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Report for the National Science Foundation on the Studies of the Midwestern Universities Research Association

The efforts of the Midwest Study Group during the time of its support by National Science Foundation funds has been divided into three main parts. First, experimental work; second, computational work; and third, theoretical work which is closely connected with the computational effort.

The experimental work has consisted of three groups at Iowa State College, Ames, Iowa. One group has been examining Hall effect magnetic field gradient detectors for use in magnetic model tests and for examination of the final magnet. Dr. Danielson, with the help of Mr. Redin, has found what the hard problems will be with this method. By the Hall method a gradient measurement is a measurement of the difference between two large quantities. There are errors which influence the two large quantities differently and which hence show up in the difference or gradient. There are ideas to be tried to correct some of the difficulties encountered, but the Hall method for gradient measurement is turning out more subject to difficulties than it was hoped.

Another Ames group, under Dr. Legvold, has been investigating the problem of the dependence of remnant field on the time cycling of idealized geometrical structures of iron.

The third Ames group, with Zaffarano, von Tersch and Brown, have made a model of a mechanically frequency modulated

R.F. cavity and they have finished testing its range of frequency and have found that more than the minimum requirement of three on frequency can be attained. The purpose of such a cavity is to provide possible simplification of the electronic problem by using just one high Q accelerating cavity instead of many low Q cavities around the accelerator. The work has been very encouraging so far. The tests for next fall will be to make a self-excited cavity and then to see if practical mechanical modulation can be achieved. Simplification is a goal worth seeking as accelerators become more complicated and more difficult to maintain. A completely electronic analogue for strong focussing is also under development by Doty of Ames.

Other experimental work was done at the University of Minnesota. The design of a linear accelerator adapted to injection has been worked on by Professor J. H. Williams and Dr. L. Johnston. The construction of a 70-Mev linear accelerator, which is well along at Minnesota, affords experience which can be applied directly to the injector linac problem. The development of strong focussing lenses to be mounted within the drift tubes is a development being carried out which is especially valuable to injectors.

An iron uniformity study was contemplated for the spring semester of 1954 at Illinois, but there was not time to start this after funds became available. However, the magnetic detection instrumentation and magnet modeling which Haxby at Purdue (now affiliated with MURA) is planning to do in our next year's program will get us into the iron testing business.

The computational work carried out by Professor Powell and Mr. Wright of the University of Wisconsin with the help of Professor J. N. Snyder and his staff of coders at the University of Illinois digital computer (Illiac), has demonstrated to an unanticipated extent how the numerical calculations and their display can strongly influence the development of the general theory of orbits. Observing the orbits described by the high speed computer is almost like having a versatile accelerator on which to test orbits.

The existence of invariant curves in the phase plane for general non-linear forces has been demonstrated by the Illiac but no mathematical proof has yet been found for our case with alternating gradients. However, assuming that these invariants exist, it has been possible to proceed with further development of orbit theory based on these invariants. One such example is Symon's treatment of adiabatic variation of parameters in our summer reports.

The systematic survey of the phase plane started by Powell near the end of 1953 is now complete for the case of the homogeneous differential equation with alternating cubic and for one dimension. During the spring and summer the inhomogeneous differential equation was examined on the digital computer in several ways. The change of amplitude of the steady state solution during secular or adiabatic changes of the parameters was observed for the purpose of learning what saturation of the iron and consequently what variation of the parameters in the differential equation could be tolerated without the steady state solution growing too much. This inhomogeneous differential equation is equivalent to the study of a machine with displaced or misaligned magnet sectors. In addition, some similar tests were made for the case of misshapen pole phases, namely n errors.

One of the most striking developments has been the discovery that it is possible to use algebraic transformations which are not too complicated to go from the initial conditions in one sector of the magnet to the final conditions in the same sector of the magnet without having to use the digital computer to compute by way of the differential equation. This now allows Powell to calculate fifty times faster than we were able to calculate using a differential equation. His transformations give results which are already close to those obtained with the differential equation and with a certain amount of perfection the transformations are expected to give us all essential details of the motion. We are able to compute all the way around a 50-sector machine in one second, including inhomogeneities.

The phase plane at one azimuth in the machine with an inhomogeneity and consequently with multiple possible equilibrium orbits has been studied easily and rapidly with the transformations, whereas with the differential equation it took a tremendous amount of time because the number of points on the phase plane is equal to the number of revolutions around the machine, so a study of stable and unstable regions took a long time. This study is important because the connections of the regions of phase space with inhomogeneities in the machine must be understood, since some equilibrium orbits disappear as a result of saturation of the magnet or secular changes or space charge influences. Apparently no work of this scope has been done by other groups, but we now understand that Brookhaven is commencing it.

We have studied some special cases of locked-in motions which occur abundantly for non-linear restoring forces. While the phase space available for one particular brand of locked-in motion is small, it is not inconceivable that a machine can be scaled to fit such a motion, should these motions turn out to be particularly stable in the face of inhomogeneities and secular changes. We have had a demonstration of the stability of one such locked-in motion over a 20% secular change of n. We have also found that by adding a little stray strength to the entrance position of each focussing lens in an alternating gradient accelerator, one can create motions which lock-in vigorously for certain wavelengths in the machine which were previously quite neutral and relatively free from tight lock-in.

Some of the experiments with the digital computer were suggested by the Michigan electromagnetic analogue of a strong focussing accelerator. The behavior of locked-in motions during secular changes of the parameters was first found on the Michigan analogue and these analogue results suggested a branch of study for the Illiac. The Michigan analogue, built by Terwilliger and Jones, has been refined during the summer period at Madison and it will be transported back to Michigan again, where its use will be continued, particularly for the study of cases of mis-aligned magnets.

The analytical and theoretical work has concentrated on the examination of perturbation methods and the influence of space charge on the design of a machine. Clarification of the application of Liouville's theorem and study of the adiabatic properties of the motion have also been undertaken as indicated in the reports.

A rigorous, or as closely rigorous as possible, examination of the influence of a tilted radio-frequency accelerating gap or of the radio-frequency accelerating gap with a voltage gradient in the radial direction, has been undertaken. Hammermesh and Crosbie, of the Argonne Laboratory, have examined the case of an infinitesimally narrow radio-frequency gap and Norman Francis and Laslett have examined some of the small quantities which may give serious observable results. There is more to be done to clarify this problem.

The employment of the so-called smooth approximation developed by Symon for analytical treatment of alternating gradient machines with non-linear forces has shown that we have a powerful tool by which to consider the difficult two-dimensional problem. It has been applied by Symon and others in the group and to a certain extent it has been tested by the digital computer.

Theoretical physicists who have been engaged in these efforts are F. T. Cole of Iowa, N. C. Francis of Indiana, E. S. Akeley of Purdue, M. Hammermesh of the Argonne Laboratory, E. A. Crosbie of the Argonne Laboratory, K. R. Symon of Wayne University and J. L. Powell of the University of Wisconsin. Stimulating visits from Drs. E. D. Courant and H. S. Snyder of Brookhaven during the summer meeting contributed greatly to the developments.

It can be said that we now have at our command methods which we have developed for rapid calculation and study of restoring forces of a more general nature than linear restoring forces; and we are taking steps to apply these methods in an effort to exploit non-linear forces to the maximum possible advantage. In this regard we have several designs on which we are in a position to start an engineering study calling on computational and theoretical methods to test the design from time to time. The specific examples which we intend to examine are, first, an accelerator with a repetition rate quite a bit higher than the usual one pulse every five seconds. This accelerator would possibly use condensers instead of the usual flywheel and its high repetition rate will simplify the production of intense beams without encountering such serious injection limitations as the low repetition rate machines find.

As already mentioned, we will test non-linear field shapes for this machine which may have several equilibrium orbits contained within its aperture for stability in the presence of randomly spaced bumps, randomly spaced field shape errors and secular changes. By a high speed transformation method of calculation, we will test the influence of straight sections on a non-linear machine.

Another specific design which we will evaluate is that of the so-called Ubangi lip type conventional machine in which the tips of the gap saturate as the magnetic field rises and as the beam size shrinks, thus enabling the magnet to be made with a small region of high field and, in consequence, with small power supply. An engineering study of this and comparison with the strong focussing machine may well determine whether or not the conventional machine can be used economically in the region between 5 and 20 Bev. Another design which should receive engineering study is the very recent suggestion by Keith Symon for a direct current ring magnet strong focussing accelerator. This accelerator has tremendous momentum compaction but it has a large circumference because alternate short sections have reverse magnetic field. This new idea has prompted a study of an effort to get more momentum compaction into the standard alternating gradient machine. Since Symon's idea came up near the end of our summer period, we do not yet know all the difficulties which might be encountered. Proper recording of the dates of this invention has been carried out by Professor Symon, and this report should be considered notification to the National Science Foundation of this invention. If hopeless obstacles arise to the utility of this ring magnet DC machine, it will not be surprising, but the study of this radical structure has already given us more insight and suggested new possibilities.

We have sent L. J. Laslett of Ames to Brookhaven for the summer to work with their accelerator development group. He returned briefly to present a report to the MURA group assembled at Madison. We also sent Haxby of Purdue to Brookhaven to study magnetic measurement devices, since he will be working with that problem for us. We are sending S. C. Wright, of the University of Chicago, to Berkeley later in the year to work closely with the bevatron.

During the summer we had a two-week visit from E. D. Courant of Brookhaven and a one-week visit from H. S. Snyder of Brookhaven.

August 14, 1954 D. W. Kerst

List of Reports of the
Midwestern Universities Research Association

MAC-1 D. W. Kerst, "An Estimate of the Effects on Non-Linear Restoring Forces for Avoiding Resonances"

EDC-3)
MAC-2) E. D. Courant, "Linear Coupling Between Vertical and Horizontal Oscillations"

MAC-3 F. Cole and F. Rohrlich, "A Perturbation Treatment on Non-Linear Restoring Forces"

SCW-1) S. C. Wright, "Adiabatic Damping of Large Phase
MAC-4) Oscillations"

LWJ-LJL L. W. Jones and L. J. Laslett, "A Study of the Feasibility of a Multi-Bev Circular Electron Accelerator"

KMT-MAC-6 K. M. Terwilliger, "Magnet Apertures as a Function of n"

MAC-7 D. W. Kerst, "An Ion Pipe, An Extreme Form of A.G. Magnet"

MAC-8 Crane, "Cranial Motion"

MAC-DWK-3 D. W. Kerst, "Magnet Power Supply"

MAC-DWK-4 D. W. Kerst, "Approximate Calculation of Non-Linear Lock-In at $\sigma = \pi$ "

MAC-DWK-5 D. W. Kerst, "Characteristics of Non-Linear Lock-In Caused by Field Inhomogeneity"

LJL(MAC)-1 L. J. Laslett, "Estimate of Possible Spacial Variation of n"

LJL(MAC)-2 L. J. Laslett, " Discussion of Space-Charge Effects in the Alternating Gradient Synchrotron"

LJL(MAC)-3 L. J. Laslett, "Possible Instability From Momentum Errors in the Alternating Gradient Synchrotron"

MAC-LWJ-1 L. W. Jones, "Note on Amplitude of Betatron Oscillations"

MAC/LWJ-KMT-1 L. W. Jones and K. M. Terwilliger, "An Electro-Mechanical Analogue for the Study of Strong Focussing Synchrotron Orbits"

MAC-JLP-1 J. L. Powell, "Non-Linearities in in AG Synchrotron"

MAC-JLP-2 J. L. Powell, "Theory of Alternating Gradient Synchrotron - Note on Discontinuity in Field Gradient"

MAC-JLP-3 J. L. Powell, "Non-Linearities in Alternating Gradient Synchrotron"

KRS(MURA)-1 K. R. Symon, "A Smooth Approximation to the Alternating Gradient Orbit Equations"

KRS(MURA)-2 K. R. Symon, "Smooth Solution to One-Dimensional Alternating Gradient Orbits with Cubic Forces"

KRS(MURA)-3 K. R. Symon, "An Adiabatic Theorem for Motions Which Exhibit Invariant Phase Curves"

KRS(MURA)-4 K. R. Symon, "An Alternative Derivation of the Formulas for the Smooth Approximation"

RGS(MURA) R. G. Sachs, "On the Application of Very High Energy Machines"

R. O. Haxby, "Report on Magnetic Measurements at Brookhaven"

Additional reports from Argonne representatives at our summer conference.