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"PATCH" OPERATION OF A FIXED FIELD ALTERNATING GRADIENT ACCELERATOR, MARK III, WITH COMMENTS ON MOMENTUM COMPACTION AND AMPLITUDES OF OSCILLATION*

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Introduction

Symon[1] has described an accelerator (referred to as Mark I) utilizing D.C. magnets with alternately directed fields. The primary disadvantage of such a design is the large increase in circumference over conventional A.G. synchrotrons. It is suggested here that the circumference of such a machine might be reduced by operating not in the conventional region of stability (the so-called "necktie") but in the first stable island "or patch" of the ψ_1 ψ_2 plane where the ratio of sector lengths is greater and the circumference is less. This is referred to as Mark III. The immediate criticism of this design is the large amplitudes of vertical or radial oscillation associated with an orbit of given initial slope. These amplitudes are computed and compared with those for other designs.

Stability Limits and Orbit Circumference

In FFAG accelerators of the Symon Mark Ib type[2], the magnetic field has the same radial dependence but opposite direction in successive magnet sectors, and the particle path lengths are different in successive sectors. For Mark Ib:

$$\frac{S_1}{S_2} = \Gamma = \frac{\psi_1}{\psi_2}$$

$$\psi_1 = \sqrt{n_1}\,\theta_1 = \sqrt{\left|\frac{H_1'}{H\rho}\right|}\,S_1$$

$$\psi_2 = \sqrt{n_2}\,\theta_2 = \sqrt{\left|\frac{H_2'}{H\rho}\right|}\,S_2$$

since $H_1' = H_2' = \frac{dH}{dr}$

$S_1 = \rho_1\,\theta_1$

$S_2 = -\rho_2\,\theta_2$

$$n = \left|\frac{\rho}{H}\,\frac{dH}{dr}\right|$$

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1) K. R. Symon, MURA-KRS 6

2) Ibid.

S is the particle path length in one sector,

ρ is the particle radius of curvature,

H' is the magnetic field gradient at the orbit in the median plane.

For this special case, it has been shown that

$$\cos\sigma_r = \cos\psi_1 \cosh\psi_2 \quad , \quad \cos\sigma_z = \cos\psi_2 \cosh\psi_1 .$$

Radial and vertical oscillations are stable for real values of σ_r and σ_z. The limits of stability are then, in the ψ_1 ψ_2 plane:

$$\cos\psi_1 \cosh\psi_2 = 1 \qquad \cos\psi_2 \cosh\psi_1 = 1$$

$$\cos\psi_1 \cosh\psi_2 = -1 \qquad \cos\psi_2 \cosh\psi = -1$$

These regions are shown in Figure 1.

Stability is also achieved for values of $\psi > \pi$ for certain values, as long as the above are satisfied.

The circumference of an accelerator is given by $N(S_1 + S_2)$ while the circumference of a circular orbit in the same field is $2\pi\rho$. Symon has shown that, for machines of the type discussed here, the circumference is larger than $2\pi\rho$ by the "circumference factor,"

$$C \quad \frac{N(S_1 + S_2)}{2\pi\rho} = \frac{\Gamma_+}{\Gamma_-} = \frac{\psi_1 + \psi_2}{\psi_1 - \psi_2} \quad (\text{where } H_1' = |H_2'|).$$

The smallest circumference is achieved for the largest ratio of ψ_s. Within the usual region of stability, the most favorable circumference is obtained in the region where

$$\psi_1 \cong 2.0 , \quad \psi_2 \cong 1.33 , \quad \frac{\psi_1}{\psi_2} = \frac{3}{2} , \quad C = 5.$$

At the center of the first "Patch,"

$$\psi_1 = 3\pi/2 , \quad \psi_2 = \pi/2 , \quad \frac{\psi_1}{\psi_2} = 3 , \quad C = 2$$

or $\psi_2 = 3\pi/2 , \quad \psi_1 = \pi/2 \quad \frac{\psi_2}{\psi_1} = 3 , \quad C = 2 .$

<u>Momentum Compaction</u>

The momentum compaction factor, α, is given, for Mark Ib, by:[3]

$$\alpha = \frac{\Delta L}{L} \Big/ \frac{\Delta p}{p} \cong \frac{1}{m\,C}$$

3) K. R. Symon, MURA-KRS 6

In general, it is assumed that α is positive and higher energy orbits lie at a larger radius. In Mark Ib designs, $\alpha \to 0$ as $C \to \infty$, or as $S_1 \to S_2$. However, the horizontal and vertical oscillation conditions remain valid. If the radial focussing sector is made shorter than the radial defocussing sector, the orbit curves around so that the high field region is on the smaller radius. α then becomes negative. In such a machine the radial focussing is weak, the vertical focussing is strong, and there will be no transition energy. Since vertical aperture is bounded by the magnet pole spacing, it is probably desirable to use stronger vertical focussing, hence negative α. This is especially true for Mark III, and the patch in Figure 1 is the one for $\psi_2 > \psi_2$ and negative α.

Amplitude of Oscillations

The more favorable circumference factor makes a "patch" design seem desirable, but large values of ψ_1 lead to very large vertical oscillations. From Courant and Snyder,[4] the maximum amplitude of oscillation resulting from a given angle, θ, of trajectory crossing the equilibrium orbit is given by:

$$A = \beta\,\theta,$$
$$k\beta^2 = \frac{\cosh\psi \sin\psi + \sinh\psi}{\cosh\psi \sin\psi - \sinh\psi}$$

in the special case where $k = |M_1| = |M_2|$, $\psi_1 = \psi_2 = \psi$.

For accelerators of the Symon Mark Ib type and the present Mark III patch type, $k = |M_1| = |M_2|$, but $\psi_1 \neq \psi_2$.

Amplitudes of A and Z oscillation are then given as follows:

$$A = \beta\,\theta \qquad k\beta^2 = \frac{\cosh\psi_2 \sin\psi_1 + \sinh\psi_2}{\cosh\psi_1 \sin\psi_2 - \sinh\psi_1}$$

Momentum Compaction

$$A_z = \beta_z\,\theta_z \qquad k\beta_z^2 =$$

4) EDC/HSS 1, pp. 15 and 20.

For purposes of comparison, Table I gives values of $\sqrt{k\beta^2}$ for different choices of ψ_1 and ψ_2. Since the amplitude is produced by injection errors, gas scattering and misalignment, the radical $\sqrt{k\beta^2}$ should serve as a reasonable index of the required aperture for machines of the same m.

TABLE I

ψ_2	ψ_1	$k\beta_z^2$	$k\beta_r^2$	$\sqrt{k\beta_z^2}$	$\sqrt{k\beta_r^2}$	
$\pi/2$	$\pi/2$	23	23	5	5	conventional A.G. synchrotron
2	1.33	45	220	7	15	Mark Ib
$3\pi/2$	$\pi/2$	30*	~11,000	5.5	105	Mark III

*here maximum amplitude results from passing through the center of a defocussing sector at angle θ. See MAC/LWJ 1.

It is seen that the amplitude of radial oscillation in a Mark III design is 21 times that in a conventional design, and 7 times that in a Mark I design. From Figure 1 it is also seen that on the "patch" all of the radial resonances are crowded together, so that to avoid a resonance the tolerance on ψ_1 is very much more severe than in conventional designs. The limits of radial stability on the patch differ by only 2.4% in ψ_2.

Conclusions

These two considerations, radial resonance and radial oscillations, seem to rule out Mark III as an acceptable accelerator design.

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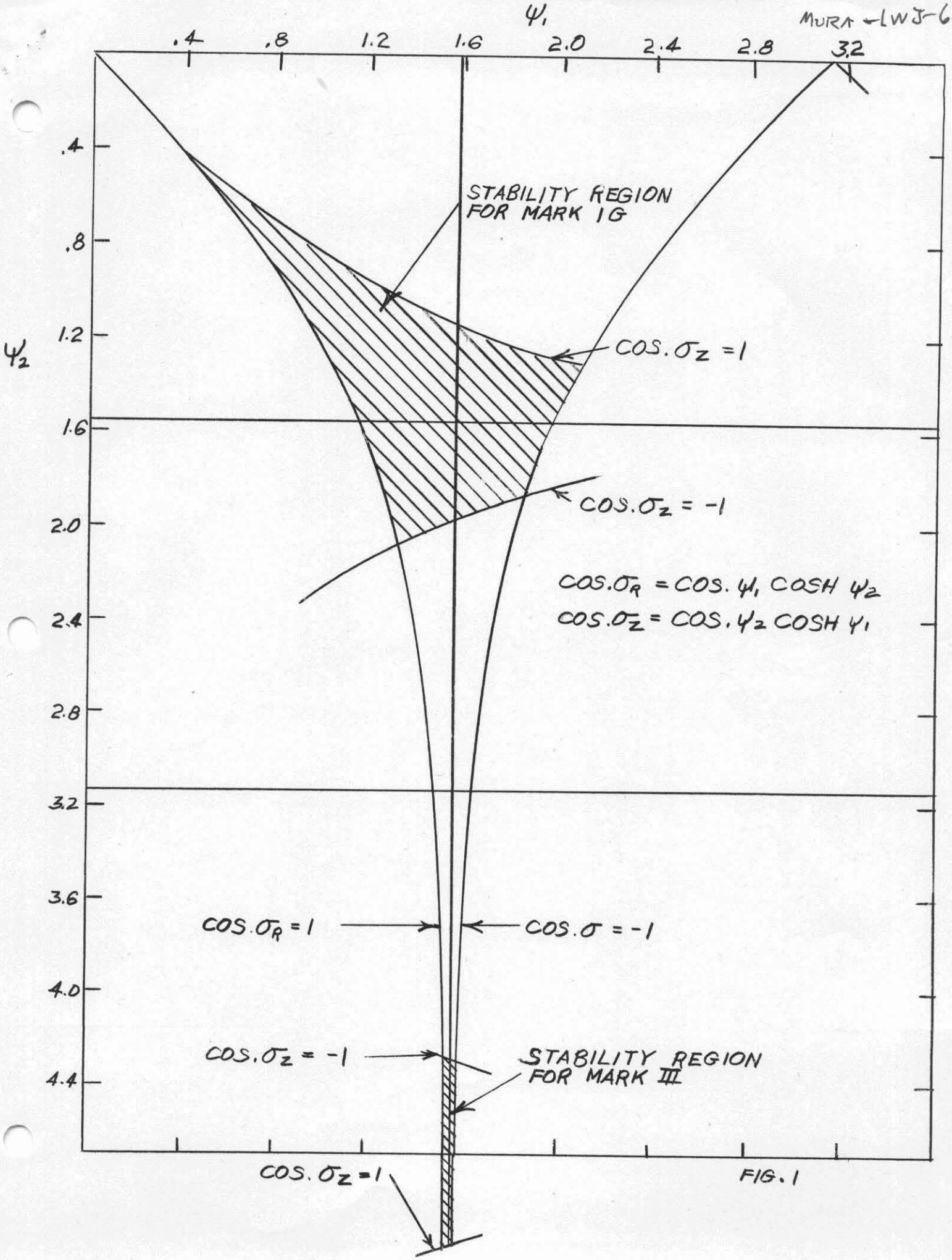

FIG. 1