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THE POMERON AND ODDERON TO ORDER α_s^3†

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ABSTRACT

We calculate the pomeron and odderon contributions to hadronic scattering to order α_s^3. We show that the structure of the hadronic form factors provides a natural mechanism through which the odderon gets suppressed at $t=0$ whereas it dominates the elastic cross section at large t. We also demonstrate that the inclusion of nonperturbative effects through a modification of the gluon propagator accelerates greatly the convergence of the log$s$ expansion, although not enough to provide agreement with the data.

It is by now a well-known fact that leading-log s resummation[1] cannot account for diffractive scattering. Taken at face value, the result of such resummation suffers from the following drawbacks:

- Its leading contribution to the hadronic amplitude goes like $s^{1+2.65\alpha_s}$, *i.e.* for any reasonable value of α_s, it grows much faster than the data, which behave[2] like $s^{1.08}$.
- At nonzero t, the differential elastic cross section has the wrong shape: its logarithmic slope at $t = 0$ is infinite and its curvature is too big.

Hence, although the perturbative answer is infrared finite, it does not seem possible to use it to describe the very-small-t region. As high-t data are very scarce, and as the small-x region is only now being probed at HERA, it is not known whether the perturbative resummation techniques apply anywhere. One is thus led to the semantic distinction of a "soft pomeron", which describes the data at low momentum transfers, and a "hard pomeron", which is supposed to arise once α_s is small enough and s is big enough. Even then, the situation is still unclear, as one must match these two QCD regimes in a somewhat arbitrary way. Several attempts have been made: one can use the soft pomeron as an initial condition for the hard pomeron[3], or mix the partonic random walk of a dual parton model and that of the BFKL pomeron[4], or continue the hard pomeron to the soft region by including gluon self-recombination[5], or try various other unitarization schemes. All of these models have appealing features, but their variety implies that no one knows what the answer should be.

One is thus in search of a theoretical method that will shed some light on these

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problems. We want here to consider a class of corrections[6] which can be derived from theory, and which point to the infrared region as the source of the problems: namely, we shall include the gluon self-energy diagrams. In the usual analysis[1,7], these diagrams, which involve gluonic loops built on each of the gluon propagators, are subleading, and either neglected, or included as terms in a running coupling constant. However, one can do much better than these leading-logs arguments, as parts of these diagrams have now been resummed by nonperturbative methods: solutions to the Dyson-Schwinger equations for the gluon propagator have been calculated by a variety of techniques[8,9,10]. Since the work of Landshoff and Nachtmann[11], it has been known that such corrections, when used in two-gluon exchange, provide a factorizing pomeron amplitude, and a finite slope for the differential elastic cross section. We shall examine their effect to order α_s^3.

Let us first remind the reader of the perturbative calculation of two-gluon exchange. We shall follow the work of Gunion and Soper[12], who have shown how to calculate the lowest-order colour-singlet exchange between hadrons, *i.e.* two-gluon exchange, assuming that the hadrons are made of their valence quarks only.

In the high-s limit, the two incoming hadrons are living on the light-cone, *i.e.* at fixed x_- or x_+. These two directions then alternatively play the role of time in the definition of the wavefunctions ψ. Transforming the remaining free + or − component to momentum space, hadrons are described by a wavefunction $\psi(\{\beta_j\},\{\vec{r}_j\})$, with β_j the longitudinal momentum fraction of quark j and $\vec{r}_j$ its impact parameter in the center-of-momentum frame of the hadron. This wavefunction is a priori unknown. One can then show, in the eikonal approximation, that multi-gluon exchange takes the following form[12]:

$$\begin{aligned}\mathcal{A}_\infty &= -2is\int d\vec{b}\; e^{-i\vec{\Delta}.\vec{b}}[\prod_{j=1,n_q}\int_0^1 d\beta_j'\int d\vec{r}_j][\prod_{l=1,n_q}\int_0^1 d\beta_l\int d\vec{s}_l]\\ &\times\; |\psi(\beta_j',\vec{r}_j)|^2|\psi(\beta_l,\vec{s}_l)|^2\left(exp\left(-i2\pi\alpha_s\sum_{i,j}\sum_{c=1,8}\lambda_i^c\lambda_j^c V(\vec{x}_i-\vec{x}_j)\right)-1\right)\end{aligned} \quad (1)$$

with $V(\vec{x}) = -\int dx^+dx^-\Delta_F(x^+,x^-,\vec{x})$. Eq. (1) implicitly assumes an ordering of the vertices in the + and − directions.

The eikonal formula for the scattering of two hadrons h_1 and h_2 containing n_1 and n_2 valence quarks via the exchange of two gluons is obtained by expanding Eq. (1) to order α_s^2. The end result depends only on the transverse components of the gluon momenta $\vec{k}_a$ and $\vec{k}_b$:

$$\mathcal{A}_2 = is\frac{8\alpha_s^2}{9}n_1n_2\int d\vec{k}_a d\vec{k}_b\frac{d\mathcal{A}_2^q}{d\vec{k}_a d\vec{k}_b}[\mathcal{E}_1^{h_1}(\vec{\Delta})-\mathcal{E}_2^{h_1}(\vec{k}_a,\vec{k}_b)][\mathcal{E}_1^{h_2}(\vec{\Delta})-\mathcal{E}_2^{h_2}(\vec{k}_a,\vec{k}_b)] \quad (2)$$

with $\vec{\Delta}$ the total momentum transfer and with the quark-quark exchange amplitude:

$$\frac{d\mathcal{A}_2^q}{d\vec{k}_a d\vec{k}_b} = \delta^{(2)}(\vec{\Delta}-\vec{k}_a-\vec{k}_b)\frac{1}{(\vec{k}_a^2+\sigma_a)}\times\frac{1}{(\vec{k}_b^2+\sigma_b)} \quad (3)$$

We have introduced two gluon squared masses σ_a and σ_b, which for now can be considered as infrared regulators. $\mathcal{E}_1$ and $\mathcal{E}_2$ are two form factors which ensure that the process is infrared finite: when gluons have large wavelengths, they average out the colour of the hadron, and hence effectively decouple. The form factors are calculated to be:

$$\mathcal{E}_1(\vec{\Delta}) = \int d\mathcal{M} e^{i\vec{\Delta}.\vec{r}_k} \tag{4}$$

and

$$\mathcal{E}_2(\vec{k}_a, \vec{k}_b) = \int d\mathcal{M} e^{i\vec{k}_a.\vec{r}_k + i\vec{k}_b.\vec{r}_l} \tag{5}$$

with $l \neq k$. The natural integration measure $d\mathcal{M}$ is defined as:

$$d\mathcal{M} = [\prod_{j=1,n_q} d\beta_j d\vec{r}_j]\delta^{(2)}(\sum_j \beta_j \vec{r}_j)\delta(\sum_j \beta_j - 1)|\psi(\beta_j, \vec{r}_j)|^2 \tag{6}$$

The same model applied to γ-hadron elastic scattering leads to the identification of $\mathcal{E}_1$ with the Dirac elastic form factor F_1. $\mathcal{E}_1$ is thus measured directly so that we know its form from experiment[13].

The form of $\mathcal{E}_2$ is more arbitrary as the only firm property that can be established from Eq. (5) is the cancellation of the infrared divergences as $\vec{k}_a$ or $\vec{k}_b \to 0$, *i.e.* $\mathcal{E}_1 \to \mathcal{E}_2$. We choose $\mathcal{E}_2$ to vary according to a functional form guaranteeing infrared finiteness:

$$\mathcal{E}_2(\vec{k}_a, \vec{k}_b) = \mathcal{E}_1(\vec{k}_a^2 + \vec{k}_b^2 - f\vec{k}_a.\vec{k}_b) \tag{7}$$

The appropriate values of f are $f = 2$ in the pion case and $f = 1$ in the proton case, corresponding to a wavefunction peaked at $\beta = 1/2$, and $\beta = 1/3$ respectively.

Two-gluon exchange sets the scale correctly: the pp cross section from Eq. (2) is calculated to be $76\alpha_s^2$ mb, which for $\alpha_s \sim 0.5 - 1$ is of the right order of magnitude. Furthermore, the quark counting rule can be reproduced, but only if we assume that the $\mathcal{E}_1 - \mathcal{E}_2$ factors are not too different when going from the proton to the pion. If we use Eq. (7) with f=2 for pions and f=1 for protons, it turns out that the ratio $\sigma_{p\pi}/\sigma_{pp} = 0.65$ is close to the experimental number 0.62. Let us point out however that this is very sensitive to our choice of form factor[6], in particular to such quantities as the pion electric radius. In perturbative QCD, the quark counting rule would then be only an accident, and the factorizability of the pomeron exchange[14] would be lost. The main problem comes from the elastic differential cross section. Its shape comes out wrong: instead of an exponential, it has too much curvature, and its logarithmic slope at the origin turns out to be infinite.

These problems can be cured by a regulation of the infrared region, coming from a subclass of diagrams (the gluon self-energy) which are resummed via nonperturbative methods (the DS equations). Although these diagrams are supposedly sub-leading $\log s$, their inclusion certainly changes the leading-log answer. In order to see that it makes sense to modify only the gluon propagator, we observe that the infrared regulators of Eq. (3) can be treated as the squared masses that enter the Källen-Lehmann representation for the propagator:

$$D(q^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{q^2 - \sigma + i\epsilon} \tag{8}$$

One can then repeat the perturbative calculation by commuting the σ and k integrals. One obtains the same results (2, 3), which then need to be convoluted with the Källen-Lehmann densities $\rho(\sigma_1)$, $\rho(\sigma_2)$. This reconstructs the propagators, and the $1/(k^2 + \sigma)$ then becomes $-D(-k^2)$ because of Eq. (8). We shall see that this property of the amplitude is encountered again at the next order of perturbation theory. Note that as far as gauge invariance is concerned, the replacement of the gluon propagator by a nonperturbative counterpart while keeping the quark propagators perturbative does not violate the Ward-Slavnov-Taylor identities, for gluon exchange diagrams.

Solutions of the Dyson-Schwinger equation for the gluon propagator have been found, solutions which are either harder than a pole[15], $D \sim 1/k^4$, or softer[8,9,10]. As the $1/k^4$ solutions do not have a Lehmann representation, we do not know how to use them in diffractive calculations. We thus assume that the other solutions, which do not have poles for $q^2 \leq 0$, are those which are relevant for diffractive scattering. These solutions have been derived in various gauges. In the following, we shall work in the Feynman gauge, but consider only gauge-invariant sets of diagrams, so that the dependence on the propagators comes from the different approximations made by the authors of references 8, 9 and 10.

Although the asymptotic forms agree with perturbative QCD at large k^2, there is a wide disagreement as to the details of their behaviour near $k^2 = 0$. We limit ourselves here to the study of three propagators[8,9,10], which represent the whole range of behaviours at the origin which can be implemented in the calculation (assuming that the propagator does not change sign for $q^2 < 0$.)

The Häbel-Könnig-Reusch-Stingl-Wigard propagator[9] vanishes at the origin. Its form has been suggested by a consistency argument in the Landau gauge, and agrees with that derived by Zwanziger[16] based on considerations related to the Gribov horizon. In the axial gauge, Cornwall[10] has derived a gauge-invariant set of diagrams defining a gluon mass. Although one might worry about simply putting this mass into an explicitly gauge dependent object, it enables us to consider the possibility of a propagator finite at the origin. Notice that in the axial gauge, a theorem due to Baker, Ball and Zachariasen[15] implies that the gauge-dependent propagator is infinite at the origin. Solutions have been found by D.A. Ross and one of us[8], which behave like a fractional power of k^2 near $k^2 = 0$.

Each of the above propagators contains an intrinsic scale μ_0: a simple dimensional argument leads to the conclusion that a modification of the infrared region means that the gluon propagator must be written $D(q^2) = (1/\mu_0^2)\ \mathcal{D}(q^2/\mu_0^2)$ with $\mathcal{D}$ a function without a single pole at the origin. As QCD is a scale-free theory, the scale μ_0 cannot be determined directly from Dyson-Schwinger equations, and must thus be determined through comparison with some dimensionful quantity. We shall assume from now on that two-gluon exchange gives the bulk of the cross sections, and that higher orders contribute to the s-dependence of the result: two-gluon exchange should then give us a total cross section of the order of 22 mb and a logarithmic slope at the origin of the order of 10 GeV^{-2}.

We show in Figure 1 the dependence of σ_{tot}^{pp} (a) and B(0) (b) on the scale μ_0 entering the propagator. As a growing μ_0 makes the propagator smaller, it is not surprising that the total cross section goes down with the propagator scale, as shown in Figure 1a. It is less obvious however that for $\mu_0 \sim 0.3 - 1.0$ GeV, one gets a large suppression factor, and thus each propagator can give a good starting value for the total cross section, of the order of 20 mb. The logarithmic slope of the elastic cross section, shown in figure 1b, also gets cured by the introduction of μ_0, and numbers of the order of 10 GeV^{-2} can be achieved for scales of the same order.

We thus see at this order that nonperturbative effects are non-negligible. The inclusion of modified propagators in the calculation provides appreciable improvements, and the improved order α_s^2 constitutes a good starting point for an expansion in $\log s$. Let us now see to which extent these improvements carry over to higher orders.

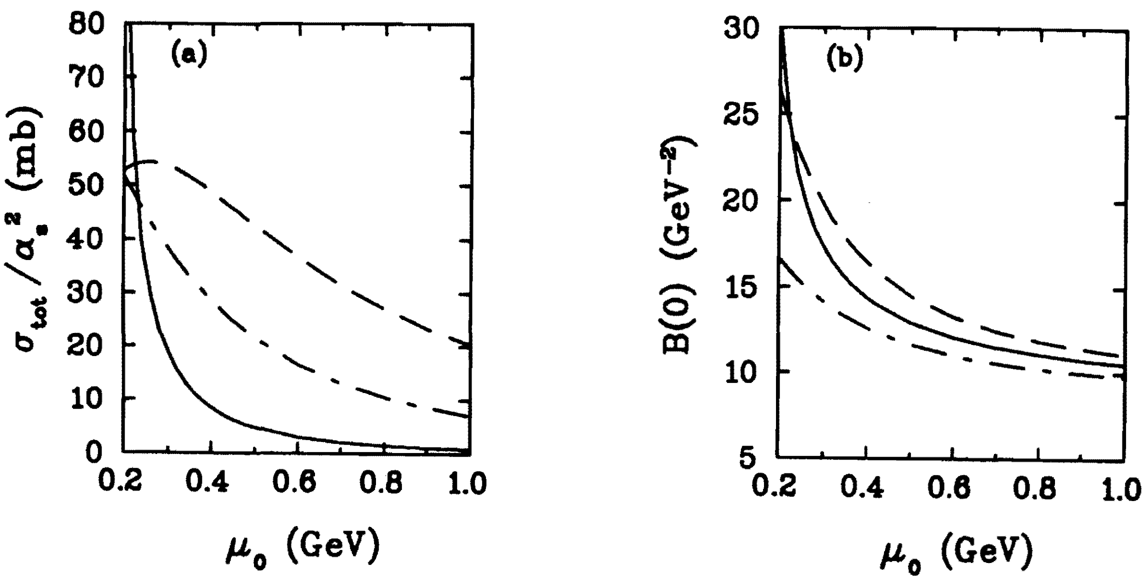

Figure 1: Two-gluon exchange results for three nonperturbative gluon propagators, Cornwall (plain curve), Häbel-Könnig-Reusch-Stingl-Wigard (dashed curve), and Cudell-Ross (dot-dashed curve). The horizontal axis gives the nonperturbative scale entering the propagator. (a) shows the total proton-proton cross section, which scales like α_s^2, (b) gives the logarithmic slope at t=0 of the elastic cross section

To calculate three-gluon exchange, we must first notice that the colour algebra involves terms like $Tr(\lambda_a\lambda_b\lambda_c)Tr(\lambda_{a'}\lambda_{b'}\lambda_{c'})$ with a', b', c' some combination of a, b, c. Using the fact that $Tr(\lambda_a\lambda_b\lambda_c) = 2(if_{abc} + d_{abc})$, we recognize that the amplitude contains two terms, one proportional to $f_{abc}f_{abc}$ and one proportional to $d_{abc}d_{abc}$. When we calculate $\bar{q}q$ scattering instead of qq, the first term flips sign while the second remains the same. As antiquarks and quarks couple to gluons with opposite signs, the first term contributes to the pomeron, while the second gives the lowest-order odderon.

Let us first consider this odd contribution. As d_{abc} is a commuting object, we can apply the QED formalism[6]. This means that the answer will not contain any $\log s$ factor. Expanding Eq. (1) to order α_s^3, and carrying out the required colour traces, we obtain no odderon contribution in $p\pi$ scattering whereas pp scattering gives:

$$\mathcal{A}_O = -\frac{10}{9\pi}\alpha_s^3 s \int d\vec{k}_a d\vec{k}_b d\vec{k}_c \frac{1}{\vec{k}_a^{\,2}+\sigma_a}\ \frac{1}{\vec{k}_b^{\,2}+\sigma_b}\ \frac{1}{\vec{k}_c^{\,2}+\sigma_c}\ \delta^{(2)}(\vec{k}_a+\vec{k}_b+\vec{k}_c-\vec{\Delta})$$
$$\times[\mathcal{E}_1(\vec{\Delta}^2) + 2\ \mathcal{E}_3(\vec{k}_a,\vec{k}_b,\vec{k}_c) - \mathcal{E}_2(\vec{k}_a+\vec{k}_b,\vec{k}_c) - \mathcal{E}_2(\vec{k}_b+\vec{k}_c,\vec{k}_a) - \mathcal{E}_2(\vec{k}_c+\vec{k}_a,\vec{k}_b)]^2 \quad (9)$$

with $\mathcal{E}_1$ and $\mathcal{E}_2$ the two form factors encountered previously in Eqs. (4, 5), and

$$\mathcal{E}_3(\vec{k}_a,\vec{k}_b,\vec{k}_c) = \int d\mathcal{M}\ e^{i\vec{k}_a.\vec{r}_k + i\vec{k}_b.\vec{r}_l + i\vec{k}_c.\vec{r}_m} \quad (10)$$

with $k \neq l \neq m$.

The third form factor $\mathcal{E}_3$ corresponds to diagrams where one gluon gets attached to each quark. As was the case for two-gluon exchange, formula (9) is infrared convergent. This is due to the fact that $\mathcal{E}_3$ reduces to $\mathcal{E}_2$ when one of its momenta vanishes: $\mathcal{E}_3(0,\vec{k}_b,\vec{k}_c) = \mathcal{E}_2(\vec{k}_b,\vec{k}_c)$ and similar conditions when $\vec{k}_b \to 0$, $\vec{k}_c \to 0$.

We shall use a parametrization of $\mathcal{E}_3$ based on the previous one for $\mathcal{E}_2$, see Eq. (7):

$$\mathcal{E}_3(\vec{k}_a,\vec{k}_b,\vec{k}_c) = \mathcal{E}_1\left(\vec{k}_a^2+\vec{k}_b^2+\vec{k}_c^2 - f(\vec{k}_a.\vec{k}_b+\vec{k}_a.\vec{k}_c+\vec{k}_b.\vec{k}_c)\right) \quad (11)$$

The fact that $f = 1$ for the proton implies that when the three quarks are scattered by the same amount in transverse space, if they have the same longitudinal momentum, they must recombine into a proton. Thus $\mathcal{E}_3(\vec{k},\vec{k},\vec{k}) = 1$. As already pointed out[17], this property means that the $\mathcal{E}_3^2$ term of Eq. (9) dominates the elastic amplitude at high-t: configurations for which the three quarks are scattered by the same amount are not affected by the sharp t-dependence coming from $\mathcal{E}_1$ or $\mathcal{E}_2$.

Notice that $\mathcal{E}_3$ is present (at this order) only for $C = -1$ contributions. Although the odderon is dominant at high-t, it is suppressed at small t, because of the structure of its form factor $\mathcal{E}_1 - 3\mathcal{E}_2 + 2\mathcal{E}_3$. In pertubative QCD the odderon contribution to the amplitude at $t = 0$ is about 18% of the two-gluon exchange amplitude.

Thus at this order, the amplitude is purely real, and contains three form factors. These are the only three possible form factors that occur at any order in pp scattering. We notice that the amplitude takes a form that involves only propagators, so that we are allowed to replace $1/(\vec{k}^2+\sigma)$ by a nonperturbative estimate $D(k^2)$ in Eq. (9), using Eq. (8). The inclusion of nonperturbative propagators does not modify the ratio of the odderon contribution to the two-gluon exchange one: both get suppressed by roughly the same factor, and we recover again a number of the order of 20%.

We can now turn to the C-even exchanges. As we have seen, the first contribution to these comes from the anticommuting part of the colour algebra of three-gluon exchange. This generates $\log s$ terms which are absent in QED. A careful application of Eq. (1), taking ordering in the x^+ and x^- directions, leads to the counterpart of Eq. (9). For pp scattering, we get:

$$\mathcal{A}_{\mathcal{P}} = \frac{12}{\pi^2}\alpha_s^3 i s \log s \int d\vec{k}_a d\vec{k}_b d\vec{k}_c \frac{1}{\vec{k}_a^{\,2}+\sigma_a}\,\frac{1}{\vec{k}_b^{\,2}+\sigma_b}\,\frac{1}{\vec{k}_c^{\,2}+\sigma_c}\,\delta^{(2)}(\vec{k}_a+\vec{k}_b+\vec{k}_c-\vec{\Delta})$$
$$\times\left[\frac{1}{2}\mathcal{F}_1^{h_1}\mathcal{F}_1^{h_2} - \frac{1}{4}(\mathcal{G}_a^{h_1}\mathcal{G}_a^{h_2}+\mathcal{G}_c^{h_1}\mathcal{G}_c^{h_2}) + \mathcal{F}_1^{h_1}(\mathcal{G}_a^{h_2}+\mathcal{G}_c^{h_2}) + \mathcal{G}_a^{h_1}\mathcal{G}_c^{h_2} + (h_1 \leftrightarrow h_2)\right] \quad (12)$$

where $\mathcal{F}_1 = \mathcal{E}_1(\vec{k}_a+\vec{k}_b+\vec{k}_c)$, $\mathcal{G}_a = \mathcal{E}_2(\vec{k}_a, \vec{k}_b+\vec{k}_c)$ and $\mathcal{G}_c = \mathcal{E}_2(\vec{k}_c, \vec{k}_a+\vec{k}_b)$.

It is interesting that once again, the structure of the answer allows the use of nonperturbative propagators, via the integration of the $1/(\vec{k}^2+\sigma)$ with a Källen-Lehmann density, as explained in the two-gluon case, see Eq. (8).

In order to compute the full order α_s^3 contributions to the pomeron, we need to add the H and Y diagrams, where the triple-gluon vertex comes in. Although the Soper-Gunion formalism does not explicitly include these, it is not very hard to convince oneself that the form factors are the same as in two-gluon exchange, so that we get:

$$\mathcal{A}_{HY} = -\frac{12}{\pi^2}\vec{\Delta}^{\,2}\alpha_s^3 i s \log s \left[\int d\vec{k}_a \frac{\left(\mathcal{E}_1^{h_1}(\vec{\Delta}) - \mathcal{E}_2^{h_1}(\vec{\Delta}-\vec{k}_a, \vec{k}_a)\right)}{(\vec{k}_a^{\,2}+\sigma)\left((\vec{\Delta}-\vec{k}_a)^{\,2}+\sigma\right)}\right] \times [h_1 \leftrightarrow h_2] \quad (13)$$

with $\mathcal{E}_1$ and $\mathcal{E}_2$ the form factors introduced in Eqs. (4,5). Notice that, once more, assuming the existence of a Källen-Lehmann density for the gluon propagator enables us to change $1/(\vec{k}^2+\sigma)$ into $D(k^2)$.

The complete oder α_s^3 formalism thus leads to a hadronic amplitude that has the following form:

$$\mathcal{A}(s,t) = \mathcal{A}_2\left\{i\left[1+\log s\left(\epsilon_0+\alpha' t + O(t^2)\right)\right] + f_{odd}(t)\right\} \quad (14)$$

It is of course tempting to see in this a first-order Taylor expansion in $\log s$ of a pomeron pole, plus a zeroth-order term from an odderon pole. There would then be a one-to-one mapping between the Regge picture of the pomeron and this calculation. As BFKL have shown[1], life is not so simple, and higher-order terms spoil the analogy. Hence in the following the terms "pomeron intercept" or "slope" must not be taken literally. As we shall now see, all the problems of the BFKL formalism[18] are already present at this low order. It is thus worth examining them in the context of the simple equations we have gathered in this paper, rather than obscuring the issues by resumming.

At this order, the normalization of the cross section, $\mathcal{A}_2$, comes from two-gluon exchange, Eq. (2); the coefficient of $\log s$, ϵ_0, is the ratio of two to three gluon exchange, Eqs. (2, 12). The odd contribution comes of course from Eq. (9). Finally, the α' contribution comes from Eq. (13) and from the Taylor expansion of Eq. (12).

The main problem here is that the pomeron intercept is much too large: $1+\epsilon_0 = 2.85\alpha_s$. One can of course lower it by using a smaller value of α_s, or equivalently by letting the coupling constant run. This can reduce the value of the intercept to $1+\epsilon_0 \approx 2.32$. Another obvious problem is the fact that, although the expression (13) is finite for $\vec{\Delta} \to 0$, it does not lead to a finite value for α': the integrands at $\vec{\Delta} = 0$ look like $(\mathcal{E}_1 - \mathcal{E}_2)/k^4$, and we thus have a logarithmically divergent α'.

As we have already mentioned, we can easily accommodate nonperturbative propagators in the expression given for the hadronic amplitude. Provided that the propagators have the usual analytic properties embodied by the Källen-Lehmann density, the prescription is simply to replace $1/(k^2+\sigma)$ by the nonperturbative prescription for the propagator, $D(k^2)$.

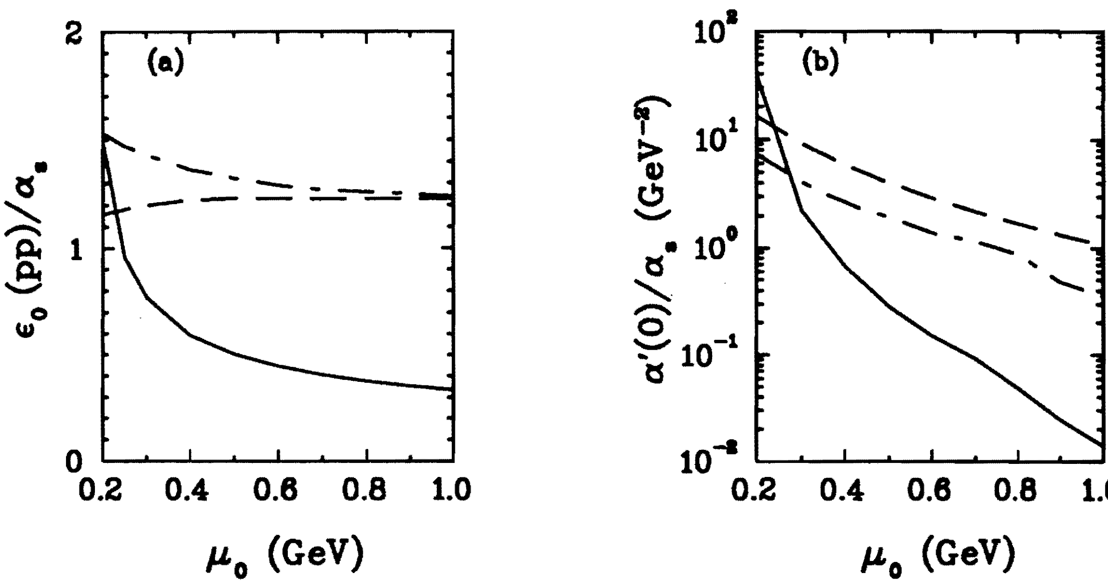

Figure 2: Order α_s^3 results for the three nonperturbative gluon propagators of Figure 1. Our convention for the curves is the same as in Figure 6. μ_0 is the nonperturbative scale entering the propagator. All graphs scale like α_s. (a) shows the ratio at $t=0$ of the coefficient of the $\log s$ term to the two-gluon exchange term; (b) the ratio at $t=0$ of the coefficient of the $t \log s$ term to the two-gluon exchange term.

Figure 2 shows the result of the use of the propagators of refs. 8, 9 and 10, as a function of the nonperturbative scale μ_0. Figure 2a shows that ϵ_0 can in principle get as low as 0.35. For values of μ_0 favoured by our previous discussion on two-gluon exchange, we see that we get values of the order of 2 for the intercept: it thus seems impossible to get acceptable numbers both for two and three-gluon exchange. Another way to look at this is to allow α_s to vary: for values around 0.1, one would indeed get an intercept compatible with data, but that would mean that although rising at the correct rate, the pomeron cross section would be much too low.

Figure 2b shows that the pomeron slope becomes finite once the infrared region is smoothed out. Values compatible with 0.25 GeV can again be achieved, but again for values of μ_0 or α_s that would suppress two-gluon exchange.

Hence there is no "golden propagator". The three propagators we have tried have met with some success, but none of them gives us a perfect fit: one cannot accommodate a sizeable two-gluon exchange amplitude together with a slowly rising third-order amplitude. At third order, the Cornwall propagator gives the most promising results. One has to point out however that this is mainly due to the fact that the α_s used is appreciably smaller than in the case of the two other propagators.

To conclude, we have shown in this paper that the Soper-Gunion formalism can be extended to include three-gluon effects, and that the proton possesses three form factors, in the valence quark approximation. The structure of these form factors naturally suppresses the odderon at small t and makes it leading at high-t.

We have demonstrated that the infrared region is important in leading $\log s$ calculations. Although the perturbative answer is infrared finite, the dominant region of k^2 remains small until t is as large as 10 GeV. The perturbative calculation can be easily recast in a form which allows for a modification of the gluon propagator, provided that there exists a Källen-Lehmann density: the complete third-order result then depends only on the t-channel propagators.The use of recent solutions to the Dyson-Schwinger equation, combined with the simple idea that it is enough to modify the gluon propagator at small momentum transfer[11], is not sufficient to diminish the value of the pomeron intercept to bring it in agreement with data. These nonperturbative effects nevertheless can change the answer by large factors, of the order of 3, and must certainly be taken into account before using the leading-log s formalism in the diffractive region.

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