MURA 50

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NEGATIVE MOMENTUM COMPACTION IN CONVENTIONAL A.G. SYNCHROTRONS

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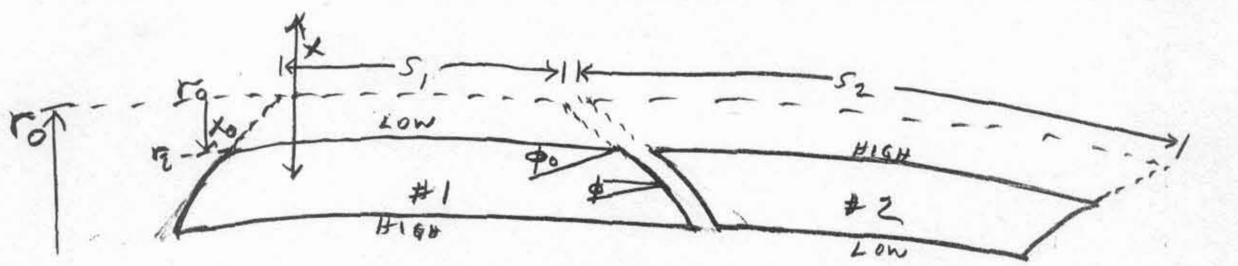
To eliminate the transition energy and to increase phase focussing, Laslett has been searching for negative momentum compaction machines. The recent results with the Argonne Avidac on Mark II FFAG suggested to Jones that one of the anular rings in the trial machines where the fields in each half sector were in the same direction might have negative momentum compaction. Such a possibility is examined here for a standard low momentum content A.G. magnet for the special case of slanting magnet ends.



It is clear that if the field $H_{10} = H_{20}$ along r_0, then $\overline{H}$ is greater at smaller radii which gives negative momentum compaction if the increase in $\overline{H}$ is more than enough to offset the naturally positive momentum compaction of an ordinary A.G. magnet operating in the main stability region. If r_0 was in the center of the sector, the length effect on the momentum compaction at $r > r_0$ would be to increase the already positive momentum compaction. Hence we must work at $r < r_0$ for negative compaction.

The questions examined here are the influence and magnitude of ϕ, the angle of the edge, and the possibility of making a magnet with σ_x and σ_y constant with R.

We will assume a constant gradient machine and constant ϕ_0 from r_0 in to $r_i = r_0 + x_0$. From r_i in farther we may specialize further,

$$\overline{H(x)}^{\theta} = \frac{1}{S_1+S_2}\left\{H_1(r_0)\left(S_1 - 2x\,\mathrm{TAN}\,\phi - \frac{S_1 m_1 x}{R} + 2x^2\,\mathrm{TAN}\,\phi\,\frac{m_1}{R}\right) + H_2(r_0)\left(S_2 + 2x\,\mathrm{TAN}\,\phi + \frac{S_2 m_2 x}{R} + \frac{2x^2 m_2\,\mathrm{TAN}\,\phi}{R}\right)\right\}$$

or

$$\overline{H} = \frac{1}{S_1+S_2}\left\{H_{10}S_1 + H_{20}S_2 + (H_{20}-H_{10})2x\,\mathrm{TAN}\,\phi + (H_{20}S_2m_2 - H_{10}S_1m_1)\frac{x}{R} + (H_{10}m_1 + H_{20}m_2)\frac{2x^2}{R}\,\mathrm{TAN}\,\phi\right\}$$

and

$$\frac{d\overline{H}}{dr} = \frac{1}{S_1+S_2}\left\{2(H_{20}-H_{10})\,\mathrm{TAN}\,\phi + \frac{(H_{20}S_2m_2 - H_{10}S_1m_1)}{R} + \frac{4x}{R}(H_{10}m_1 + H_{20}m_2)\,\mathrm{TAN}\,\phi\right\} \qquad (1)$$

from x_0 INWARD LET $(m_1+m_2)\,x\,\mathrm{TAN}\,\phi = (m_{10}+m_{20})\,x_0\,\mathrm{TAN}\,\phi_0$

Also choose one special case to examine where $H_{20} = H_{10} = H_0$.

$$\alpha \equiv \frac{\Delta r/r}{\Delta p/p} = \frac{\overline{H}}{H_0}\,\frac{S_1+S_2}{(S_2m_2 - S_1m_1) + 4(m_1+m_2)\,x\,\mathrm{TAN}\,\phi} = \frac{1}{\frac{d\overline{H}}{dr}\frac{R}{\overline{H}}} \qquad (2)$$

In addition if we make $S_2 m_2 = S_1 m_1$ then

$$\alpha = \frac{\overline{H}}{H_0}\,\frac{2\pi R}{4N(m_1+m_2)\,x_0\,\mathrm{TAN}\,\phi_0} \qquad \text{IF } x_0 = -5\text{ cm},\ R = 10^4,\ m_1+m_2 = 500,\ N = 60,\ \overline{H}/H = .88 \qquad (3)$$

THEN $\alpha \cong -\dfrac{.045}{\mathrm{TAN}\,\phi_0}$

The α due to scalloping in an ordinary A.G. magnet is

$\alpha_{sc} = +\dfrac{1}{\nu_x^2} \sim .03$ so it appears that edges might be able to eliminate the transition energy if $\phi \sim 70^0$

The effect of edges on σ_x is to decrease it: IF $f \equiv$ FOCAL LENGTH $-\frac{1}{f} = \frac{(H_1 - H_2) \text{TAN}\phi}{\rho}$. Since there are two such edges per sector

$$-\frac{1}{f_{TOTAL}} = \frac{2(H_1 - H_2)\text{TAN}\phi}{\bar{H} R}$$

. For such lenses

$$\sigma = \sqrt{\frac{S_1 + S_2}{f_{TOTAL}}} = \sqrt{\frac{2\pi R}{N f_{TOTAL}}}$$

thus

$$\sigma_{EDGES} = \sqrt{-\frac{2\pi R}{N} \frac{2(H_1 - H_2)\text{TAN}\phi}{\bar{H} R}} = \sqrt{\frac{4\pi}{N} \frac{H_0}{\bar{H}} (m_1 + m_2) \frac{x_0}{R} \text{TAN}\,\phi_0}$$

BY (3)

$$\sigma_{EDGES} = \frac{\pi}{N}\sqrt{\frac{2}{\alpha}} \tag{4}$$

combination of σ's follows the rule

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{EDGES} + \sigma^2_{SECTORS}}$$

where σ_{edge} can be imaginary if X_0 is negative.

If we make $S_1(x)\, H_1'(x)$ constant and $S_2(x)\, H_2'(x)$ constant in the aperture, σ_{sectors} will be constant.

We have four relations:

$$S = S_1 + S_2$$

possible if $\frac{\Delta S_1}{S_1} \ll 1$

$S_1 H_1' = S_2 H_2' = B$ everywhere in aperture. Keeps σ_{sectors} constant. FOR $\sigma_{SEC} < \frac{\pi}{4}$.

$(H_1' + H_2') x \,\text{TAN}\,\phi = A$ " " " Keeps σ_{edge} and α constant

$$= (H_{10}' + H_{20}') X_0 \,\text{TAN}\,\phi_0$$

from which we can determine the law for the edge shape.

Thus:

$$H_1' = \frac{S_2 H_2'}{S - S_2} \qquad \text{so}$$

$$A = (H_{10}' + H_{20}')\, X_0 \tan\phi_0 = H_2' \left(\frac{S}{S - S_2}\right) X \frac{dS_2}{dX}$$

Then

$$A\,\frac{dX}{X} = -H_2' S\,\frac{dS_1}{S_1} = -\frac{H_2' S_2}{S_2 S_1} S\,dS_1 = -\frac{B S\,dS_1}{(S - S_1) S_1} = -B\left\{\frac{dS_1}{S_1} + \frac{dS_1}{S - S_1}\right\}$$

integrating

$$\left(\frac{X_0}{X}\right)^{A/B} = \frac{S_1 (S - S_1)}{S_{10}(S - S_{10})}$$

Simplifying:

$$\frac{S_1 S_2}{S_{10} S_{20}} = \left(\frac{X_0}{X}\right)^{\frac{(H_{10}' + H_{20}') X_0 \tan\phi_0}{H_{10}' S_{10}}} \cong \left(\frac{X_0}{X}\right)^{-.005 \tan\phi_0} \tag{5}$$

SUBSCRIPT 0 REFERS TO VALUE AT X_0

Thus $S_1 S_2 \sim$ constant for $\phi_0 \sim$ ~~45°~~ 75°

This makes

$$\tan\phi = \frac{dS_2}{dX} = \frac{A}{X}\frac{S_1}{H_2' S} = \frac{A}{X}\frac{S_1 S_2}{B S} \cong \frac{1}{X} \tag{6}$$

or exactly

$$\tan\phi = \left\{\frac{(H_{10}' + H_{20}') X_0 \tan\phi_0}{H_{10}' S_{10}}\right\} \frac{1}{X}\left(\frac{X_0}{X}\right)^{\{\ \}} \tag{7}$$

So it seems that $\sim$ ~~45°~~ 75° cuts would give a substantial raising or elimination of the transition energy and that σ would stay about constant if tan $\phi \sim \frac{1}{X}$ provided the scallops do not increase enough with the variation in sector length with radius to eliminate the effect of the r^{-k} or r^{-k} dependence of average field.

The circumference of such an A.G. machine is bigger than one with $H_1 = H_2$ in the aperture, but the specialization of $H_{20} = H_{10}$ at r_0 and $S_2 H_2' = S_1 H_1'$ may not be optimum.

For example suppose at r_0 we are on the diagonal with $\sigma_x = \sigma_y = \frac{\pi}{4}$ and $H_1 = H_2 = H_0$. LET $X_0 = -5$ cm, $m_1(r_0) = m_2(r_0) = 500$
$r_0 = 10^4$ cm, $N = 60$ SECTORS.

$\frac{\Delta H_1(x_0)}{H_1} = +m_1 5 \times 10^{-4} = .25$. SO IF $H_0 = 10,000$ GAUSSES,
$H_1(x_0) = 12,500$ GAUSSES, $H_2(x_0) = 7,500$ GAUSSES. AND AT
$X = -10$ cm $H_1(-10) \cong 15,000$ GAUSSES AND $H_2(-10) \cong 5,000$ GAUSSES
IF TAN $\phi_0 = 3$, $\alpha = -.015$, $\phi_0 = 71°$. AT $X = -10$ cm $\phi = 51°$
AT $X = -15$ cm $\phi = 45°$.

BY (4)
$$\sigma_x = \sqrt{-\frac{\pi^2}{N^2}\frac{(2)(3)}{.045} + \left(\frac{\pi}{4}\right)^2} = \frac{\pi}{4}(.64) \sim \frac{\pi}{6}$$
$$\sigma_y = \sqrt{+\frac{\pi^2}{N^2}\frac{(2)(3)}{.045} + \left(\frac{\pi}{4}\right)^2} = \frac{\pi}{4}(1.26) \sim \frac{\pi}{3}$$

which is still comfortably within the stable region. The field at the orbit ($x = -10$ cm) is 15,000 gausses in magnet #1 and 5,000 gausses in magnet #2. The circumference factor is $>$ 1.5. If we combine the α calculated here with the $\alpha_{scallop} = 1/\nu_x^2$, by algebraically adding the reciprocals of α's (to add their dH's)

$$\nu_x^2 - \frac{\tan\phi_0}{.045} = \frac{1}{\alpha_{TOTAL}}$$

IF TAN $\phi_0 = 3$ AND $\nu_x = 6$

$$\alpha_{TOTAL} = \frac{1}{36 - 67} = -.032$$

which is about equal to the conventional α but of the opposite sign and there is no transition energy. If the focussing and defocussing sectors are subdivided, smaller angles can be used on the edges.

If we slant the edges the other way, the transition energy can be raised somewhat.