desired to estimate the extent to which fine-grained variations of significant extent may be present in the "field index", n, due, for example, to roughness in machining of the pole tips. This information should be of value not only in fixing machining tolerances but also for determining the requirements which should be met in an instrument designed to measure field gradients. It is assumed for the present discussion that the limits on the tolerable variation in n are given by the semi-lateral or semi-vertical widths of one of the diamonds near the center of the necktie diagram, so that

$$(\Delta n)/n < 0.4/n^{1/2}.$$

This requirement may be unnecessarily extreme if it can be shown that no serious effects on the orbit result if a particle passes rapidly in and out of regions with "resonant" values of "n" as it progresses through the magnet sectors and as it executes betatron oscillations--hence the condlusion reached here concerning the spatial extent of a gradient measuring device represents a lower limit to the requisite size. For a useful vertical aperture of three-quarters of the pole-separation, it is found (paragraph 3) that a gradient measurement device of extent no greater than about one-twentieth of the semi-aperture (i.e., possibly 2 mm) is required.

2. We consider a magnetic scalar potential of the form (suitable in a two-dimensional problem, curvature of the equilibrium orbit being neglected)

$$V = H_{oo}\left[n(\frac{x}{r_o} - 1)y + a \sin kx \sinh ky\right],$$

with $a \ll r_o/n$.

Then the field in the median plane is given by

$$-H_o = H_{oo}\left[n(\frac{x}{r_o} - 1) + ak \sin kx \right]$$

and is of interest in the range $r_o < x \leqslant r_o(1 + \frac{2}{n})$, within which the field strength will vary from zero to $2H_{oo}$ when $a = 0$.

The relative gradient is

$$S \equiv (r_o/H_{oo})(d(-H_o)/dx) = n + ar_o k^2 \cos kx,$$

representing a relative ripple in slope (in the median plane) of $\pm ar_o k^2/n$ with a wave-length $2\pi/k$ or $(2\pi n/r_o k)\cdot$(horizontal semi-aperture).

The pole tips for which $V = \pm 2H_{oo}r_o/n$ have, in the absence of the ripple (i.e., in the case $a = 0$), hyperbolic contours for which

$$y = \pm(2r_o/n^2)/((x/r_o)-1)$$

and thus at $x = r_o(1 + \frac{2}{n})$ are distant from the median plane by the horizontal semi-aperture r_o/n.

Due to the perturbing term introduced into the potential, the y-coordinate of the pole-tips will differ somewhat from the foregoing; this difference, if small, may be estimated as follows:

$$\Delta y = -(a/n) \sin kx \sinh ky \;/\; ((x/r_o)-1),$$

$$\Delta y/y = -(an/2r_o) \sin kx \sinh \left[(2kr_o/n^2)/((x/r_o)-1)\right] .$$

The smallest amplitude ripples in the pole tips, for a given "a", will occur at $x = r_0(1 + \frac{2}{n})$ and will amount to

$$\Delta y/y = \mp(an/2r_0)\ \sinh\ (kr_0/n)$$
$$= \mp 1/2 \left[\text{Rel. ripple in Slope}\right]_0 \left(\frac{\lambda}{\text{Semi-Aperture}}\right)^2 \times \sinh\left(\frac{\text{Semi-Aperture}}{\lambda}\right).$$

Thus, if we are interested only in situations for which the relative ripple in slope is greater than $1/2 \times 10^{-3}$ (a rather small value) and if we can achieve $\Delta y/y \leqslant 10^{-3}$, the only wave-lengths of interest are those for which

$$0.18 \times (\text{Semi-Aperture}) < \lambda < 3.96 \times (\text{Semi-Aperture})$$

(the lower limit being of the greater interest in the present context) and gradient measurements over intervals no larger than one-fifth of the semi-aperture (i.e., possibly a range of 8 or 10 mm) would appear satisfactory for surveying fields <u>in the median plane</u>. Similarly, for a relative ripple in slope of 2.2×10^{-3} (and $\Delta y/y \simeq 10^{-3}$, as before), $\lambda = 0.52 \times$ (Semi-Aperture) and a measurement interval of a couple of centimeters would appear acceptable.

3. The requirements are considerably more demanding if one is interested in the field-shape away from the median plane. Thus along the surface for which

$V = (3/2)H_{00}r_0/n$, corresponding to $y = (3/4)(r_0/n)$ at $x = r_0(1 + \frac{2}{n})$,

we have a field obtainable as follows:

$|H| = \left|\frac{dW}{dz}\right|$, where

$W = H_{00}\left[(n/2r_0)(z-r_0)^2 - a \cos kz\right]$ and

$z = x + iy.$

For $k \ll n/r_o$ (or $\lambda \ll$ semi-aperture), the relative ripple in field gradient is then found to be approximately

$$\pm(ar_o k^2/n) \cosh ky = \pm(ar_o k^2/n) \cosh (3r_o k/4n)$$

at $x = r_o(1 + \frac{2}{\pi})$, $y = (3/4)(r_o/n)$.

Accordingly, along the surface considered, the variation in "n" is related to the ripple in the pole-tip profile by

$$\Delta y/y = \mp 1/2 \left[\text{Rel. ripple in Slope}\right]$$

$$\times \frac{\left(\frac{\lambda}{\text{Semi-Aperture}}\right)^2 \sinh \left(\frac{\text{Semi-Aperture}}{\lambda}\right)}{\cosh \left(\frac{3}{4} \frac{\text{Semi-Aperture}}{\lambda}\right)}.$$

This equation, and that developed in Section 2, are plotted in Figure 1.

For a relative ripple in slope of $1/2 \times 10^{-2}$ and $\Delta y/y = 10^{-3}$, we now obtain

$$0.049 \times (\text{Semi-Aperture}) < \lambda \gtrsim 0.51 \times (\text{Semi-Aperture}).$$

A gradient measurement device of extent no greater than about one-twentieth of the semi-aperture (i.e., possibly 2 mm) is thus desired.

4. If attention is directed at points $x = r_o(1 + \frac{2}{n})$, $y = fr_o/n$, one may proceed similarly with

$$\Delta y/y = \mp 1/2 \left[\text{Rel. ripple in Slope}\right]$$

$$\times \frac{\left(\frac{\lambda}{\text{Semi-Aperature}}\right)^2 \sinh \left(\frac{\text{Semi-Aperture}}{\lambda}\right)}{\cosh \left(f \frac{\text{Semi-Aperture}}{\lambda}\right)}.$$

5. An alternative analysis might be made by using as the unperturbed potential

$$V = \frac{H_{oo}}{n-1} \frac{r_o^{\,n}}{(x^2+y^2)^{\frac{n-1}{2}}} \sin\left[(n-1)\tan^{-1}\frac{y}{x}\right] = \frac{H_{oo}}{n-1}\frac{r_o^{\,n}}{r^{n-1}}\sin(n-1)\theta$$

$$= -\frac{H_{oo}}{n-1}\,\mathcal{Im}(r_o^{\,n}/z^{n-1}),$$

for which the field in the median plane is $-H_{oo} r_o^{\,n}/x^n$;

or by using $V = (H_{oo}/2k_o)\, e^{k_o x} \sin k_o y = (H_{oo}/2k_o)\,\mathcal{Im}(e^{k_o z})$, for which $H_o = \frac{H_{oo}}{2} e^{k_o x}$.

The latter potential has been examined, and similar results obtained.

6. Similar methods may be applied to examine the effect of positioning errors; it appears that a tilt of the order of 1/2 (Semi-Aperture)·(tolerable frac. error in n) is important.

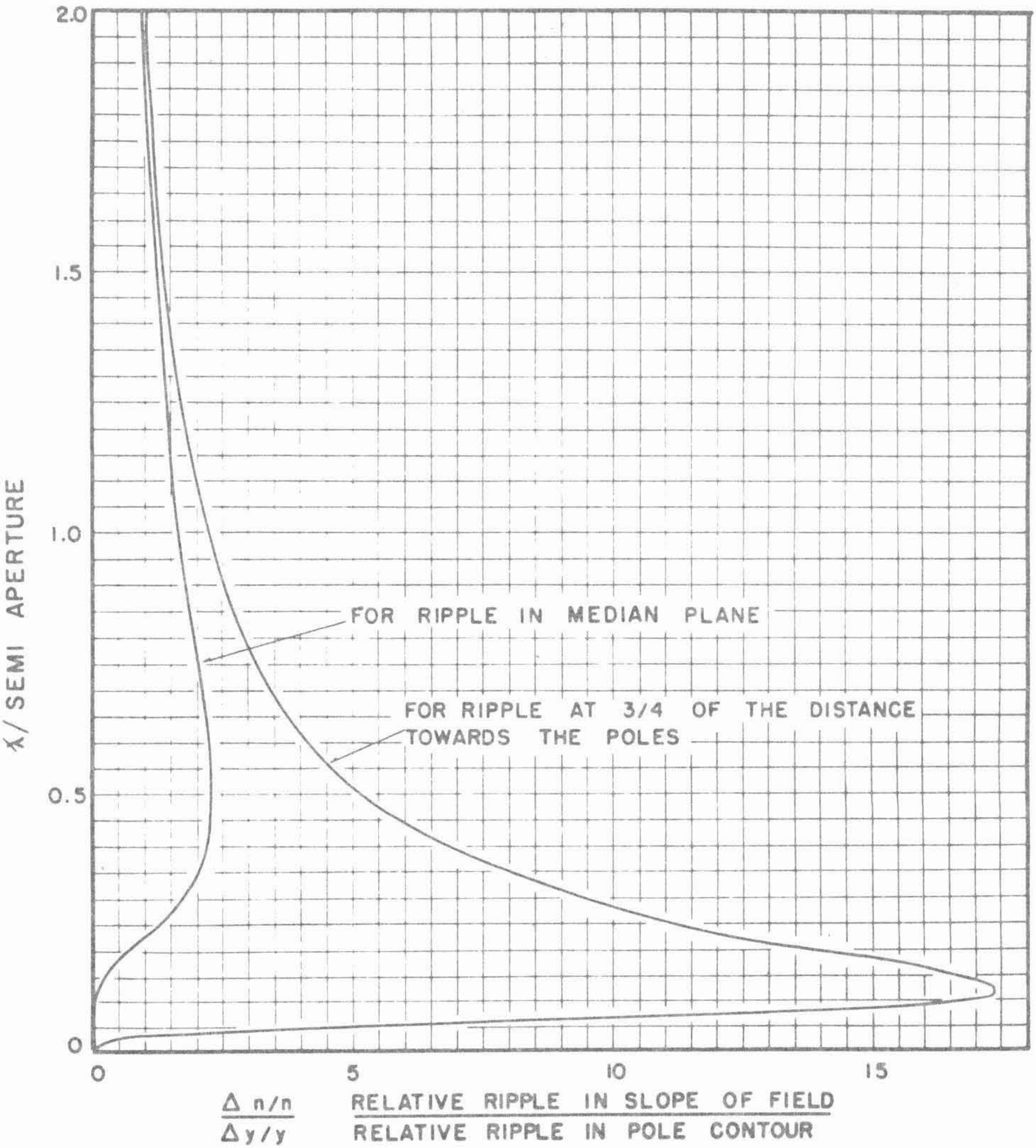

FIG.1 PLOT OF POSSIBLE SPACE-WAVELENGTHS ($\div 2\pi$) OF RIPPLE IN MAGNETIC FIELD. THE LOWER BRANCH OF THE CURVE IS RELEVANT IN JUDGING HOW SMALL A GRADIENT MEASURING DEVICE SHOULD BE.