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A FIXED FIELD ALTERNATING GRADIENT SYNCHROTRON WITH A SMALL CIRCUMFERENCE FACTOR*

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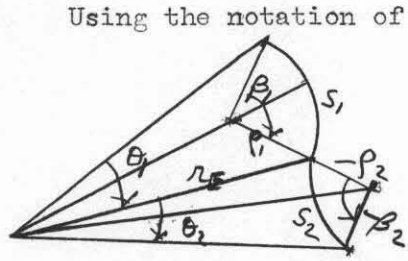
The Mark I FFAG accelerators[1] have a large circumference due to negative field magnets over 2/5 of the particle path length. The relatively long negative sectors were needed to give both radial and vertical stability with the same μ. However, since ψ (the parameter in the focussing expressions) is determined by $\sqrt{n}$ S, (n is the field index, S the path length) one might have shorter negative field sectors if the gradient in those sectors were sufficiently increased. A higher gradient in the negative magnets causes orbits of different radius to experience different absolute field strengths in successive sectors. For equilibrium orbits of all momenta to close, and for small radial and vertical oscillations about all equilibrium orbits to have the same wavelength (same σ_r and σ_z) it will be shown that the sector boundary may no longer be straight and radial from the center of the machine as in Mark I. Since the practical problem is to obtain as great a magnetic field as possible for the maximum energy particle, H_o is chosen to have the same absolute value in both focussing and defocussing sectors.

Using the notation of Reference 1:



$$\gamma = -\frac{\beta_1}{\beta_2}$$

$$\Gamma = \frac{\theta_1}{\theta_2} = \frac{S_1}{S_2}$$

$$S_1 = \beta_1 \rho_1 = r_E \theta_1$$

$$S_2 = \beta_2 \rho_2 = r_E \theta_2$$

$$\beta_1 + \beta_2 = 2\pi/N = S_1/\rho_1 + S_2/\rho_2$$

$$\psi_1 = \frac{\sqrt{1-n_1}\, S_1}{\rho_1} \qquad \psi_2 = \frac{\sqrt{n_2}\, S_2}{\rho_2} \qquad -\frac{\frac{\partial H_2}{\partial r}}{\frac{\partial H_1}{dr}} = \zeta$$

$$\therefore \frac{S_1 H_1 + S_2 H_2}{H\rho} = \frac{2\pi}{N} \qquad 1.$$

$$S_1 + S_2 = \frac{2\pi r_E}{N} \qquad 2.$$

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1) K. R. Symon, MURA-6.

$$\cos \sigma_x = \cos \psi_1 \cosh \psi_2 + \frac{\rho - 1}{2\sqrt{\rho}} \sin \psi_1 \sinh \psi_2 \qquad 3.$$

$$\cos \sigma_z = \cos \psi_2 \cosh \psi_1 - \frac{\rho - 1}{2\sqrt{\rho}} \sin \psi_2 \sinh \psi_1 \qquad 4.$$

$$\sqrt{\rho} \; \frac{\psi_1}{\psi_2} = \Gamma$$

Equations 1., 2., 3. and 4. together with the definitions of ψ_1, ψ_2, ρ and Γ completely determine the problem. Since, to avoid resonances, σ_x and σ_z should be held constant, and the number of sectors N must be constant for any real machine, the above relationships relate S_1, S_2, H_1, H_2, and H_ρ as functions of r. For a machine to be designed from these relationships, one would choose a σ_x, σ_z, N, and a function $\Gamma(r)$. For the smallest synchrotron, H_1 at r_o and H_2, at r_o should be as great as possible, and, therefore, equal, and Γ at r_o should be large so that the circumference factor, C, will be small.

$$C = \frac{\Gamma(r_o) + 1}{\Gamma(r_o) - 1}$$

Illiac tables have been made up for values of $\psi_1, \psi_2, \rho, \frac{\psi_1}{\psi_2}, \sigma_x, \sigma_z$ for different values of ρ.[2] From the values of σ, Γ, H_1, H_2 and H_ρ at the maximum energy radius r_o, equations 1. and 2. and the Illiac tables may be used to compute ψ_1, ρ, etc., at a new radius Δr from r_o. The value of Γ and other parameters may then be found at this new radius and the tables used to find values at a point Δr beyond. The procedure can be repeated for new r_s until H_ρ has been worked down to the injection energy. Then $\psi_1(r)$, $\psi_2(r)$, $\rho_1(r)$ and $\rho_2(r)$ obtained from their point values may be used to arrive at $H_1(r)$, $H_2(r)$ and pole face contours.

This procedure has not yet been carried through in detail with the new tables. Symon has briefly looked at some solutions using the smooth approximations; and it appears that for $\Gamma(r_o)$ to be larger than 2 or 3, $\Gamma(r)$ is such a strong function that at the injection radius $\Gamma(r_i) \ll 1$. For magnet sectors several meters long and of only a meter radial extent

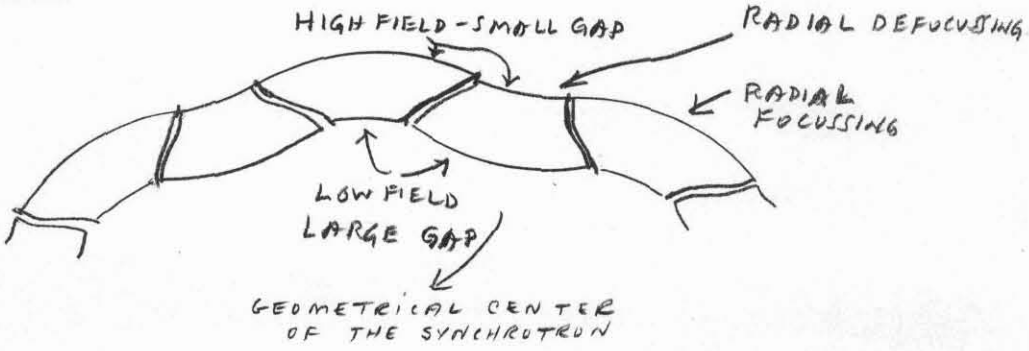
2) σ tables J. N. Snyder and D. Hutchinson.

this would mean sector boundaries almost tangent to the particle trajectories. Since an impulse approximation shows that the effective focal length of a magnet edge is given by

$$f = \frac{\rho}{\tan\theta}$$

where ρ is the radius of curvature of the particle in the magnetic field and θ is the angle between the particle path and the normal to the magnet boundary. f has one sign for radial focussing and the opposite sign for vertical focussing. A Mark II design with positive momentum compaction factor, α, and large $\Gamma(r_o)$ has strong radial focussing and quite weak vertical focussing. The direction of taper of the magnets (Γ increasing with r) is such as to increase radial focussing and decrease vertical focussing, and the edge effect on the structures looked at hurriedly was about as great as the effects of the alternating gradients themselves.

Mark II machines may be designed with good vertical focussing and negative momentum compaction as in Mark I, but here the fringe fields strengthen vertical focussing and weaken radial focussing.

It is important to design several trial machines using the Illiac tables and the methods above describes and the matrix for the edge effects before more concrete statements are made about the gain in circumference factor over a Mark I.

A sketch of a portion of a possible Mark II machine with positive α is given below: