#9

MAC-8

CRANIAC MOTION

Excerpt from a letter from H. R. Crane to Midwest Accelerator Conference, Madison, Wisconsin, August 18, 1953

"...Upon thinking about the consequences of having n vary across the orbit, it seems to me that we have the possibility of a new kind of stability, or focussing, which we might make interesting use of. Consider first the situation of resonance, where we have an oscillation which repeats on each revolution.

Imagine a purturbing force at a fixed position which, if acting at phase A' will damp the oscillation, and if acting at A" will drive the oscillation to higher amplitude. Let us say it is at phase A'. Now let us assume (and this is something to be looked into) that magnet can be designed such that large amplitude oscillations will have a slightly higher frequency than low amplitude ones. If so, then the damping of the oscillation by A' will increase the wavelength of the oscillation, and cause the phase to shift toward A". The oscillation will then be built up again, shortening the wavelength and shifting it back toward A', and so on. Thus, we have an oscillation stable as to phase and amplitude <u>at a resonance</u>. If an accidental purturbation appears at some other point, the phase will merely shift so as to compensate -- within limits, of course.

"Several interesting consequences follow, all provided we have the magnet properties (anharmonic motion) mentioned.

1. The presence of any purturbing field or misalignment will "pull" the motion into resonance if it is anywhere near resonance to start with. Therefore, it may not only be undesirable, but practically impossible, to work between resonances, if they are closely spaced.

2. We need no longer have all phases present in the orbits, but can have a single "standing wave" orbit (same path for all particles on all revolutions). Incidentally, calculations become simpler.

3. Without any added purturbation, there may be strong lock-ins at all the resonances.

4. If we should go the whole way and <u>use</u> resonances, the king of them all should be considered, namely that in which the particle goes alternately through the Z-focussing section when Δr is 0, and through the r-focussing section when Z=0.

5. If it should turn out that Maxwell's equations permit us to make the field only such that, say larger amplitude in Z increases the Z frequency and larger amplitude in r decreases the r frequency, stable phases will still exist for both. Also, the Z and r resonances need not be the same order, since the motions are nearly independent.

"All in all, we are now talking about a machine which is quite different from the one originally planned..."

MEMO

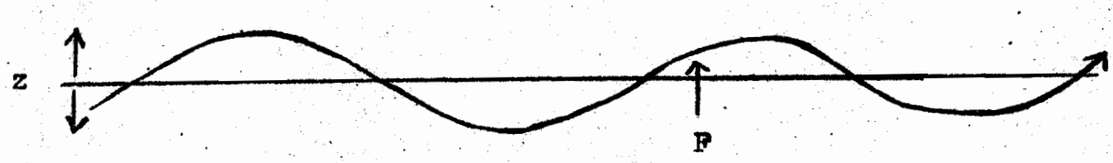
To: D. W. Kerst, Chairman, Midwest Accelerator Conference

From: H. R. Crane, University of Michigan - September 4, 1953.

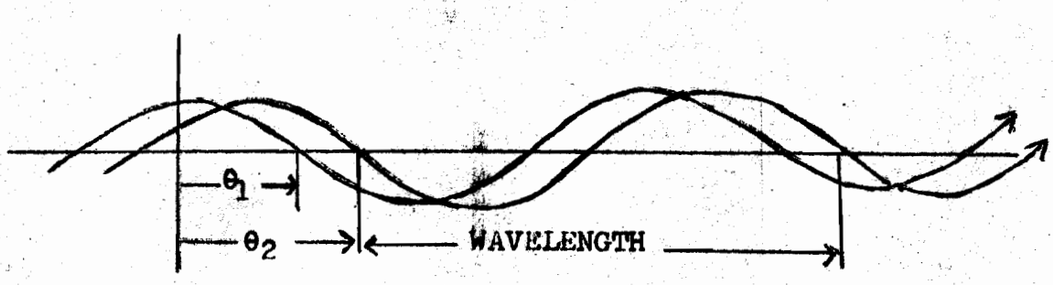
I.

The consideration of the effects of odd-power non-linearities in the restoring force in the alternating gradient synchrotron upon the motions of the particles brings to light a kind of orbit which is stably locked into resonance with either the regular alternations of the field gradient or with superimposed irregularities. Preliminary indications are that the lock-in tendency may in some cases be quite powerful. The question immediately arises, then, as to whether the stable lock-in property (which exists only in non-linear fields) possesses advantages and possibilities which put a new light upon resonances--perhaps making it desirable to use certain resonances as operating points, rather than to try to avoid them. Needless to say, an adequate answer to such a question seems quite a distance away, inasmuch as the theory of the non-linear A. G. machine is an order of magnitude more difficult than that of the linear one, and calculations are only in a beginning stage. Some of the more accessible consequences of lock-in were explored by the group at the Midwest Accelerator Conference at Madison, Wisconsin, August 21 to September 4, 1953. Following an introductory discussion of the lock-in phenomenon itself, this memo will give the pertinent points about it which came out at the conference. Finally, a suggestion for an electrical-mechanical analog suitable for the study of stability questions in the non-linear A. G. machine will be outlined.

The type of lock-in we wish to consider can best be introduced by describing a simplified, though somewhat unrealistic case. Consider a circular machine with a magnetic field which makes a particle move with an approximately sinusoidal z oscillation about the median plane. We shall consider only the z motion. Assume that the z oscillation is at resonance with the fundamental frequency of revolution, i.e., that on successive revolutions the particle goes over and over the same path. A piece of the path can be pictured as follows:

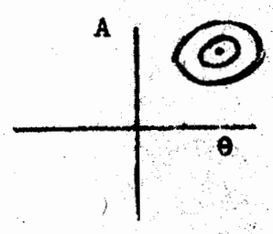


We will at no time in the discussion be interested in where the particle is at any instant. We will be interested only in the path that it follows. In view of this, we must be very clear as to what is meant by the three quantities we will be dealing with: amplitude, phase and wavelength. There is no ambiguity as to what is meant by amplitude, since the amplitude of the path is that of the particle. By phase we shall always mean the phase of the path, with respect to some fixed reference direction in the machine, and it will have nothing to do with where the particle may happen to be along the path. Paths having two different phases are sketched below. θ is the phase angle, which can be measured, say, from a fixed point in the donut to one of the crossings of the median plane ($z = 0$). Wavelength is self-evident, but is indicated on the sketch.



Kinds os motion will be discussed in which we allow the phase, θ, to drift to the left or right. With definitions out of the way let us proceed.

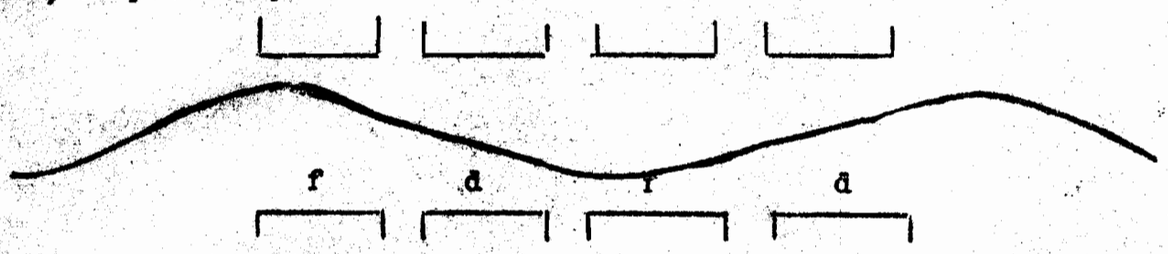
Assume that the field contains cubic terms of such a character that if the amplitude increases, the wavelength decreases. Assume, also, that there is present a force F at a fixed place (due to the magnetic field being slightly higher at a local spot, say) and that the phase of the path happens to be such that F drives the oscillation, increasing the amplitude slightly at each time around (see first diagram). The increase in amplitude results in a decrease in wavelength, so the phase begins to drift, moving the sine wave path to the left. This eventually changes the phase far enough so the bump F damps the oscillation, instead of driving it. The whole process then reverses: amplitude starts to decrease, increasing wavelength, making the wave drift to the right, until the bump again begins to drive the oscillation. This hunting process continues and is stable. The amplitude and phase will, in general, go round and round one of a family of loops in a phase-amplitude diagram, as shown below.



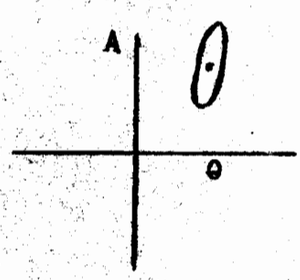
The dot inside the loops is the equilibrium orbit. It is interesting that the equilibrium orbit is one having finite amplitude of oscillation, and that an equilibrium orbit of zero amplitude as we are used to thinking of in the betatron or synchrotron, does not exist here, in the presence of the perturbing force F. The point of greatest interest in this example concerns the behaviour of the orbit when, in addition to F, an accidental bump (misalignment in the magnet, for example) is introduced. The operating point merely takes up motion around a different closed loop in the A-θ diagram, which has within it a point representing a new equilibrium orbit. In other words the motion is stable, within limits, against the appearance of accidental bumps or built-in errors in the field. This is in marked contrast to the behaviour of a particle moving at resonance in a machine with linear restoring force. In the latter case any bump whatever leads to unlimited growth of amplitude.

While the foregoing discussion has served to illustrate a principle, it has been unrealistic in that it does not deal with a set of conditions which could be extended to three dimensions. A smooth circular field which would make the orbit repeat in z on every revolution as we postulated would, according to the simple betatron conditions, be defocussing in r, or vice-versa. We therefore proceed to look for the possibility of similar lock-in in the A. G. field, where we know that resonant orbits which follow over the same path on every revolution are possible in both r and z, separately or simultaneously.

Consider the resonant orbit in the sketch below, in which f and d stand for focusing and defocusing sections of the A. G. field, respectively.



The orbit goes over the same path on every revolution. If the phase is shifted slightly, the amplitude will change, and in the case drawn, a drift of the wave to the left will increase the amplitude. If, now, the restoring force is non-linear, so that an increase in amplitude results in an increase in wavelength and a consequent drift to the right, the motion will be stable and will hunt around an equilibrium point, as indicated in the amplitude-phase diagram below.



Both the r and z motions may be locked in as indicated, but of course the presence of non-linearities in the restoring force brings in cross terms connecting the r and z motions. How serious the cross terms would complicate matters is an open question and depends, for one thing, upon how much non-linearity would be necessary to insure lock-in.

What has just been said about lock-in at resonance in the A. G. field applies to a field without errors or bumps. The big question now is: If we introduce a bump in the field, will the operating point merely take up motion around a new closed loop in the A-θ diagram, or will the amplitude blow up? In other words does the locked-in motion gain us anything in terms of stability of the machine against built-in errors or other kinds of errors in the field, or are the same old bad consequences of resonance still present without any real compensating advantages?

The question just asked can be answered with little more than a guess. It appears at first sight (to this writer) that there is stability against field bumps, in principle; but that from the quantitative standpoint the situation does not look very promising. In the motion described the amplitude will surely be a steep function of the phase angle. This will mean that the lock-in will be "stiff" and will permit the phase to hunt back and forth over only a narrow range. Suppose there is a fixed bump in the field, at such a location that it drives the oscillation to greater amplitude. The phase will then start to drift, but if the range of drift permitted is narrow, as we have reason to believe it will be, the drift might not be able to proceed far enough to put the bump into the damping phase. This is the pessimistic view. On the other hand, if there is reason to believe that the motion is stable against bumps in principle, one can hope to devise a field, or choose a resonance condition that will be more advantageous than the one we have just discussed.

A special consequence of locked-in motion is the opportunity of using a feed-back, or servo, loop for stabilizing the amplitude of oscillation. The fact that in locked-in motion all of the particles are moving over the same path (within the rather narrow limits of phase hunting) makes it possible (a) to continuously ascertain the phase and amplitude, and (b) to keep them under control through a variable magnetic bump. A minimum of two sensing electrodes or pickups would be necessary to measure the phase and amplitude. The information could be processed by a circuit, which probably would not have to be very complex, and the output signals (control currents) would have to go to at least two correcting coils in the magnet (again because two variables would be involved). It should be emphasized again that servo control of the oscillation amplitude becomes a possibility in principle only because of the phase bunching which has been postulated to exist in the machine with non-linear restoring force. A servo controlled orbit would have the following advantages:

(1) The strongest of all resonances could be used, namely the one for which there are two focusing and two defocusing sections per wavelength. This is the one which would allow the largest range of injection angles for a given donut size, that is, it is the one which has the strongest focusing.

(2) Since the effects of misalignment and of field errors and bumps would be "servoed out" it should be possible to relax the tolerance requirements.

The most obvious disadvantage of servo control, aside from just the additional complexity of the apparatus required, is that it requires a certain minimum intensity of beam in order to work, and it might therefore present a problem of starting.

The foregoing discussion has necessarily been qualitative only, because we know so little about the complex effects of non-linear restoring forces in the A. G. machine. It is hoped however that, if nothing more, the discussion has pointed up the fact that the introduction of non-linearity greatly widens the variety of machines and modes of operation that are possible, along with making the calculations an order of magnitude tougher. In the linear restoring force machine, resonance is synonymous with catastrophy. Non-linearity first came under consideration because it offered a hope of dodging the resonance trouble--a rise in amplitude at resonance would change the frequency of oscillation and detune the motion off resonance. Now it appears that resonance may not be entirely a bad thing. At any rate resonance, in conjunction with non-linearity and the possibility of servo control opens a new and complex array of possibilities for machines, which merits further exploration.

II.

In the course of the discussions at Madison a number of points were brought out by various members in regard to the kinds of motion just outlined. Brief mention of the pertinent ones will be made here, although more details will probably be given in memos by other members of the conference.

(1) A point was made about the stability against "n-bumps" and "B-bumps"--the former being errors in n at local places in the magnet, and the latter being local errors in B. The one would have a symmetrical effect on the two sides of the donut space, and the other not. It was pointed out that if an odd number of focusing and an odd number of defocusing sectors were used, the motion would repeat only after two revolutions, and on alternate revolutions the displacements would be reversed in sign. This would average out the B, but not the n, bumps. The mirroring of the displacements on alternate revolutions would not affect the phase-amplitude hunting described earlier.

(2) The optical model consisting of a series of alternately converging and diverging thin lenses with aberration (the latter to simulate the non-linear restoring force) was used for some rough computations, both graphical and analytical. A ray having an initial angle to the axis was found, as expected, to cross and recross the axis of the lens system. Furthermore, the indications were that, in the presence of aberration, the crossing point drifted or hunted backward and forward, relative to the coordinates of the lenses. Further work will be necessary, but the lock-in at resonance seemed to be substantiated with this model.

(3) The point came out that the frequency of hunting is only required to be smaller than the frequency of passing through the A. G. sections, and can, therefore, be either higher or lower than the fundamental frequency of rotation around the machine. Some practical considerations point toward a frequency lower than the fundamental as being better, particularly with servo control.

(4) In view of the fact that the particle paths would be bunched into a band resembling a screw, the question was asked why the donut should not be made in screw shape to fit it--or better yet, to impose a magnetic field such as to straighten out the screw. No answers were forthcoming.

(5) The question of secular changes in n was raised. This would change the wavelength of the oscillation, making it jump out of one lock-in, and perhaps fall into another. The loss of beam in such a jump, if the jump is possible at all, is difficult to guess. (A memo on the effects of secular changes in n has been circulated by D. W. Kerst.)

(6) There was some discussion of the possibility of lock-in at resonances other than the one specified earlier in this memo. It appears reasonable to suppose that there will be a lock-in tendency at every resonance, but that the strength of the lock-in will vary greatly.

(7) Courant pointed out that there is an interesting special solution for the non-linear A. G. field, giving a path which lies entirely on one side or the other of the center of the donut, and not crossing and recrossing, as is the usual case. It is stable, and hunts in phase and amplitude.

III.

Analytical solutions for the A. G. machine with non-linear restoring force being as difficult as they are, there is an obvious need for an analog computer, by means of which experiments pertaining to stability can be tried quickly and easily. A motor with a fly-wheel has many of the properties required. The torque exerted by a d.c. motor is, qualitatively proportional to the product of the currents in the armature and stator. The current in the armature could be made to conform to the desired non-linear function of the angular displacement. This could be accomplished in a variety of ways, but one would be to have a light beam reflected from a mirror on the fly wheel fall on a photo-cell, then amplify the photo-cell current, and feed it back to the armature, perhaps with an adjustable non-linear electrical circuit in series. The introduction on any desired non-linearity is simple, when we are dealing with an electric current. The stator of the motor would have a constant current which would be reversed periodically, or on any desired time schedule, to represent the passage of the particle through alternately focusing and defocusing sectors of the A. G. machine. It could also be cut off at intervals to represent the passage through the field free

sectors (straight sections). This would be externally programmed. It would also be simple to program in a bump or error in the restoring force occurring only once per revolution. This would be an analog computer which, for two dimensional motion in the machine, would contain all the essential elements. For three dimensional motion in the machine--r and z oscillations together, with cross coupling effects, two such analog computers could be used, with electrical interconnections: armature #1 to stator #2 and vice-versa. The currents in the two stators would of course be reversed, etc., simultaneously. Mechanically, the analog computers would not look much like ordinary motors with fly wheels. They would have to be made with as small friction as practicable, and the amplitude of rotation would probably be small, particularly if a light beam were used for translating deflection into current. It seems to the writer that such a device might be of considerable value in answering the many questions which arise in connection with the non-linear restoring force machine.