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Values of α Off the Diagonal of the Stability Diagram

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The question arose in recent discussions as to whether the transition energy could be greatly shifted or the momentum compaction improved by operating an alternating gradient synchrotron with an n_1 quite different from n_2. The parameter α is defined by

$$\frac{\Delta L}{L} = \alpha \frac{\Delta P}{P} ,$$

and the transition energy is given by

$$E_{TR} = \frac{E_0}{\sqrt{\alpha}} .$$

From the Argonne notes of Hammermesh,

$$\alpha = \frac{2-n_1+n_2}{2(1-n_1)(1+n_2)} + \frac{N}{\pi} \frac{(n_1+n_2)^2}{(n_1-1)(n_2+1)^{3/2}} \frac{1}{\left\{\left[(1+n_2)^{1/2} \coth \frac{(n_1-1)^{1/2}\pi}{2N} - (n_1-1)^{1/2} \cot \frac{(n_2+1)^{1/2}\pi}{2N}\right]\right\}}$$

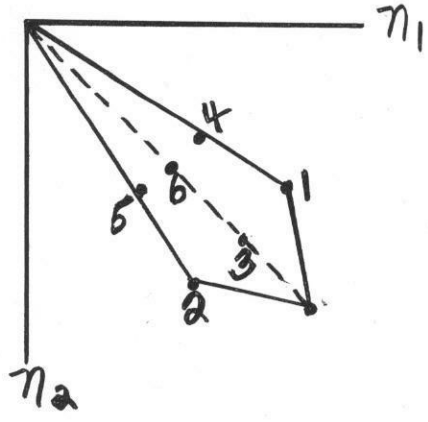
which can be simplified, for large n, to

$$\alpha = \frac{n_1-n_2}{2n_1 n_2} + \frac{N}{\pi} \frac{(n_1+n_2)^2}{(n_1 n_2)^{3/2}} \frac{1}{\left(n_2^{1/2} \coth \frac{\psi_1}{2} - n_1^{1/2} \cot \frac{\psi_2}{2}\right)}$$

An arbitrary large n machine was picked and α compared for six points on the "necktie." The results are shown in the table below, for N = 100.

	n_1	n_2	$\psi 1$	$\psi 2$	$\sigma 1$	$\sigma 2$	α
1	3360	2170	1.821	1.465	π	$\pi/25$	0.1
2	2170	3360	1.465	1.821	$\pi/25$	π	$.8\times10^{-4}$
3	2765	2765	1.650	1.650	$\sim\pi/1.8$	$\sim\pi/1.8$	17.5×10^{-4}
4	2120	1580	1.446	1.248	$\pi/2$	$\pi/25$	0.1
5	1580	2120	1.248	1.446	π	$\pi/2$	15.2×10^{-4}
6	1850	1850	1.350	1.350	$\sim\pi/3$	$\sim\pi/3$	26.2×10^{-4}

These points appear on a sketch of the necktie roughly as below.



Lines of constant α appear to parallel lines of constant radial focussing force, with α approaching infinity at the limit of radial stability. It appears that α changes more rapidly as the operating point is moved perpendicular to the diagonal of the stability diagram than along the diagonal. The general behavior of α for different n's and N's is probably as above, since in the general, simplified expression the second term is a term proportional to n/ψ average times the trigonometric terms.

Therefore, for the usually considered alternating gradient machine, some gain in momentum compaction and higher transition energy can be obtained by operation off the diagonal; however, the changes are not dramatic.

For example if the off-diagonal point 2 is used instead of the on-diagonal point 3 the momentum compaction; $\frac{1}{\alpha} = \frac{\Delta P/P}{\Delta L/L}$, is about doubled and the transition energy is raised by $\sqrt{2}$.