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AN ELECTROMECHANICAL ANALOGUE FOR THE STUDY OF STRONG FOCUSING SYNCHROTRON BETATRON ORBITS

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Although exact analytical solutions to the strong focusing equations are possible[1] in the linear case, non-linear fields have been suggested[2] as a possible means of avoiding resonance problems. Although the equations of motion in non-linear fields are not readily soluble analytically, they are being studied on some of the large electronic digital computers. In addition to these computer programs, it was thought desirable to have a more flexible method of studying these problems. To this end the electromechanical analogue at Michigan has been built.

The analogue, following a suggestion by Crane,[3] is essentially an A.C. galvanometer, where the coil current is made proportional to angular displacement of the coil. The field in which the coil moves is produced by a field coil on a laminated iron yoke. The motion of the coil is given as follows for constant field:

$$\tau = BiA\cos\phi, \qquad i = K\phi$$

$$\ddot{\phi} + \omega^2\phi = 0 \quad \text{for small } \phi. \qquad \phi = a\sin\omega t + b\cos\omega t, \ B \text{ POS.}$$

$$\omega^2 = \frac{BKA}{I} \qquad \phi = c\,e^{\omega t} + d\,e^{-\omega t}, \ B \text{ NEG.}$$

Where

τ is the torque on the coil
ϕ is the angle between the plane of the coil and the field
i is the coil current in ampere turns
I is the coil moment of inertia
A is the coil area
B is the magnetic field.

By reversing the field with a period T, strong focusing motion is achieved, where ωT in the above corresponds to the parameter ψ of C. and S.[1] The data are analyzed by comparison with the corresponding sinusoidal oscillation.

The mechanical structure of the analogue is illustrated in figure 1. The rotor of a variable capacitor is attached to the coil shaft.

The circuit block diagram for the analogue is shown in figure 2. Coil current is made proportional to coil displacement through the top set of equipment. The communications receiver is tuned to a

frequency f_o corresponding to the oscillator frequency when the coil is at $\phi = 0$. Rotation of the armature detunes the oscillator producing a voltage signal from the F.M. detector proportional to $f - f_o$. This signal is amplified and applied to a pair of 6L6's which in turn control the coil current. The output of the D.C. amplifier is also applied to an oscilliscope for observation and photographic recording of the coil motion on moving film.

The field current is controlled by a bank of 6SN7 tubes. A pulser is used to trigger a standard scaler, and the square wave signal from the scaler is used to drive the 6SN7's to produce a square wave current in the field winding.

The versatility and reliability of the present form of the analogue are limited by three factors. First, the friction in the jewel bearings produces a constant damping force. Future plans include rebuilding of the mechanical structure of the analogue with a torsion suspension to minimize this friction. Second, due to the coil's inductance, a phase shift is introduced between the coil current and the derived voltage signal. If this current lags the voltage, a constant driving force results in the sine wave case. This phase shift may be balanced out by placing inductance also in the B+ leads to the 6L6's and then adjusting the resistors R_1 and R_2 in the circuit for any desired value of phase shift. As presently used, the phase shift driving is adjusted to balance frictional damping for a sine wave amplitude near the center of the range of interest. The third limitation at present is the maximum frequency of field reversal possible without greatly distorting the square wave field current pattern, due to the inductance of the field winding. For field reversal frequencies of 50-75 cps, the distortion is not serious, and as this proves to be a convenient frequency for other reasons, no change here is contemplated.

Non-linearities may be introduced into the armature feedback loop at the D.C. amplifier input (X on the block diagram of figure 2). However, as presently operated, the 6L6 V_g-i_p characteristics can be used to provide up to 10% cubic non-linearity by adjusting R_2 to a low value, and increasing the gain of the D.C. amplifier. With this, the analogue has been used to study solutions to the equation:

$$x'' + m(\theta)x - \alpha x^3 = 0 \quad , \text{ where } \frac{\alpha}{m} = \text{Constant}$$

and $\dfrac{\alpha x^2_{max.}}{m} = 5-10\%$

The amount of non-linearity present is determined by using a constant field and observing the frequencies of the cubic "sine" motion resulting from the equation of motion

$$\ddot{x} + \omega_0^2 x - \alpha x^3 = 0$$

McLachlan[4] gives a solution to this equation where the observed ω (= 2π f observed) is given as follows:

$$\omega^2 = \omega_0^2 - \frac{3}{4}\alpha x^2_{MAX}$$

Or, for small α,

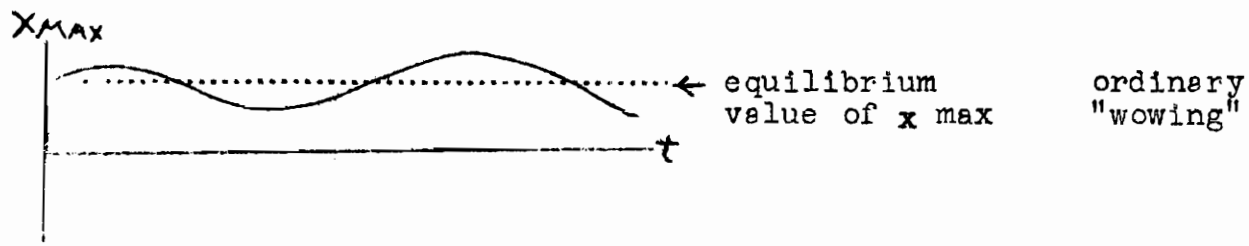
$$\omega \pm \omega_0 \left(1 - \frac{3}{8}\frac{\alpha}{\omega_0^2} X_{MAX}^2\right)$$

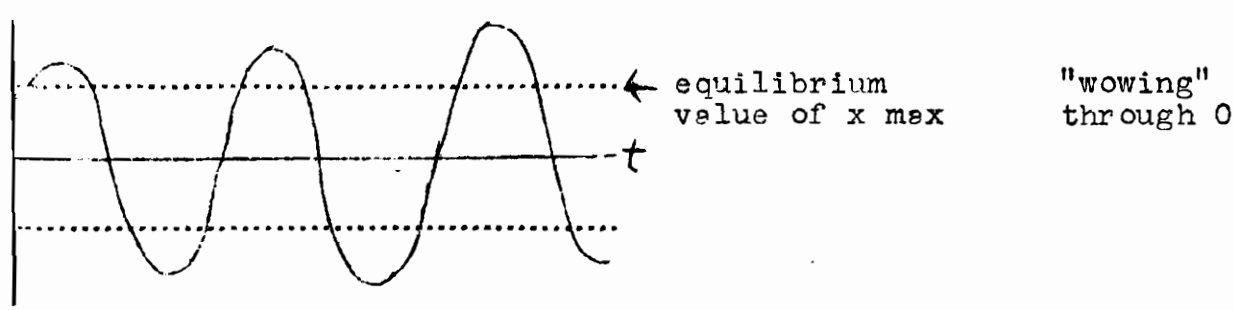
By finding the frequency of oscillation versus maximum amplitude, α/ω_0^2 may be found. This will be the same as $\alpha(\theta)/m(\theta)$ for the corresponding strong focusing motion.

Crane has suggested[2] that for certain modes of oscillation, the above cubic strong focusing equation might provide exceptional stability by "locking" onto a resonance. Here the equilibrium orbit would resemble an orbit at an integral resonance of the linear theory and discplacements from this would result in oscillations in phase and amplitude about this orbit. Courant[5] and Kerst[6] have observed such locked-in motions on the large digital computers, and Kerst[6] has analyzed the frequency with which such oscillations will occur about this equilibrium orbit.

These locked-in orbits are observed with the analogue; in particular, the orbit corresponding to the $\sigma = \pi$ resonance of the linear theory has been studied. An attempt has been made to find the range of pulser frequencies over which the locked-in motion is stable. Preliminary results indicate that the field alternation frequency may be varied through 5% before the amplitude of the locked-in equilibrium orbit grows above a value of X for which $\alpha X^2/n$ reaches 7.6%. Since $\omega \propto f \propto \sqrt{n}$, a 5% change in pulser frequency corresponds to a 10% change in n in a synchrotron. Due to the uncertainty of the effects of friction and phase shift, these results should be regarded as tentative.

Another problem which has been studied for this mode of motion is the amplitude of the "wow" about the equilibrium orbit. Orbits have been observed and photographed where the amplitude vs. time of the maximum goes through zero to the opposite phase, and yet the orbit does not "blow up." This appears on a photograph of the orbit as an apparent phase "slip" of 180° in the betatron motion. The effect can be illustrated in the following graphs:





Future plans for the analogue include, in addition to further study of the above problems, study of cubic motion with the opposite sign of non-linearity, and study of lock-in motion on other integral resonances.

References:

1. Courant and Snyder, Brookhaven internal report EDC/HSS 1. M. Hamermesh "The Alternating Gradient Synchrotron." (Midwest Accelerator Conference). G. Luders, various CERN reports.

2. H. R. Crane, MAC 8 and Memo to D. W. Kerst, Sept. 4, 1953.

3. H. R. Crane, Memo to D. W. Kerst, Sept. 4, 1953.

4. McLachlan, N. W., "Ordinary Non-Linear Differential Equations in Engineering and Physical Sciences." Oxford. Clarendon Press 1950.

5. E. D. Courant, Private communication to D. W. Kerst, Nov. 12, 1953.

6. D. W. Kerst, MAC/DWK. 4.

ELECTROMECHANICAL ANALOGUE

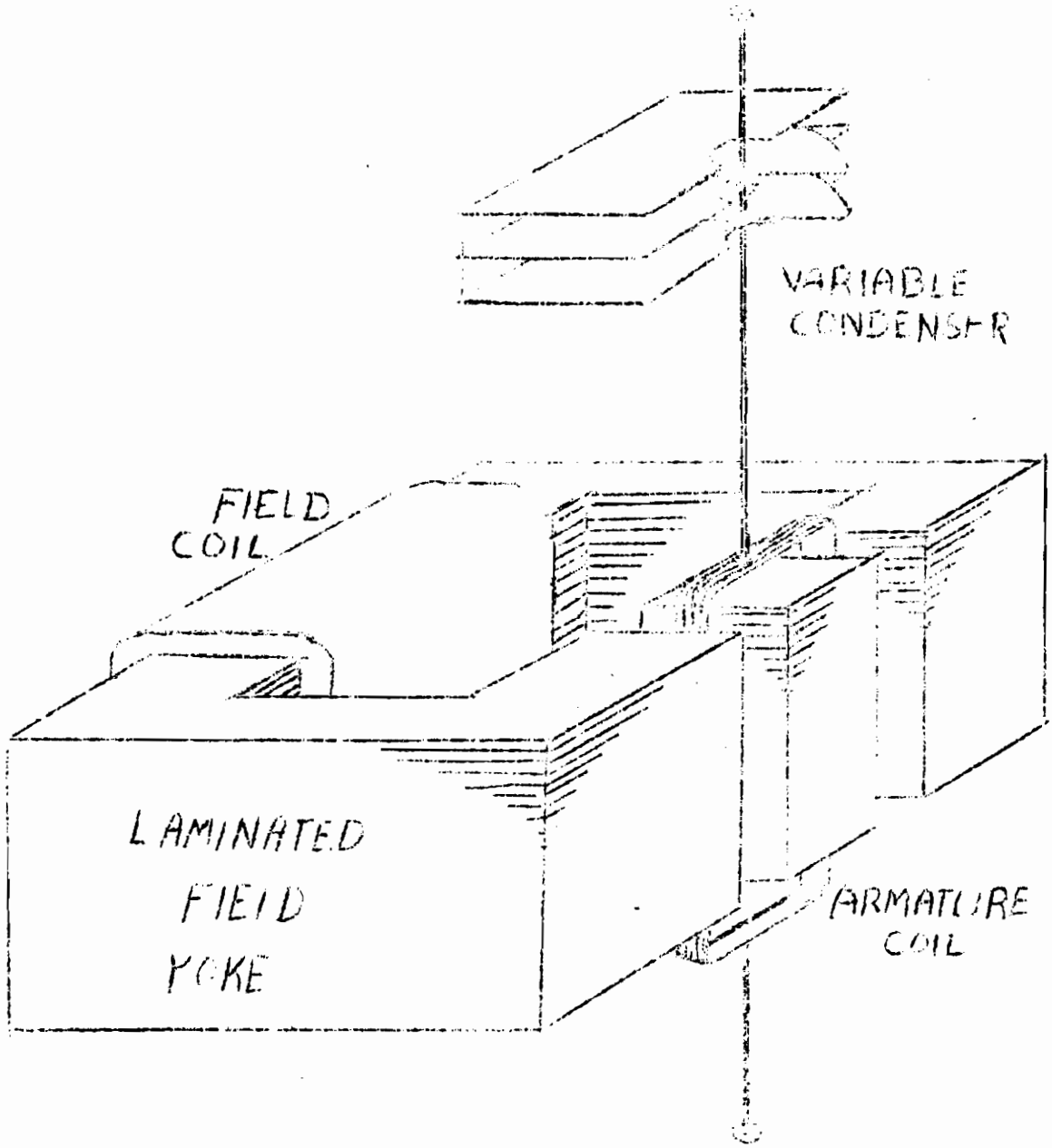


FIGURE 1.

ANALOGUE BLOCK DIAGRAM

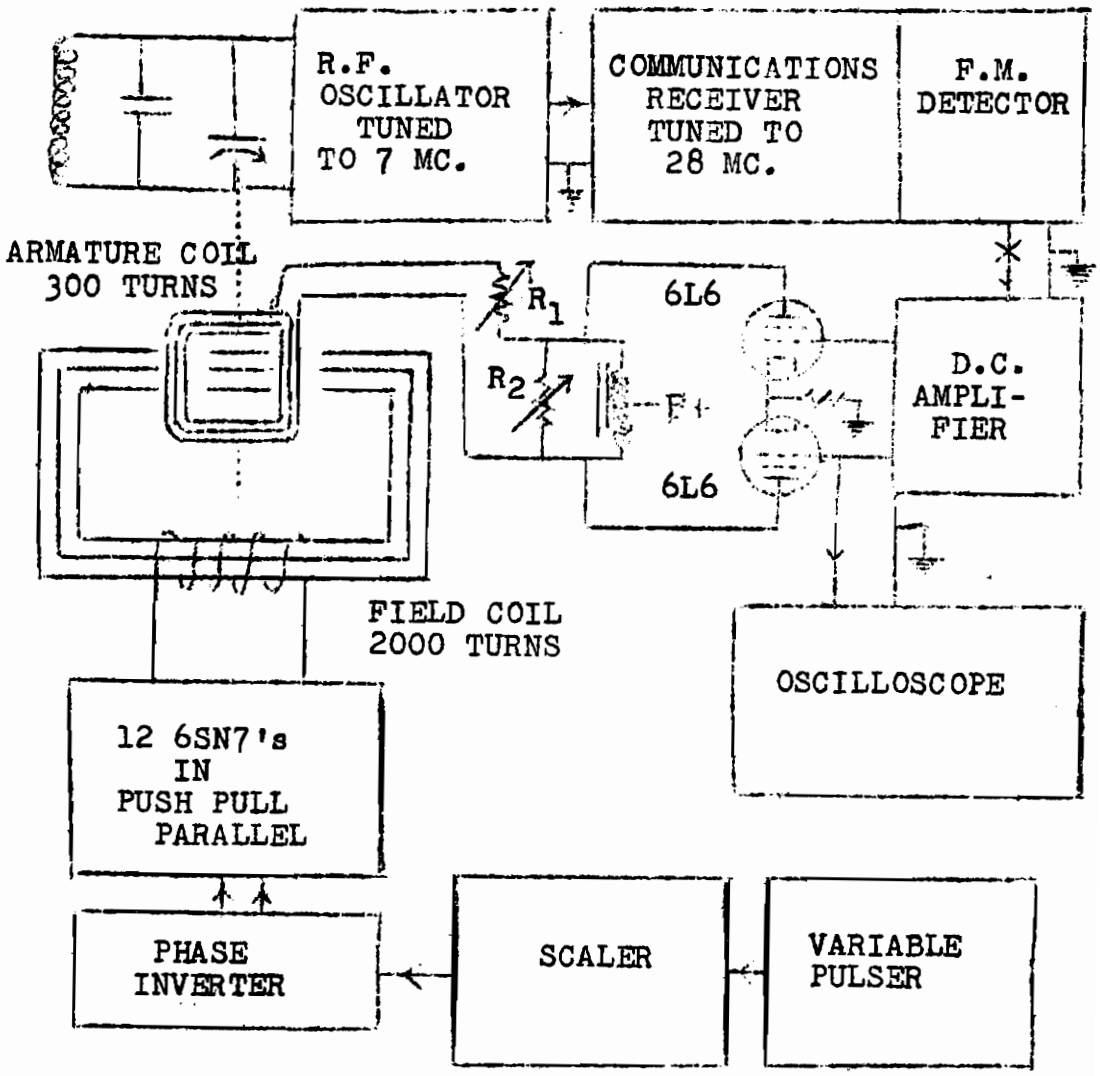


FIGURE 2