April 21st 1993 MITH 93/10

MITH 93-10

Fermilab Library
0 1160 0029007 4

Comment on " Savvidy ' Ferromagnetic vacuum ' in three-dimensional lattice gauge theory "

Luca Gamberale[a,b], Giuliano Preparata[a,b] and She-Sheng Xue[a]

a) INFN, section of Milan, Via Celoria 16, Milan Italy

b) Physics Department, University of Milan, Italy

Abstract

We make some comments on the recent article " Savvidy ' ferromagnetic vacuum ' in three-dimensional lattice gauge theory " by H. D. Trottier and R. M. Woloshyn, that appeared in Physical Review Letters 70 (1993) 2953.

PACS code: 11.15Ha

In a recent letter [1] H. D. Trottier and R. M. Woloshyn present the results of a Monte Carlo simulation of a three-dimensional SU(2) Euclidean lattice gauge theory in a constant abelian chromomagnetic background field. This problem, which was first consider by G. K. Savvidy [2], is of great interest for QCD for the crucial informations it may provide on the structure of the non-perturbative QCD ground state. By analyzing the behaviour of the effective potential as a function of the constant abelian chromomagnetic field H, the authors conclude that their calculation for the first time provides evidence for the "ferromagnetic nature" of the ground state of a non-abelian gauge theory, as first suggested by Savvidy's work [2].

We wish to point out the incorrectness of the above conclusions, which originate from a faulty assessment of the extant literature, in particular of Ref.[7] of their article. Indeed it has been shown [3] that the "unstable Landau modes", whose neglect by Savvidy Nielsen and Olesen [4] were the first to point out, are responsible for a striking phenomenon of complete "diamagnetism" that occurs in non-abelian gauge theories, by which the "classical" contribution to the effective potential, $\frac{1}{2}H^2$, is completely cancelled by a non-perturbative contribution originating from such "unstable modes". We believe that this phenomenon, which is completely general [5], is clearly indicated in the calculation that Trottier and Woloshyn report in their letter [1]. Indeed, by carrying out a calculation à la Savvidy, i.e. computing the continuum theory one-loop effective potential neglecting the "unstable Landau modes", one can easily obtain ($E(H)$ denotes the effective potential as a function of the abelian chromomagnetic field)

$$\Delta E(H) = E(H) - E(0) = \frac{1}{2}H^2 + k(gH)^{\frac{3}{2}}, \tag{1}$$

where $k = -3.8 \cdot 10^{-2}$. If the effect of the neglect of the "unstable Landau modes" were negligible, as assumed by Savvidy, for large $\beta = \frac{4}{g^2 a}$ and small $\frac{H}{g^3}$ one would predict the continuous curve reported in Fig. 1, which one clearly sees to be inconsistent with the MC data [6]. The dramatic reduction of the classical contribution $\frac{1}{2}H^2$ in the $SU(2)$ case is, as remarked by the authors, completely at variance with what happens for $U(1)$, where of course there exist no "unstable Landau modes". On the other hand due to the general

phenomenon of complete screening [5], inclusion of these modes (in 2 space-dimensions there are only two unstable Landau modes) modifies (1) as

$$\Delta E(H) = k'(gH)^{\frac{3}{2}}, \tag{2}$$

where $k' > k$ originates from what is left over from the **complete** cancellation of the classical term. The fit of the low $\frac{H}{g^3}$ points is reported in Fig. 2, yielding a good $\chi^2/dof = 0.47$ and a value $k' = (-2.4 \pm 0.2) \cdot 10^{-2}$, rather close to the one computed in the one-loop approximation.

In conclusion we would like to stress that, far from corroborating Savvidy's picture of the ground state of a non-abelian gauge theory, the calculation by H. D. Trottier and R. M. Woloshyn is a most relevant confirmation of the **complete** screening of the classical term, demonstrated in the works of Refs. [3] [5] [7].

References and Footnotes

[1] H. D. Trottier and R. M. Woloshyn, *Phys. Rev. Lett.* **70** (1993) 2053.

[2] G. K. Savvidy, *Phys. Lett.* **B71** (1977) 133.

[3] G. Preparata, *Nuovo Cim.* **A96** (1986) 366.

[4] N. K. Nielsen and P. Olesen, *Nucl. Phys.* **B134** (1978) 376.

[5] As discussed in G. Preparata, *The magnetic instability of the perturbative Yang-Mills vacuum*, in *Variational Calculations in Quantum Field Theories*, edited by L. Polley and D. E. L. Pottinger (World Scientific, Singapore, 1988), the general reason for the cancellation of the classical magnetic energy was discovered by the late R. P. Feynman, and is related to the peculiar fact that in a non-abelian gauge theory there are two gauge-inequivalent ways to build up a constant magnetic field, one through the curl of the vector potential and the other via the commutator of the vector potentials. Thus the non-perturbative contribution from the " unstable Landau modes " can be arranged to exactly cancel the classical abelian field, and in this way the effective potential gets automatically minimized.

[6] From the MC data one can clearly notice that for the β-values considered, one cannot compare them with the continuum theory for $\frac{H}{g^3} > 1$. Indeed in this region lattice artifacts are quite evident, as one can see from the remarkable dependence of the MC points on the lattice size.

[7] L. Gamberale, G. Preparata and S.-S. Xue, *Nuovo Cim.* **A105** (1992) 309.

Figure Captions

Fig. 1 Monte Carlo data taken from Fig.2 in [1] compared with the Savvidy prediction (solid line) eq.(1), and our prediction (dashed line) eq.(2).

Fig. 2 Fit of the data of Fig.4 in [1] with eq.(2). The best-fit gives $k' = -(2.4 \pm 0.2) \cdot 10^{-2}$ to be compared with the one-loop value $-3.8 \cdot 10^{-2}$. The data with $\frac{H}{g^3} > 1$ have been neglected (see Footnote [6]).

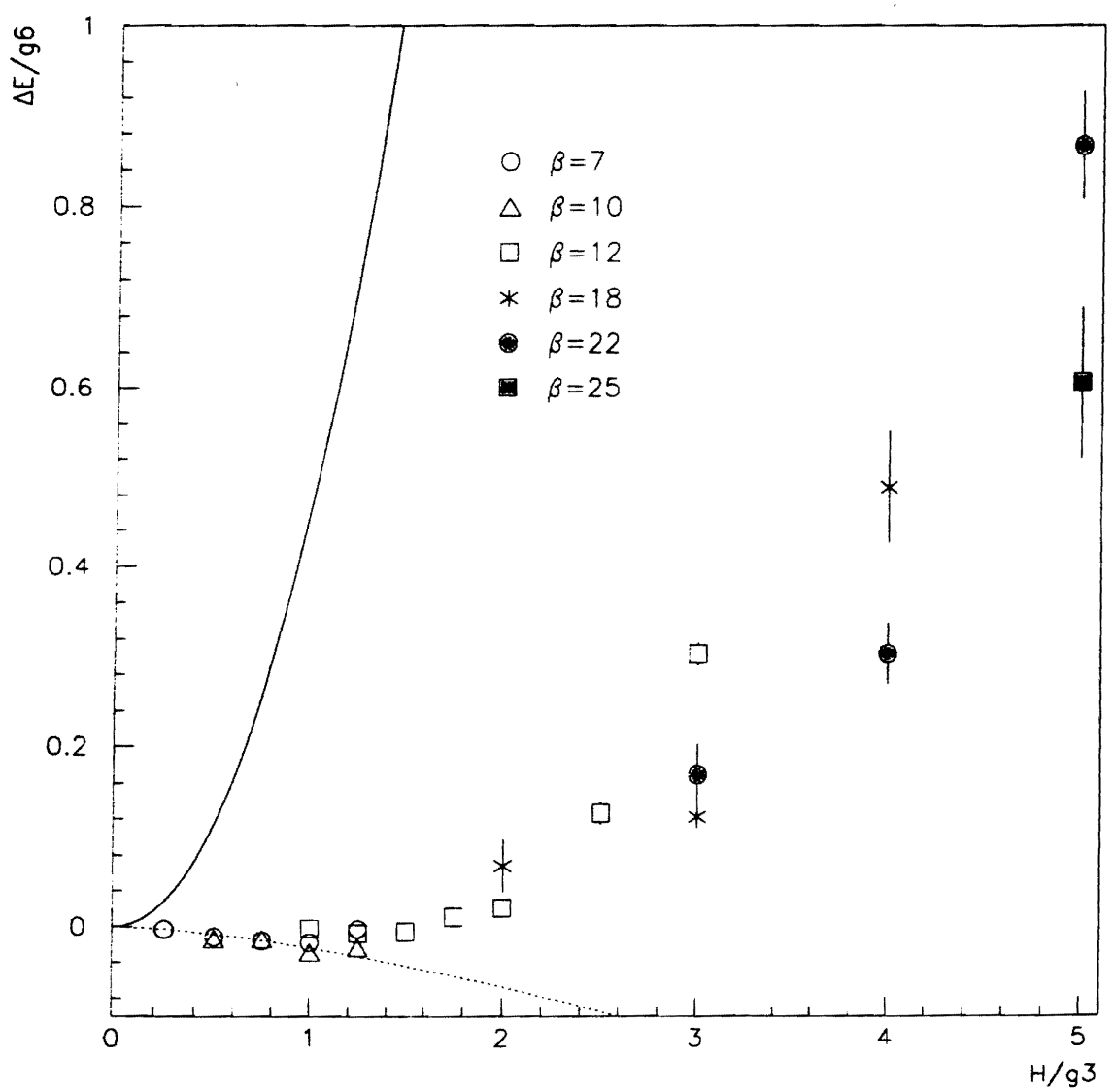

Fig. 1

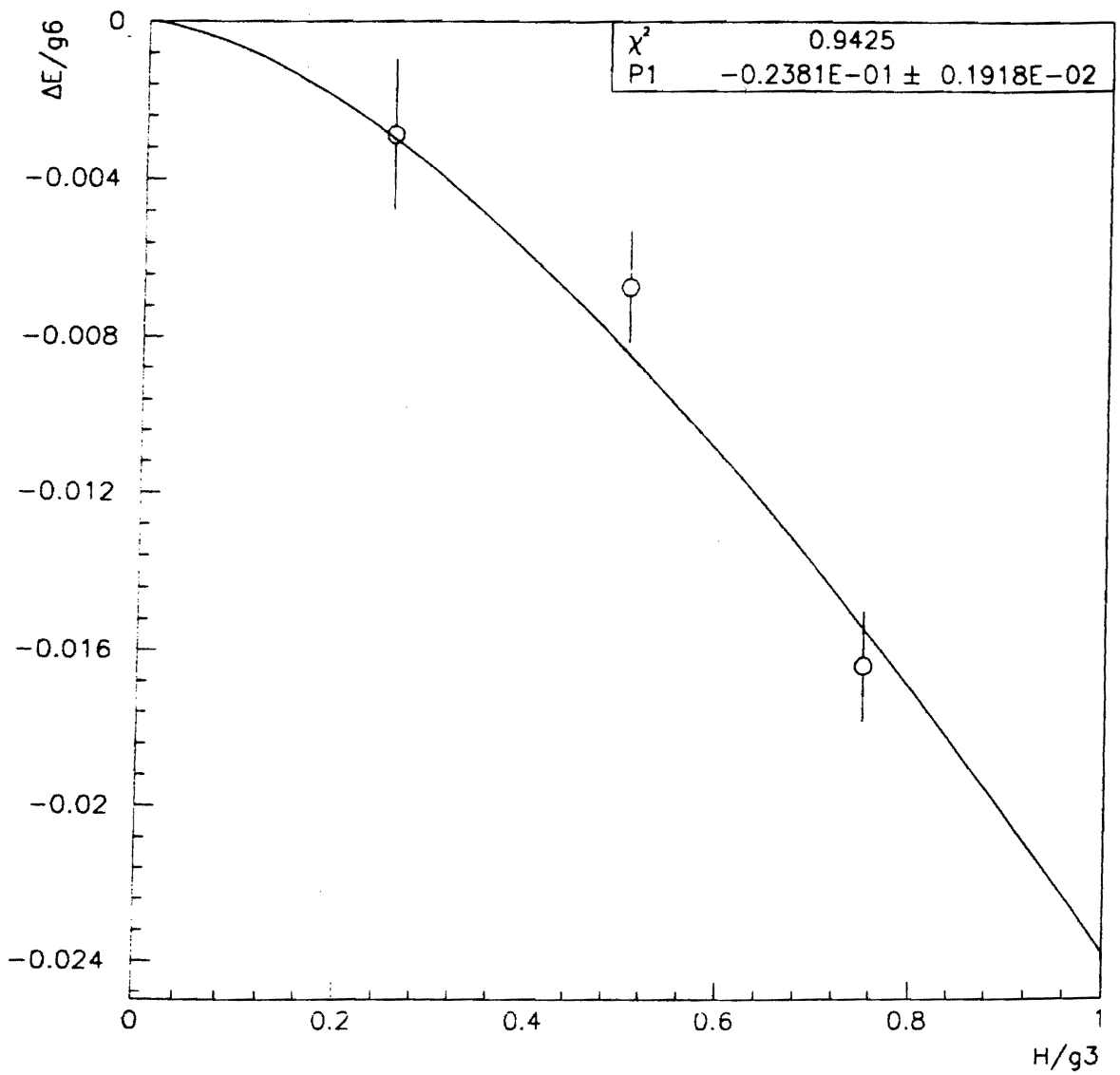
ΔE/g6
0
−0.004
−0.008
−0.012
−0.016
−0.02
−0.024
0
0.2
0.4
0.6
0.8
1
H/g3
χ²
0.9425
P1
−0.2381E−01 ± 0.1918E−02

Fig. 2