Fermilab Library
0 1160 0038406 9

MURA A-48

MURA - LWJ-7
Supplement

EDGE EFFECTS IN FFAG, Mark II*

L. W. Jones, University of Michigan and
Midwestern Universities Research Association

December 30, 1954

It was pointed out in MURA-LWJ-7 that for a particular choice of parameters (e.g. $N = 63$, $\sigma_x = \frac{5\pi}{6}$, $\sigma_z = \frac{\pi}{6}$) the effect of focusing from magnet edges was so important that a Mark II must have a circumference factor almost as great as a Mark Ib if the fields in half the sectors are not to reverse direction.

One questions whether this is due to an unfortunate choice of parameters or whether the results are general to all Mark II types. In particular; if the number of sectors, N, and the number of betatron wavelengths, ν, are varied but the values of σ held constant, are the effects of edges in Mark II designs always as serious as indicated above?

The arguments below indicate an affirmative answer, and imply that conclusions from the previous specific calculations on Mark II apply generally.

We rephrase the question as follows:
If the σ's from strong focusing neglecting edges are held constant as N is changed, will the σ due to edges alone vary?
Let r_0, $\Gamma(p)$, σ_x, and σ_z be held constant.
For edges alone,

$$\sigma_{edge} = \sqrt{8T \tan\theta/\rho} \propto \sqrt{T \tan\theta},$$

if ρ is constant. T is the half-sector length and θ is the angle between a normal to the magnet edge and the equilibrium

*Supported by the National Science Foundation and MURA Universities.

orbit.

From the smooth approximation, neglecting edges,

$$\frac{P}{P_0} \cong \frac{\bar{H}}{\bar{H}_0} = \left(\frac{r}{r_0}\right)^k$$

$$2k = \nu_x^2 - \nu_z^2 - 1$$

$$2G = \nu_x^2 + \nu_z^2 - 1$$

where $G = \overline{\left[\int g(r,\theta)\, d\theta\right]^2}$

ν_x and ν_z are the numbers of horizontal and vertical betatron oscillations around the machine.

If σ_x and σ_z are held constant, the ratio ν_x/ν_z remains constant also since

$$\nu_x = \frac{N\sigma_x}{2\pi}, \quad \nu_z = \frac{N\sigma_z}{2\pi}.$$

Let

$$\nu_x/\nu_z = u.$$

$$2k = \nu_x^2(1-u^2) - 1$$

$$2G = \nu_x^2(1+u^2) - 1$$

therefore if $\nu_x^2 \gg 1$ and either $u^2 \gg 1$ or $u^2 \ll 1$,

$$k \propto \nu_x^2$$

So for strong focusing alone,

$$k \propto N^2 \sigma_x^2$$

$$\sigma_x \propto \sqrt{\frac{k}{N}}$$

In the case that tan θ is constant,

$$\tan\theta = \frac{S_{1o} - S_{1i}}{2(r_o - r_i)}$$

where S_1 is the arc length of equilibrium orbit in a radial focusing sector.

If tan θ and ρ are not constant, the same arguments may be made for each differential, dr.

Since we are holding r_o and $\Gamma(p)$ constant,

$$S_1 \propto \frac{1}{N},$$

$$\tan\theta \propto \frac{1}{N(r_o - r_i)},$$

$$T \propto \frac{1}{N}.$$

Thus,

$$\sigma_{edge} \propto \sqrt{T\tan\theta} \propto \frac{1}{N}\sqrt{\frac{1}{r_o - r_i}}.$$

The ratio

$$\frac{\sigma_x}{\sigma_{edge}} \propto \sqrt{k(r_o - r_i)}$$

If the same range of momenta are contained in different machines,

$$\frac{P_o}{P_i} = \text{constant},$$

$$\therefore k \ln\left(\frac{r_o}{r_i}\right) = \text{constant}.$$

In practical FFAG designs, $r_o - r_i$ is much less than r_o, and we may write $\frac{r_o}{r_i} = 1 + \Delta$ where $\Delta = \frac{r_o - r_i}{r_o}$

In this case

$k \ln(1+\Delta) \cong k\,\Delta$ for small Δ,

$k \ln(r_o/r_i) \cong k\left(\frac{r_o - r_i}{r_o}\right)$, and $k(r_o - r_i) \cong$ constant

Therefore under the above restrictions, σ_x/σ_{edge} is approximately constant. Similar arguments hold for σ_z/σ_{edge}. Some of the above approximations break down when k becomes small (less than about 10 or 20), however, then so do the small angle approximations used in the original formulation of Mark I and Mark II, and the entire system must be reformulated. Since a variety of forms of $\Gamma(p)$ were tried in specific calculations, it does not appear that in general Mark II offers any significant reduction in circumference factor over Mark I.

The above treatment and conclusions do not cover accelerators of the extreme Mark II type where reverse field sectors are deleted and all focusing in one dimension is accomplished by edges.

The author wishes to acknowledge the many helpful discussions with and suggestion of K. M. Terwilliger on the material included above and in MURA-LWJ-7.