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Determining α_s from $R_{e^+e^-}$

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Abstract

Recent determinations of α_s from global, perturbative QCD, fits to the data on e^+e^- annihilation into hadrons, including the LEP data at the Z^0 pole, are discussed and shown to be mutually compatible despite apparent discrepancies. Indeed the difference in conclusions obtained by the various authors is traced to two elements: (*i*) the inclusion of an exceedingly large and erroneous third-order α_s correction to the e^+e^- hadronic cross-section and (*ii*) the use of variable normalisations to combine data from different experiments, which gives rise to a systematic shift in the data. We argue that such a procedure biases the outcome of the fit and thus conclude that all analyses are consistent with a strong interaction correction to $R_{e^+e^-}$ larger by almost a factor two than that predicted by perturbative QCD using other experimental determinations of α_s.

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Introduction

One of the simplest and most direct methods, both conceptually and in practice, for determining the strong coupling constant, α_S, is via the measurement of the e^+e^- annihilation cross-section into hadrons. However, there are in the literature apparently conflicting analyses [1–6] of the vast amount of data now available for this process. On the one hand the data have been interpreted as evidence for a strong-interaction correction rather larger than expected [1–3] and on the other it has been claimed that a good description is obtained from the perturbative QCD (PQCD) correction and that the extracted value of α_S is in agreement with other determinations [4–6]. In the following we shall attempt to elucidate this conflict by examining in detail the procedure adopted in ref. [4]. In particular we shall discuss the *rôle* of variable normalisations when combining data from different experiments and examine in detail the values obtained in ref. [4]. In addition, the results of the analysis are compared with recent LEP data in order to check the consistency of the high- and low-energy behaviour of the hadronic corrections.

Theoretical formalism

Since we shall be addressing the question of the *rôle* of strong interaction effects to the ratio $R_{e^+e^-} = \sigma(e^+e^- \to \text{hadrons})/\sigma_{\mu\mu}$ we assume the electro-weak corrections to be understood and lump them together in $R^{(0)}_{e^+e^-}$, the ratio of the e^+e^- hadronic to $\mu^+\mu^-$ cross sections in zeroth-order QCD. (For a discussion of E-W corrections the reader is referred to, *e.g.*, refs. [1–5].) Thus the PQCD corrected form for $R_{e^+e^-}$ is expressed in the $\overline{\text{MS}}$ scheme as, ref. [7]:

$$R_{e^+e^-}(s) = R^{(0)}_{e^+e^-}\left[1 + a(s) + 1.41a^2(s) - 12.8a^3(s) + \ldots\right], \tag{1}$$

where $R^{(0)}_{e^+e^-}$ contains all radiative corrections apart from those deriving from strong interactions and $a(s) = \alpha_S(s)/\pi$. Correspondingly, in third-order $\overline{\text{MS}}$ formalism, α_S is given by

$$\alpha_S(Q^2) = \frac{12}{(33-2f)\ell n(Q^2/\Lambda^2_{QCD})}\left[1 - \frac{6(153-19f)}{(33-2f)^2}\frac{\ell n\left(\ell n(Q^2/\Lambda^2_{QCD})\right)}{\ell n(Q^2/\Lambda^2_{QCD})} + \right.$$
$$\left. + \left(\frac{6(153-19f)}{(33-2f)^2}\frac{\ell n\left(\ell n(Q^2/\Lambda^2_{QCD})\right)}{\ell n(Q^2/\Lambda^2_{QCD})}\right)^2 + \right.$$

$$-\frac{\frac{(153-9f)^2}{3}\left(\ell n\left(\ell n(Q^2/\Lambda^2_{QCD})\right)+1\right)-\frac{(33-2f)}{4}\left(2587-\frac{5033}{18}f+\frac{325}{54}f^2\right)}{\pi(33-f)^3\ell n^2(Q^2/\Lambda^2_{QCD})}\Bigg], \qquad (2)$$

where f is the number of excited flavours; in our analysis $f = 5$. From deep-inelastic scattering processes Λ_{QCD} is typically constrained to lie in the range $\Lambda_{QCD} = 100-250\,\mathrm{MeV}$ [8] and thus the PQCD correction is expected to amount to about 4-5% at the energies of interest.

Radiative corrections

Fitting data from different experiments is always a delicate procedure. In the case at hand one potential difficulty for our analysis is the experimental inclusion of radiative corrections. Initial-state radiative corrections and propagator loop-corrections, depend on the various Standard Model parameters: Z^0, top-quark, Higgs boson masses *etc.*, which have varied with time and so too from experiment to experiment. From the analysis of refs. [3,5], to which the reader is referred for a more complete discussion, one sees that our neglect of such effects leads to an *under*-estimate on our part of α_S (see Table 1 of ref. [5]) since, with newer input values for the standard model parameters, the values for $R_{e^+e^-}$ obtained are invariably higher than those originally quoted by the experimental groups themselves. Indeed the value obtained in [5] for α_S is somewhat larger than, for example, that of [4].

In fact the specific results of [5] have not been taken into consideration here as the value extracted for the Z^0 mass was particularly low (89.3 GeV) and would have been even lower for a higher top mass than assumed there. Indeed we would make the common observation that the mass of the Z^0 as extracted experimentally (essentially from its *rôle* in controlling the electroweak contributions) is invariably *lower* than the value quoted by the four LEP experiments. This has the obvious effect of partially hiding any discrepancy, at least in the higher energy region.

While it is often claimed that the systematic errors involved are at the percent level, to quote the TOPAZ collaboration [9]: "*the total deviation* [is] *less than 1.5%*", this is true only over the full possible range of parameter values. However, the present LEP experiments have certainly pinned down the mass of the Z^0 and also provide limits on the top quark mass, thus neither of these can vary sufficiently to cause significant deviations from the corrections

adopted. Note moreover that the Higgs-mass dependence partially cancels that of the top mass as these masses are forced towards higher values by experimental limits. Therefore we feel safe in claiming that the effect of radiative corrections on our conclusions is likely to be negligible and indeed should only strengthen them.

Fitting procedure

The main problem in extracting a value of α_S (or more properly Λ_{QCD}) from a number of different experiments at differing energies is to account for the overall systematic normalisation errors quoted by each experiment. Now if one believes that the theory to hand provides a good description of the data and that the exercise is simply to extract a value of α_S, then the procedure (as adopted in [4]) is straight-forward: for each experiment one introduces an absolute normalisation parameter, N_i, and thus an extra term into the χ^2 in the following manner

$$\chi^2 = \sum_i \left[\sum_j \left(\frac{N_i R_{ij} - R_{th}}{N_i \sigma_{ij}} \right)^2 + \left(\frac{1 - N_i}{\sigma_i} \right)^2 \right], \tag{3}$$

where R_{ij} and σ_{ij} are the jth data value and error of the ith experiment and σ_i is the corresponding overall normalisation error.

It is not difficult to see that in the simpler case of a constant quantity the above procedure essentially reduces to taking the average value for each individual experiment with its statistical error, adding in quadrature the normalisation error and then averaging over all experiments.

On the other hand, if one wishes to uncover any possible disagreement between the theory and data an alternative approach is to parametrise and, by so doing, attempt to determine such a difference. In ref. [1] the proposal was that perhaps PQCD underestimates the radiative corrections and thus the strategy adopted was to parametrise this in terms of a single, constant parameter k, multiplying the entire PQCD radiative correction. This is a reasonable starting point since, in PQCD, $k = 1$ and therefore one initially wishes to check by how much it might differ from unity. A possible later refinement could then be to examine any energy dependence of such a parameter, should it be found to differ significantly

from unity. So the alternative procedure adopted in ref. [1] was to determine, experiment-by-experiment, the value of k and then to combine all experiments including their overall systematic errors.

Normalisations

It is self-evident that the normalisations of the first procedure and the k-factors of the second are related. Indeed for any given fixed energy there is a direct relationship, unfortunately, owing to the energy-scale dependence of α_S this relationship is not constant. At a given energy the relationship would be

$$N_i = \frac{1 + \delta_{PQCD}}{1 + k\,\delta_{PQCD}}, \tag{4}$$

where δ_{PQCD} is the second-order PQCD correction to $R_{e^+e^-}$. Thus in fact even the first procedure does permit the evaluation of possible differences between data and theory via examination of the normalisations obtained, which are of course treated as parameters extracted from the data. If one treats the normalisations simply as an alternative k-factor (*i.e.*, one which multiplies $R_{e^+e^-}$ overall) then one can use the results presented in [4] to make a direct evaluation of global normalisation factor.

Fit results

The result of the best fit reported in ref. [1] yielded

$$k = 2.0 \pm 0.3, \tag{5}$$

for $\Lambda_{QCD} = 150\,\mathrm{MeV}$ and gave $\chi^2/\mathrm{DoF} = 0.53$. This fit was performed using a certain amount of TRISTAN data [9] which was not available for [4]. Indeed the TRISTAN data has been considerably extended and improved and does in fact now give a somewhat lower average value. Including all the most recent TRISTAN data [10] and a recent MARK II determination [11] and including also the third-order expression eq. (1) we now obtain

$$k = 1.55 \pm 0.2, \tag{6}$$

with $\chi^2/\mathrm{DoF} = 0.5$ for a total of 109 data points. This corresponds to $\Lambda = 150^{+140}_{-80}$ MeV. Alternatively, to obtain $k = 1$ we should require $\Lambda_{QCD} = 1150^{+600}_{-400}$ MeV, an untenably large value. Rather similar results have also been obtained via a slightly different method [2,3].

In order to make comparison between refs. [1–4] possible we shall consider only the results coming from the data for which $E_{cm} > 14\,\mathrm{GeV}$. From table 2 of ref. [4] we obtain a global average normalisation factor

$$N_R = 0.981 \pm 0.005, \tag{7}$$

where we note that the extracted value of $\Lambda_{QCD} = 410 \pm 340\,\mathrm{MeV}$ is already rather larger than other evaluations. This is also true of the value obtained for $\sin^2\theta_W = 0.236 \pm 0.015$, which helps compensate the large R values at the higher energies. We should point out that account has not been taken of possible correlations between the different values, as the full covariance matrix was not presented. However, since there are strong correlations between the N_i and α_S, a better strategy would be to fix α_S whence also the N_i would decouple from each other without, we suspect, greatly altering the errors quoted. Taking into account the effect of the larger value of α_S extracted in ref. [4] we estimate that the k-factor equivalent to the normalisation N of eq. (7) is

$$k = 1.7 \pm 0.2, \tag{8}$$

which is perfectly compatible with the values quoted above. Even using all the data considered (*i.e.*, even down to very low center-of-mass energies) one obtains a very similar value.

Recently, in ref. [6], the analysis carried out by us in ref. [1], is claimed incorrect. The arguments presented are the absence of third-order α_S corrections (which we have now included) and an analysis [12] in which the k-factor is found to agree with PQCD, but this analysis comes before the LEP determination of Z^0 mass and $\sin\theta_W$; in fact a lower mass is used in the fit thus enhancing the electroweak correction, as previously discussed.

The LEP determination

In ref. [1] the results obtained from $R_{e^+e^-}$ are compared with the related quantity $\Gamma^{Z^0}_{had}$, the hadronic width of Z^0. We now show that the evidence of an hadronic correction larger than

expected is confirmed at LEP by analyzing:

$$R = \frac{\Gamma^{Z^0}_{had}}{\Gamma^{Z^0}_{\ell\ell}} = \frac{\sigma^h}{\sigma^l}, \tag{9}$$

where $\Gamma^{Z^0}_{had}$, $\Gamma^{Z^0}_{\ell\ell}$ ($\sigma^{h,l}$) are the hadronic and leptonic widths (cross-sections). This quantity has the important features of being luminosity independent and having the same strong-interaction correction as $R_{e^+e^-}$. The mean value obtained from the four LEP experiments is [13]

$$R_{exp} = 20.85 \pm 0.07, \tag{10}$$

while the Standard Model (SM) value (including PQCD correction) foreseen is

$$R_{PQCD} = R_{SM}(1 + \delta_{PQCD}) = 20.71 \pm 0.11, \tag{11}$$

with [14]

$$R_{SM} = 19.97 \pm 0.03 \quad \text{and} \quad \alpha_S(M_Z) = 0.11. \tag{12}$$

In terms of a k-factor, the hadronic correction is thus:

$$1 + k\,\delta_{PQCD} = \frac{R_{exp}}{R_{SM}} = 1.044 \pm 0.005, \tag{13}$$

which gives

$$k = 1.22 \pm 0.15. \tag{14}$$

Fixing the k-factor equal to unity, one requires a value of the strong coupling constant:

$$\alpha_S(M_Z) = 0.130 \pm 0.012. \tag{15}$$

Thus, as we have seen, the LEP results match the evidence manifested by $R_{e^+e^-}$ at lower energies.

In ref. [6] criticism is made of our conclusion that the hadronic decays of the Z^0 are not in agreement with PQCD predictions, but the argument presented is based on a value for the strong correction of

$$1 + k\,\delta_{PQCD} = 1.037, \tag{16}$$

which does not agree with the data.

We notice also that our observation is confirmed by a leptonic cross-section, σ_l, lower than expected, in fact:

$$\sigma_l = \frac{12\pi}{M_Z^2} \frac{1}{(3 + 3\Gamma_{\nu\nu}^{Z^0}/\Gamma_{\ell\ell}^{Z^0} + R)^2}, \tag{17}$$

and since the experimental value of $\Gamma_{\nu\nu}^{Z^0}/\Gamma_{\ell\ell}^{Z^0}$ is in good agreement with the expected number of light neutrinos, we see that an increase in R corresponds to a lower value of σ_l, in fact

$$\sigma_l(R_{PQCD}) = 2.004 \pm 0.015\,\text{nb} \quad \text{and} \quad [15] \quad \sigma_l^{exp} = 1.974 \pm 0.013\,\text{nb}. \tag{18}$$

Conclusions

We have considered two rather different strategies for fitting the data on $R_{e^+e^-}$: the first [1–3] specifically designed to bring to light any disagreement between the data and the predictions of the theory, the second [4] aimed at extracting in the most accurate manner the parameters of a theory tacitly assumed to be correct.

In both approaches it is seen that of utmost importance in obtaining a good fit to the data is the possibility of allowing a change in normalisation. However, the way in which this is achieved is conceptually different and leads to an apparent difference in the final outcome. The attitude adopted in ref. [1] was to take the experimental values and associated errors at face value and to ascribe the serious discrepancy ($\sim 3\sigma$) to a shortcoming of the theoretical framework via a phenomenological k-factor. In ref. [4] it was merely noted that it "*...gives an indication of possible residual systematic effects in the analysis.*", indeed the conclusion there was that "*...we now have a good description of the R continuum ...*". Let us remark that, where available [11,16], the estimated systematic shifts due to missing higher-order QED corrections would go only a very small way to eliminating the difficulty we have elucidated and that in general the discrepancy has been compensated by the use of superceded SM parameter values in calculating the electroweak effects.

In conclusion then we wish to underline the consistency with which entirely independent and conceptually very different analyses require (implicitly or otherwise) a relative renormalisation between theory and data in $R_{e^+e^-}$. Such a large renormalisation ($\sim 3\%$) has yet to be explained within the standard theoretical framework although we would call the reader's attention to a recent calculation [17] in which the non-trivial nature of the QCD vacuum is explicitly taken into account and which leads to $k = 1.9$ at the one-loop level.

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