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PATHS OF PARTICLES IN CIRCULARLY SYMMETRIC FRINGING FIELDS

STÖRMER METHOD

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For a particle of mass m_0 and charge e, moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$

$$\frac{d\vec{p}}{dt} = \frac{e}{c}[\vec{v} \times \vec{B}]$$, where $\vec{p}$ is momentum

Denote $\frac{du}{ds}$ by u'

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1-\beta^2}}$$

$$\frac{d\vec{p}}{dt} = \vec{p}' \frac{ds}{dt}.$$

Since $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dt}$, $\vec{p}' = \frac{m_0}{\sqrt{1-\beta^2}} \frac{d}{ds}\left(\frac{d\vec{r}}{dt}\right)$

$$\text{so } \frac{d\vec{p}}{dt} = |v| \frac{m_0}{\sqrt{1-\beta^2}} \frac{d}{ds}\left(\frac{d\vec{r}}{dt}\right) = |v| \frac{m_0}{\sqrt{1-\beta^2}} \frac{d}{ds}\left(\vec{r}' |v|\right)$$

$$= |v|^2 \frac{m_0}{\sqrt{1-\beta^2}} \vec{r}''$$

$$\therefore \vec{r}'' = \frac{e}{c}[\vec{v} \times \vec{B}] \frac{\sqrt{1-\beta^2}}{|v|^2 m_0} = \frac{e}{c|p||v|}[\vec{v} \times \vec{B}]$$

$$= \frac{e}{c|p|}\left[\frac{\vec{v}}{|v|} \times \vec{B}\right] = \frac{e}{c|p|}[\vec{r}' \times \vec{B}]$$

$$\frac{d}{ds}[\vec{r} \times \vec{r}'] = \vec{r} \times \vec{r}'' = \vec{r} \times \left\{\frac{e}{c|p|}[\vec{r}' \times \vec{B}]\right\}$$

$$= \frac{e}{c|p|}\left\{\vec{r} \times [\vec{r}' \times \vec{B}]\right\}$$

$$= \frac{e}{c|p|}\left\{\vec{r}' [\vec{r} \cdot \vec{B}] - \vec{B}[\vec{r} \cdot \vec{r}']\right\}$$

IN THE MEDIAN PLANE, $\vec{r}\cdot\vec{B} = 0$

THUS, $$\left[\vec{r} \times \vec{r}\,'\right]_{r_1}^{r_2} = -\frac{e}{c|p|}\int_{r_1}^{r_2} \vec{B}\left[\vec{r}\cdot d\vec{r}\right]$$

Let $d_i = r_i \sin\theta$, where θ is the angle between the direction of the particle and the radial vector.

$$\therefore\ d_2 - d_1 = (r_2 - r_1)\sin\theta = -\frac{e}{|p|c}\int_{r_1}^{r_2} B r\, dr$$

To use the Störmer results, take a point where the position and direction of the particle are known. Call the distance from the center of symmetry of the field r_0. D_0 is the vector which has its origin at the center of symmetry of the field and is perpendicular to the velocity vector of the particle.

To find the position of the particle when its distance from the center is r_1, evaluate the integral from r_0 to r_1. D_1 can now be computed. Take a right angle with leg A having length D_1. Rotate it about the center until leg B passes through the position of the particle at r_0. Then plot the line segment from r_0 to r_1 along leg B. This line segment then approximates the path of the particle between r_0 and r_1. Repeating this process will carry the particle from r_1 to r_2, r_2 to r_3, etc.