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A Strong-Focussing Accelerator With A DC Ring Magnet
(Preliminary report for internal circulation only)

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It is proposed to design an accelerator using D.C. magnets around a roughly doughnut-shaped vacuum chamber, so designed that there exist within the doughnut stable equilibrium orbits at all energies between the injection energy and the final energy. The frequency of revolution will depend on the energy, and the particle will be accelerated by one or more r.f. gaps, the accelerating voltage frequency being modulated so as to carry the particles from the orbit corresponding to injection energy to the orbit corresponding to the final energy, as in a synchro-cyclotron.

In order to achieve the required orbit system, we propose to use the alternating gradient principle, arranging the magnets so that the mean magnetic field increases with radius (or perhaps decreases). The orbit radius will then increase (or decrease) with energy. In the radial focussing sectors, the field will have perhaps the form

$$H_z = H_0 \left(\frac{r}{r_0}\right)^n , \tag{1}$$

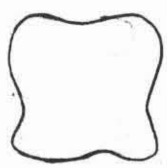
where r is the radius of the circular arc around which an ion moves in this sector. The n-value for a particle moving at radius r would then be

$$\frac{r}{H}\frac{dH}{dr} = n \tag{2}$$

so that n is independent of r if we use the field (1). In order to achieve an opposite gradient in the vertical focussing sectors, we use either a field

$$H_z = -\beta H_0 \left(\frac{r}{r_0}\right)^n \tag{3}$$

or else we use a field equal and opposite to (1) in a sector of length β times the length of the focussing sectors. The ratio of $\partial H_z/\partial r$ in radial and vertical focussing sectors will then be equal to β and should probably be independent of r if we want to stay at the same point in the necktie diagram. For stability, β cannot be smaller than about 2/3. The orbit will now be a series of concave and convex circular arcs. In the radial focussing sectors, the tangent to the orbit turns by an angle θ, and in the vertical focussing sectors, by an angle $-\beta\theta$, so that the number of (double) sectors is



$$N = \frac{2\pi}{(1-\beta)\theta} \tag{4}$$

If we use equal and opposite fields with half-sectors of different lengths, then the orbit length is

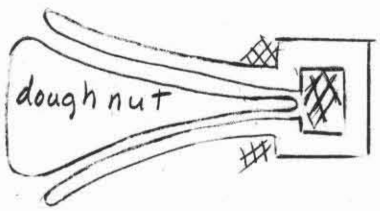
$$N(1+\beta)\theta r = 2\pi r \left(\frac{1+\beta}{1-\beta}\right) \tag{5}$$

(we assume no straight sections, but the analysis is not significantly complicated by straight sections.) The increase of orbit length over a circular machine is by a factor

$$\frac{1+\beta}{1-\beta} = 5, \tag{6}$$

if $\beta = 2/3$.

The magnets will have a shape something like this:



The doughnut shape takes advantage of adiabatic damping so that the space available for betatron oscillations is reduced at high energies. There will be a transition energy if $\beta < 1$, but the transition problem is less severe here, since the magnetic field is constant in time.