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Further Studies of the Influence of Energy Calibration and Scanning Strategy on the Γ_Z Measurement at LEP.

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Abstract

The expected errors on Γ_Z are compared for various choices of the off-peak energy points and the strategy for energy calibration. The comparisons are made at fixed total cost in number of collected Z^0s. A refinement with respect to a previous estimate is that the variation both of the cost of performing energy calibrations and of the resulting error on the LEP energy scale as the scanning and calibration strategy is varied are taken into account in calculating the expected Γ_Z error. In addition, a number of improvements have been made in the estimation of the energy calibration error.

These refinements do not significantly affect, however, the basic result obtained previously: that the Γ_Z accuracy achieved when data are taken at ± 3GeV from the Z^0 peak is essentially equal to that achieved if data were to be taken at the ± 2GeV points used in the previous scan in 1993.

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1 Introduction

In a previous note [1] the expected errors on Γ_Z from a scan at LEP in 1995 were evaluated. Consistent results have been obtained by other authors [2, 3]. The possibility that data be collected in 1995 at different off-peak energy points[1] from those at ± 2GeV used in the 1993 scan was also considered [1]. Once the uncertainty due to the LEP energy scale and the precision of the on-peak cross-section measurement were taken into account, it was found that the Γ_Z accuracy from data taken at ± 3GeV was essentially equal to that obtained from ± 2GeV.

For the central results of reference [1] a set of simple assumptions concerning the 1995 energy calibration were made. Irrespective of the choice of off-peak energy points, or of the amount of integrated luminosity to be collected off-peak, or of any variation in the possible calibration strategy to be adopted, it was assumed that the accuracy of the LEP energy calibration would be 1MeV and that the amount of integrated luminosity lost because of the need to perform frequent energy calibrations would be 6pb^{-1}. Some indication was given of the sensitivity of the results to variations in the assumed energy calibration accuracy.

The proposal to scan in 1995 has now been accepted [4]. It is the aim of the current note to reevaluate the expected Γ_Z errors using more refined estimates of the accuracy of the energy calibration and the associated overhead. The input assumptions are presented in section 2; these represent the current best guesses from the energy working group, guided by the performance achieved in 1993 and 1994 [5, 6, 7, 8]. Parameters whose value cannot be predicted with any confidence are varied over reasonable ranges.

Comparisons between different choices of off-peak energy points and calibration strategies are made at fixed cost in the total number of collected Z^0s. As different scanning and calibration strategies are considered, the variation in the cost due to energy calibrations is taken into account in addition that arising from the reduced off-peak cross-section (section 3).

The calculation of the expected systematic error on Γ_Z due to the LEP energy scale is described in section 4. The contribution of uncalibrated fills to the uncertainty in the average energy depends on the fraction of fills that are calibrated and on the total number of fills. As different scanning and calibration strategies are considered the variation in these quantities is taken into account. The fact that all fills do not have the same integrated luminosity is taken explicitly into account in calculating the energy error; this represents a refinement with respect to previous treatments. The energy error also has a fixed component that is independent of the scanning and calibration strategy adopted.

As is explained in section 5, the expected statistical error on Γ_Z for each scenario is obtained by extrapolation from the values given in reference [1].

The results of the analysis are presented in section 6. The variation of the expected error on Γ_Z as changes are made to the input assumptions and the strategy for scanning and energy calibration are discussed in some detail.

The results presented in section 6 correspond to the expected accuracy obtained from combining

[1] In the following the terms $\pm$nGeV refer to the points above and below the Z^0 by $n \times 0.9$GeV; these correspond to the energies at which polarization can be achieved in LEP and which, therefore, can be calibrated.

the 1994 and 1995 data of the four LEP experiments. In section 7 the expected accuracy on Γ_Z when these data are combined with the measurements already made in 1990–1993 is given.

Brief conclusions are given in section 8.

An appendix gives the derivation of the improved formula for the contribution of uncalibrated fills to the uncertainty in the average energy.

2 Assumptions about 1995 Running

2.1 Fixed Assumptions

The following assumptions have been made concerning the performance of LEP in 1995:

- Average length of physics fills: T^{fill}=10hours. (In 1994 T^{fill} was 9.4 hours [5].)
- Fraction of fills lost during the setup phase between the end of the squeeze and the start of physics: F^{setup}=0.23. (In 1994 60/257=0.23 of fills were lost in the setup phase [5].)
- Fraction of fills lost during physics running: F^{lost}=0.23. (In 1994 45/197=0.23 of fills were lost during physics [5].)
- Time required for an energy calibration by resonant depolarization with separated beams at the end of a physics fill: T^{eof}=2hours [5, 6].
- Time required for an energy calibration by resonant depolarization with separated beams at the beginning of a physics fill: T^{bof}=3hours. (The feasibility of performing energy calibrations with the high beam currents present at the beginning of physics fills is yet to be demonstrated. The required operations are expected to be rather more delicate than with the low beam currents typical for the end of fill calibrations that are currently performed [5, 6].)
- Fraction of fills lost during an energy calibration at the end of fill: F^{eof}=0.26. (In 1994 5/19=0.26 calibration attempts failed [6].)
- Fraction of fills lost during an energy calibration at the beginning of fill: F^{bof}=0.26.
- Residual scatter of the E_{cm} values measured by resonant depolarization with respect to the best available model for E_{cm} variations: σ_{scat}=6.0MeV. (This is consistent with the values observed in 1993 [7] and 1994 [6, 8])
- Other sources of systematic uncertainty are assumed to make a fixed contribution of 0.5MeV to the error on E_{cm}, irrespective of the scenario considered. (This would represent a modest improvement with respect to the uncertainty achieved in the 1993 and 1994 running. Some progress is confidently expected in 1995, for example, in the understanding of the NMR measurements. However, it would seem prudent to make some allowance for the possibility that new uncertainties may arise in 1995. These may result, for example,

from the RF and beam loading effects associated with the new bunch train scheme, or from having exposed our understanding of the energy variations within single fills to more stringent tests than were available previously.)

- The fractional variation in the integrated luminosity of fills that reach the scheduled end of physics: $\frac{\sigma_{\mathcal{L}}}{\mathcal{L}}$=0.45. (This is consistent with the variations seen during the stable running in the middle of 1994.)

2.2 Variable Assumptions

The following assumptions concerning the performance of LEP in 1995 and the amount of time to be devoted to the scan have been varied:

- Fraction of fills that by choice are not calibrated: $F^{choice} = -0.1, 0.0, 0.1, 0.2, 0.3$. (The variation of this parameter can also be understood as allowing the assumed fraction of failed end of fill energy calibrations (F^{eof}) to be modified. A negative value of F^{choice} corresponds, therefore, to a more optimistic value of F^{eof} than the default given above.)
- Average integrated luminosity collected per physics fill: $\mathcal{L}^{fill} = 0.6, 0.8, 1.0, 1.2, 1.4 pb^{-1}$.
- Loss in collected Z^0s, expressed as a cost in on-peak equivalent luminosity: $\mathcal{L}^{on-peak}_{cost} = 30, 15 pb^{-1}$. ($\mathcal{L}^{on-peak}_{cost}$ is defined precisely in the next section[2].)

3 Comparisons for Equal Cost in Number of Collected Z^0s

Fewer Z^0s are collected during a scan than when running on-peak. This is because of the reduced off-peak cross-section and the need to perform frequent energy calibrations. The total reduction in the number of collected Z^0s will be expressed in terms of the cost in on-peak equivalent luminosity per experiment:

$$\mathcal{L}^{on-peak}_{cost} = \mathcal{L}^{off-peak}\left(\frac{\sigma^{on-peak} - \sigma^{off-peak}}{\sigma^{on-peak}}\right) + \mathcal{L}^{cal}$$

where $\sigma^{off-peak}$ is the average off-peak cross-section, $\sigma^{on-peak}$ is the on-peak cross-section and $\mathcal{L}^{cal}$ is the integrated luminosity lost due to the time spent performing energy calibrations.

For end of fill energy calibrations the total loss in integrated luminosity is given by:

$$\mathcal{L}^{cal} = T^{eof}\frac{\mathcal{L}^{fill}}{T^{fill}}2N^{fill}\left(1 - F^{lost}\right)\left(1 - \frac{F^{eof}}{2} - F^{choice}\right)$$

[2]For ease of comparison with reference [1] it may be noted that for ±2GeV the values $\mathcal{L}^{on-peak}_{cost} = 30, 15 pb^{-1}$ correspond quite closely to the scans with total off-peak luminosity of 40, $20 pb^{-1}$ considered there.

where N^{fill} is the number of fills at either of the off-peak energy points. It is assumed that failed attempts to perform energy calibrations will take, on average, half the time of successful calibrations.

For beginning of fill energy calibrations the total loss in integrated luminosity is given by:

$$\mathcal{L}^{cal} = T^{eof}\frac{\mathcal{L}^{fill}}{T^{fill}}2N^{fill}\left(1+F^{setup}\right)\left(1+\frac{F^{bof}}{2}\right)$$

The errors on the measured Γ_Z will be compared for various choices of the off-peak energy points and the strategy for energy calibration. The comparisons will be made at fixed total cost, $\mathcal{L}_{cost}^{on-peak}$, taking into account the variation of both the statistical and systematic errors on the Γ_Z measurement. Because the cross-section at ± 3GeV is lower than at ± 2GeV a smaller amount of off-peak luminosity can be collected at ± 3GeV for the same $\mathcal{L}_{cost}^{on-peak}$.

4 Energy Systematic Error

The error on Γ_Z arising from the uncertainty on the LEP energy scale is given to a good approximation by the formula:

$$\Delta\Gamma_Z = \sqrt{2}\Delta E_{cm}\Gamma_Z/(E_1 - E_2), \tag{1}$$

where ΔE_{cm} is the uncertainty on the energy scale of one of the off-peak points (only sources that are uncorrelated between the two energy points contribute); E_1 and E_2 are the centre-of-mass energies of the two off-peak scan points. The measurement of Γ_Z using scan points at ± 3GeV therefore has a sensitivity to the energy systematic errors smaller than that at ± 2GeV by a factor of 2/3.

Let us first consider the strategy of performing energy calibrations at the end of physics fills. In this case a significant contribution to the energy error is made by uncalibrated fills. This is because the energy of uncalibrated fills must be obtained from a model that is known to describe variations in the LEP energy only imperfectly [7].

The fraction of uncalibrated fills is given by:

$$1 - F^{uncal} = (1 - F^{lost})(1 - F^{eof} - F^{choice})$$

In reference [7], the contribution of uncalibrated fills to the uncertainty in the average energy of the off-peak scan points is shown to be:

$$\Delta E_{cm}^{scat} = \sigma_{scat}\frac{F^{uncal}}{\sqrt{N^{fill}}(1 - F^{uncal})} \tag{2}$$

Equation 2 is derived under the assumption that all fills have equal luminosity. In the appendix it will be shown that if this assumption is relaxed the expression unfortunately becomes rather more complicated. Qualitatively there are two main effects:

- When the fills do not all have the same luminosity statistical fluctuations become larger than $\sqrt{N}$. This increases the error by typically 10%.
- When a large fraction of fills are calibrated then fills lost before the end of physics make up a large fraction of the uncalibrated fills. These fills have a lower than average integrated luminosity, which decreases the error by typically 20%.

The more accurate expression for ΔE_{cm}^{scat} is:

$$\Delta E_{cm}^{scat} =$$

$$\frac{\sigma_{scat}}{\sqrt{N^{fill}}(1-F^{lost}/2)}\sqrt{K^{eof}(F^{eof}+F^{choice})(1-F^{lost})+\frac{K^{lost}F^{lost}}{4}+\frac{\{(F^{eof}+F^{choice})(1-F^{lost})+F^{lost}/2\}^2}{(1-F^{eof}-F^{choice})(1-F^{lost})}} \quad (3)$$

where

$$K^{eof} = 1+\left(\frac{\sigma_{\mathcal{L}}}{\overline{\mathcal{L}}}\right)^2$$

$$K^{lost} = 1.33+\left(\frac{\sigma_{\mathcal{L}}}{\overline{\mathcal{L}}}\right)^2$$

Other sources of systematic uncertainty are assumed to make a fixed contribution of 0.5MeV to the error on E_{cm}, irrespective of the scenario considered. The total E_{cm} error is then given by the sum in quadrature of these two contributions:

$$\Delta E_{cm} = 0.5 \oplus \Delta E_{cm}^{scat} \text{ (MeV)}$$

An alternative strategy that is considered is to perform energy calibrations with separated beams at the beginning of physics fills. In this way, all off-peak data would be collected in calibrated fills, ΔE_{cm}^{scat} would be zero, and $\Delta E_{cm} = 0.5$MeV

5 Statistical Error on Γ_Z

The expected statistical error on Γ_Z for each scenario is obtained by extrapolation from the values given in section 5 of reference [1], which were derived from fits using the ZFITTER package [9] to data representing the LEP combined cross-section measurements for the years 1994 and 1995. It is assumed that both hadronic and leptonic cross-sections are used to extract the Z^0 lineshape parameters. The relevant values are summarized in table 1. For a particular off-peak luminosity $\mathcal{L}^{off-peak}$, the appropriate statistical error on Γ_Z is calculated using:

$$\Delta\Gamma_Z = \Delta\Gamma_Z^{ref}\sqrt{\frac{\mathcal{L}^{ref}}{\mathcal{L}^{off-peak}}} \quad (4)$$

where $\Delta\Gamma_Z^{ref}$ and $\mathcal{L}^{ref}$ are the reference values given in table 1 for the statistical error and off-peak luminosity, respectively. In reference [1] it was pointed out that the uncertainty on the peak cross-section has a non-negligible effect on the Γ_Z error. This means that, for example, the

Scan	$\mathcal{L}^{ref}$ (pb^{-1})		
	40	32	20
±2	1.88		2.52
±3		2.09	2.60

Table 1: Reference values ($\Delta\Gamma_Z^{ref}$) for the statistical error on Γ_Z (in MeV) for various values of integrated luminosity collected off-peak [1].

values of $\Delta\Gamma_Z$ given for different values of $\mathcal{L}^{ref}$ in table 1 are not exactly related by equation 4. However, for the rather small extrapolations with respect to the reference values that are required in this note, equation 4 provides a perfectly adequate approximation; the tedium involved in generating for each variation in scan strategy the appropriate pseudo data and performing a full fit can thus safely be avoided.

6 Results

For $\mathcal{L}_{cost}^{on-peak} = 30\text{pb}^{-1}$ the open points in figure 1 show the variation of the total error on Γ_Z as a function of F^{choice}, the fraction of fills that by choice are not calibrated at the end of the fill. Table 2 gives the details of the calculation[3]. It can be seen that at the ±5% level the Γ_Z accuracy is equal for scans at ±2GeV (open squares) and ±3GeV (open circles) and is independent of the value of F^{choice}, as long as F^{choice} remains below about 0.2.

Looking in more detail it can be seen that whether the ±2GeV or the ±3GeV scan gives the smaller overall error on Γ_Z depends on the value of F^{choice}, with the cross-over being around F^{choice}=0.05. For a scan at ±2GeV (open squares) the Γ_Z error increases more rapidly with F^{choice}. For such a scan it would be, therefore, desirable to calibrate the maximum possible number of fills. The variation of the Γ_Z error with F^{choice} is still interesting, however, because it shows by how much the error will be degraded if the assumptions made about the fraction of calibrated fills turn out to have been overoptimistic. The result is reasonably encouraging; even for F^{choice}=0.2, which corresponds to 58% of fills being uncalibrated, the fractional increase in the Γ_Z error is only 8%, with respect to the default assumption F^{choice}=0.0.

For a scan at ±3GeV (open circles) it can be seen that the Γ_Z error is even less dependent on the value of F^{choice}. This is because for a scan at ±3GeV the contribution of the energy error is smaller than at ±2GeV due to the "lever-arm" effect in equation 1. The statistical error on Γ_Z decreases with increasing F^{choice} (since less time is spent calibrating more off-peak luminosity can be taken whilst maintaining a fixed total cost). This effect partially cancels the corresponding increase in the energy systematic. (For $F^{choice} \leq 0.2$ the variation in $\Delta\Gamma_Z$ is within ±2.5%.) Requesting a smaller number of calibrations would reduce the

[3]The number of significant digits given in the table is to allow the small differences among the various scanning scenarios to be seen without being confused by the effects of rounding errors. It is clear that the uncertainties in the assumptions about, for example, the delivered luminosity and energy error that will be achieved in 1995 cause a much larger uncertainty common to all the scenarios than the small differences among them.

load placed on the polarization team. In addition, a number of physics measurements that are not sensitive to the energy calibration can be made with off-peak data. For example, table 2 shows the expected error on the measurement of the energy dependence of the lepton pair forward-backward asymmetry ($\Delta(\partial A_{FB}/\partial E)$). Such measurements clearly improve with increasing F^{choice}. If a scan is performed at ± 3GeV it may, therefore, be appropriate to choose not to calibrate every off-peak fill[4].

Also shown in figure 1 and table 2 are the expected Γ_Z errors if every physics fill is calibrated at the beginning of the fill instead of at the end. It can be seen that for small values of F^{choice} the loss due to the extra time spent in calibration outweights the benfits of reducing the scatter error to zero. We can, therefore, conclude that it would not be economic to perform energy calibrations at the beginning of every fill. However, it is highly desirable that energy calibrations be performed at the beginning and end of a subset of fills, in order to further our understanding of energy variations within single fills.

Figure 2 and table 3 show the variation of the error on Γ_Z as a function of F^{choice} for $\mathcal{L}_{cost}^{on-peak} = 15pb^{-1}$. The basic pattern is the same as for $\mathcal{L}_{cost}^{on-peak} = 30pb^{-1}$. The values of $\Delta\Gamma_Z^{scat}$ are larger for $\mathcal{L}_{cost}^{on-peak} = 15pb^{-1}$ because there are a smaller number of off-peak fills. The cross-over point at which the expected Γ_Z errors for ± 2GeV and ± 3GeV are equal occurs at F^{choice}=0.2. This is slightly higher than for $\mathcal{L}_{cost}^{on-peak} = 30pb^{-1}$, because as the off-peak luminosity decreases the statistical errors (for which ± 2GeV is favoured) increase by a larger amount than the systematic errors (for which ± 3GeV is favoured)[5].

Figure 3 and table 4 show for a fixed total cost $\mathcal{L}_{cost}^{on-peak}$=30pb^{-1} the expected variation of the error on Γ_Z as a function of $\mathcal{L}^{fill}$, the average luminosity collected per fill. It can be seen that the variation is only $\pm 5\%$ for the wide variation of $\mathcal{L}^{fill}$ considered. This is also true for table 5, which shows the expected variation of the error on Γ_Z as a function of $\mathcal{L}^{fill}$ for a cost $\mathcal{L}_{cost}^{on-peak}$=15pb^{-1}. However, it is interesting to compare with the result of the 1993 scan, for which $\mathcal{L}^{fill}$=0.25pb^{-1}. About 35 fills at each off-peak energy point resulted in the collection of $\mathcal{L}^{off-peak}$=18pb^{-1} and although F^{uncal}=0.66 in 1993 $\Delta\Gamma_Z^{scat}$=1.3MeV. If we wanted to perform a scan in 1995 with a low total off-peak luminosity then with $\mathcal{L}^{fill}$=0.8pb^{-1} we would need to achieve F^{uncal} of about 0.4 at ± 2GeV to match the 1993 scatter error of $\Delta\Gamma_Z^{scat}$=1.3MeV (see table 3).

7 Combination with Data Collected in 1990-1993

The data collected by the four LEP experiments during the period 1990–1993 give a measurement of Γ_Z with an error of:[6] $3.1 = 2.7$ (stat.) $\oplus$ 1.5 (syst.) MeV. To a good approximation

[4]Since it is very difficult to predict in advance the success rate for energy calibrations and the stability of the measured energies, it would probably still be prudent to start the scan calibrating every possible fill. If the success rate turns out to be high and the variations in the measured energies small and understood, then at some point the decision could be taken to reduce the frequency of calibrations. One lesson learned from the 1993 scan is that a fast evaluation of the calibration data is required so that the calibration frequency can be increased again if large or poorly understood variations in the energy scale occur.

[5]This is because of the fixed 0.5MeV contribution to the energy error.

[6]It is assumed that the uncertainty in the energy spread of the beams, which currently contributes a systematic error on Γ_Z of 1MeV, will be reduced to a negligible level by further study.

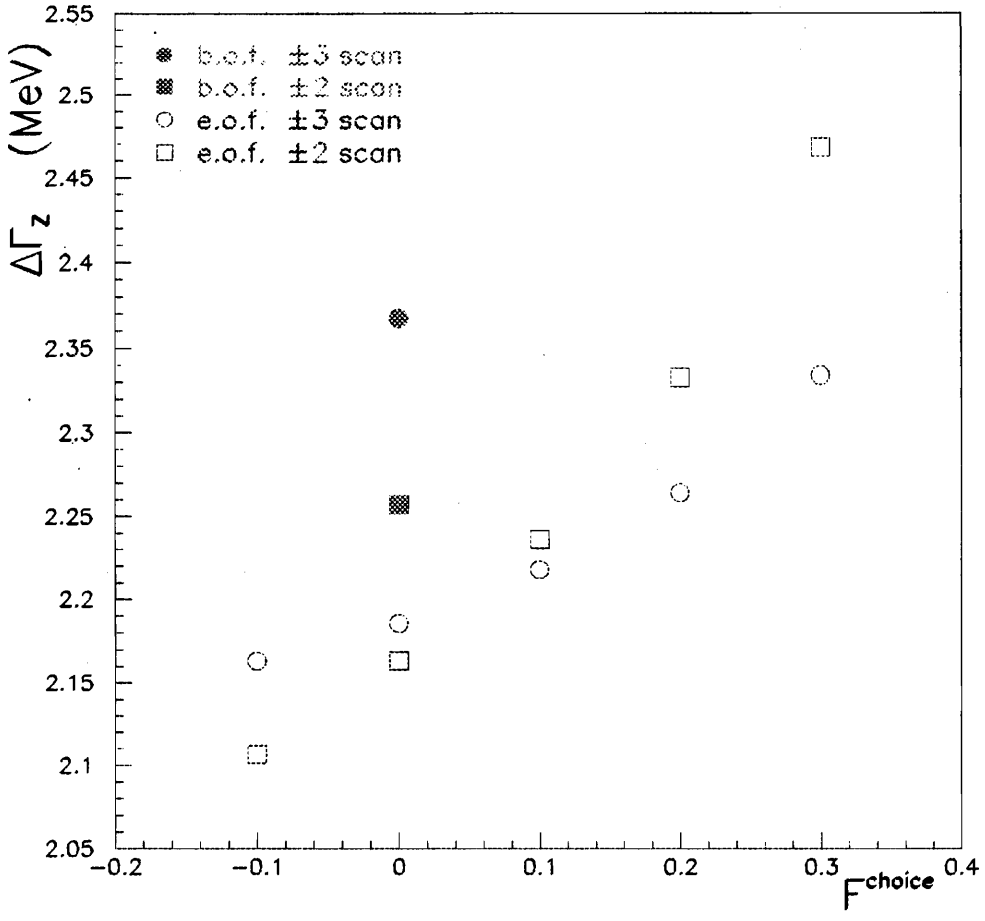

Figure 1: Variation of the error on Γ_Z as a function of F^{choice}, the fraction of fills that by choice are not calibrated. Results are given for scans at ± 2GeV and ± 3GeV. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}$=30pb^{-1}. The average luminosity per fill is assumed to be 0.8pb^{-1}.

$\mathcal{L}_{cost}^{on-peak}$=30pb^{-1} $\mathcal{L}^{fill}$= 0.8pb^{-1}

Scan at ±2GeV							
F^{choice}	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	$\Delta(\partial A_{FB}/\partial E)$ (GeV^{-1})	N^{fill}	$\mathcal{L}^{off-peak}$ (pb^{-1})	$\mathcal{L}^{cal}$ (pb^{-1})	F^{uncal}	$\Delta\Gamma_Z^{scat}$ (MeV)
-.1	2.11 = 1.89 ⊕ .92	.00355	24.7	39.5	5.9	.35	.78
.0	2.16 = 1.87 ⊕ 1.08	.00352	25.2	40.3	5.4	.43	.96
.1	2.24 = 1.85 ⊕ 1.25	.00348	25.7	41.1	4.9	.51	1.15
.2	2.33 = 1.83 ⊕ 1.44	.00344	26.3	42.0	4.3	.58	1.35
.3	2.47 = 1.81 ⊕ 1.67	.00341	26.8	42.9	3.8	.66	1.60
b.o.f.	2.26 = 2.20 ⊕ .50	.00413	18.2	29.2	12.2	.00	.00
Scan at ±3GeV							
F^{choice}	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	$\Delta(\partial A_{FB}/\partial E)$ (GeV^{-1})	N^{fill}	$\mathcal{L}^{off-peak}$ (pb^{-1})	$\mathcal{L}^{cal}$ (pb^{-1})	F^{uncal}	$\Delta\Gamma_Z^{scat}$ (MeV)
-.1	2.16 = 2.06 ⊕ .66	.00355	20.6	32.9	4.9	.35	.57
.0	2.19 = 2.04 ⊕ .78	.00352	20.9	33.5	4.5	.43	.70
.1	2.22 = 2.03 ⊕ .90	.00349	21.3	34.1	4.0	.51	.84
.2	2.26 = 2.01 ⊕ 1.05	.00346	21.7	34.7	3.6	.58	.99
.3	2.33 = 1.99 ⊕ 1.22	.00343	22.1	35.3	3.1	.66	1.17
b.o.f.	2.37 = 2.34 ⊕ .33	.00404	15.9	25.4	10.6	.00	.00

Table 2: Variation of the error on Γ_Z and other related quantities as a function of F^{choice}, the fraction of fills that by choice are not calibrated. Energy calibration at the end of fill is assumed except for the row labelled 'b.o.f.', which corresponds to the case where all fills are calibrated at the beginning of fill, before physics running. For Γ_Z the three errors given are, respectively, the total error, the LEP-combined statistical error and the error arising from the uncertainty in the LEP energy scale. The residual scatter of the energy values measured by resonant depolarization with respect to the model makes a contribution to the systematic error of $\Delta\Gamma_Z^{scat}$. (See text for the meaning of all the other symbols.) Results are given for scans at ±2GeV and ±3GeV. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}$=30pb^{-1}. The average luminosity per fill is assumed to be 0.8pb^{-1}.

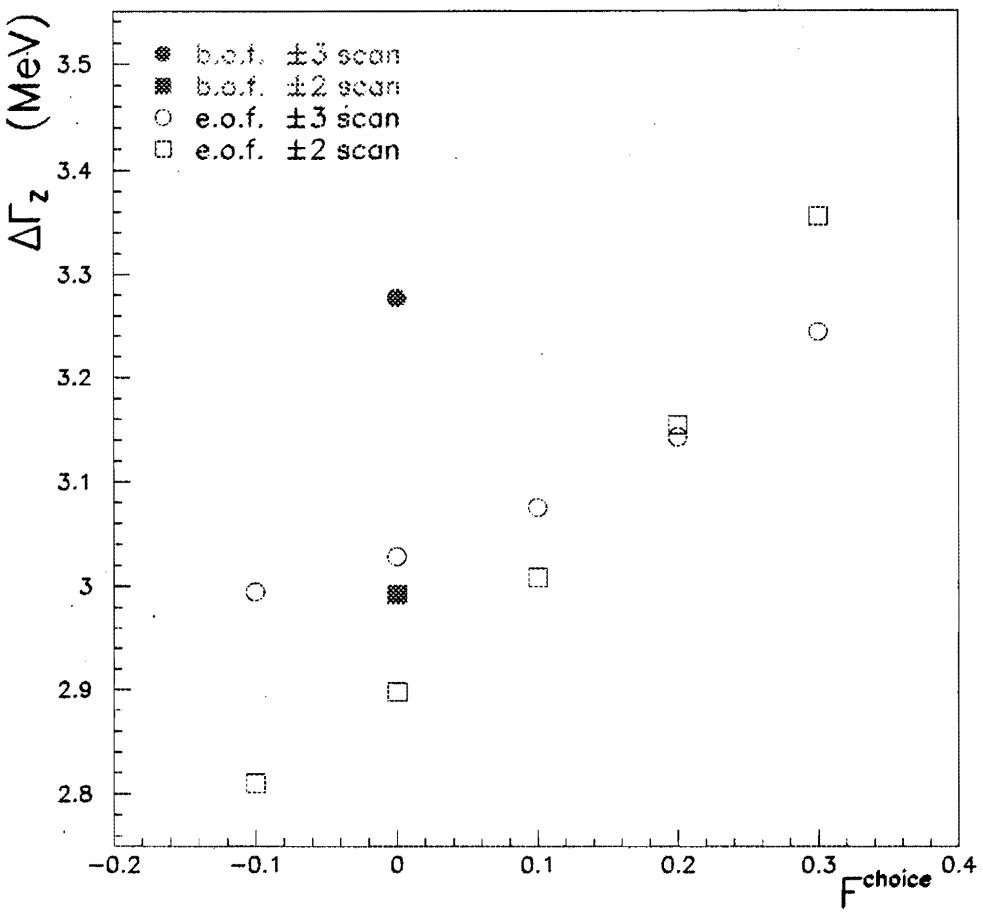

Figure 2: Variation of the error on Γ_Z as a function of F^{choice}, the fraction of fills that by choice are not calibrated. Results are given for scans at ± 2GeV and ± 3GeV. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}=15pb^{-1}$. The average luminosity per fill is assumed to be $0.8pb^{-1}$.

$\mathcal{L}_{cost}^{on-peak}$=15pb^{-1} $\mathcal{L}^{fill}$= 0.8pb^{-1}

Scan at ±2GeV							
F^{choice}	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	$\Delta(\partial A_{FB}/\partial E)$ (GeV^{-1})	N^{fill}	$\mathcal{L}^{off-peak}$ (pb^{-1})	$\mathcal{L}^{cal}$ (pb^{-1})	F^{uncal}	$\Delta\Gamma_Z^{scat}$ (MeV)
-.1	2.81 = 2.54 ⊕ 1.21	.00502	12.3	19.7	2.9	.35	1.10
.0	2.90 = 2.51 ⊕ 1.44	.00497	12.6	20.1	2.7	.43	1.36
.1	3.01 = 2.49 ⊕ 1.70	.00492	12.8	20.6	2.4	.51	1.62
.2	3.15 = 2.46 ⊕ 1.98	.00487	13.1	21.0	2.2	.58	1.91
.3	3.36 = 2.43 ⊕ 2.31	.00482	13.4	21.5	1.9	.66	2.26
b.o.f.	2.99 = 2.95 ⊕ .50	.00584	9.1	14.6	6.1	.00	.00
Scan at ±3GeV							
F^{choice}	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	$\Delta(\partial A_{FB}/\partial E)$ (GeV^{-1})	N^{fill}	$\mathcal{L}^{off-peak}$ (pb^{-1})	$\mathcal{L}^{cal}$ (pb^{-1})	F^{uncal}	$\Delta\Gamma_Z^{scat}$ (MeV)
-.1	2.99 = 2.87 ⊕ .87	.00502	10.3	16.5	2.5	.35	.80
.0	3.03 = 2.84 ⊕ 1.05	.00497	10.5	16.7	2.2	.43	.99
.1	3.08 = 2.82 ⊕ 1.23	.00493	10.6	17.0	2.0	.51	1.19
.2	3.14 = 2.79 ⊕ 1.44	.00489	10.8	17.3	1.8	.58	1.40
.3	3.24 = 2.77 ⊕ 1.69	.00484	11.0	17.7	1.5	.66	1.66
b.o.f.	3.28 = 3.26 ⊕ .33	.00570	8.0	12.7	5.3	.00	.00

Table 3: Variation of the error on Γ_Z and other related quantities as a function of F^{choice}, the fraction of fills that by choice are not calibrated. Energy calibration at the end of fill is assumed except for the row labelled 'b.o.f.', which corresponds to the case where all fills are calibrated at the beginning of fill, before physics running. For Γ_Z the three errors given are, respectively, the total error, the LEP-combined statistical error and the error arising from the uncertainty in the LEP energy scale. The residual scatter of the energy values measured by resonant depolarization with respect to the model makes a contribution to the systematic error of $\Delta\Gamma_Z^{scat}$. (See text for the meaning of all the other symbols.) Results are given for scans at ±2GeV and ±3GeV. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}$=15pb^{-1}. The average luminosity per fill is assumed to be 0.8pb^{-1}.

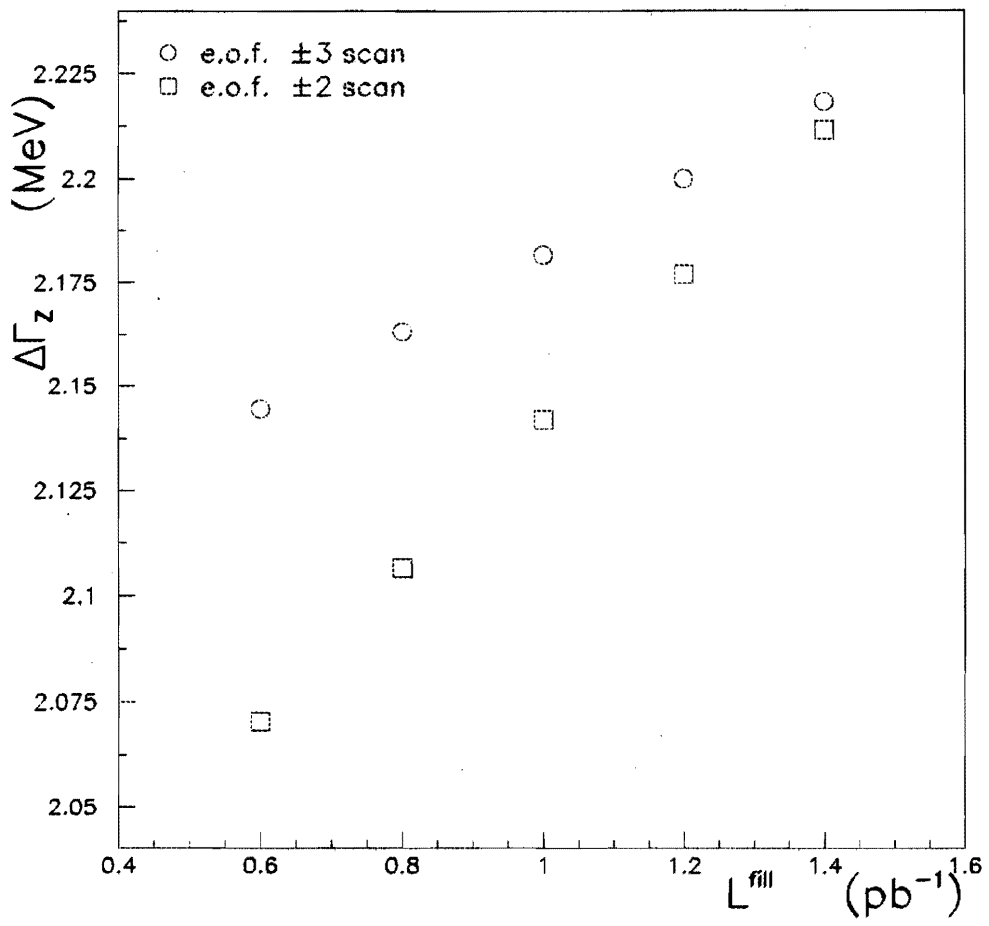

Figure 3: Variation of the error on Γ_Z as a function of $\mathcal{L}^{fill}$, the average luminosity collected per fill. Results are given for scans at $\pm$2GeV and $\pm$3GeV. Energy calibration at the end of fill is assumed. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}$=30pb^{-1}and a value of F^{choice}=0.0.

$\mathcal{L}_{cost}^{on-peak} = 30.pb^{-1}$ $F^{choice}=0.0$

Scan at $\pm$2GeV			
$\mathcal{L}^{fill}$ (pb^{-1})	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	N^{fill}	$\Delta\Gamma_Z^{scat}$ (MeV)
.6	$2.11 = 1.87 \oplus .97$	33.6	.83
.8	$2.16 = 1.87 \oplus 1.08$	25.2	.96
1.0	$2.22 = 1.87 \oplus 1.18$	20.1	1.07
1.2	$2.27 = 1.87 \oplus 1.28$	16.8	1.17
1.4	$2.32 = 1.87 \oplus 1.36$	14.4	1.27
Scan at $\pm$3GeV			
$\mathcal{L}^{fill}$ (pb^{-1})	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	N^{fill}	$\Delta\Gamma_Z^{scat}$ (MeV)
.6	$2.16 = 2.04 \oplus .69$	27.9	.61
.8	$2.19 = 2.04 \oplus .78$	20.9	.70
1.0	$2.21 = 2.04 \oplus .85$	16.7	.78
1.2	$2.24 = 2.04 \oplus .92$	14.0	.86
1.4	$2.27 = 2.04 \oplus .99$	12.0	.93

Table 4: Variation of the error on Γ_Z and other related quantities as a function of $\mathcal{L}^{fill}$, the average luminosity collected per fill. For Γ_Z the three errors given are, respectively, the total error, the LEP-combined statistical error and the error arising from the uncertainty in the LEP energy scale. (See text for the meaning of all the other symbols.) Results are given for scans at $\pm$2GeV and $\pm$3GeV. Energy calibration at the end of fill is assumed. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}$=30pb^{-1}and a value of F^{choice}=0.0.

$\mathcal{L}_{cost}^{on-peak}$=15.pb^{-1} F^{choice}=0.0

Scan at ±2GeV			
$\mathcal{L}^{fill}$ (pb^{-1})	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	N^{fill}	$\Delta\Gamma_Z^{scat}$ (MeV)
.6	2.82 = 2.51 ⊕ 1.28	16.8	1.17
.8	2.90 = 2.51 ⊕ 1.44	12.6	1.36
1.0	2.98 = 2.51 ⊕ 1.60	10.1	1.52
1.2	3.05 = 2.51 ⊕ 1.73	8.4	1.66
1.4	3.13 = 2.51 ⊕ 1.86	7.2	1.79
Scan at ±3GeV			
$\mathcal{L}^{fill}$ (pb^{-1})	$\Delta\Gamma_Z$ (MeV) (tot.) (stat.) (syst.)	N^{fill}	$\Delta\Gamma_Z^{scat}$ (MeV)
.6	2.99 = 2.84 ⊕ .92	14.0	.86
.8	3.03 = 2.84 ⊕ 1.05	10.5	.99
1.0	3.07 = 2.84 ⊕ 1.16	8.4	1.11
1.2	3.11 = 2.84 ⊕ 1.26	7.0	1.21
1.4	3.15 = 2.84 ⊕ 1.35	6.0	1.31

Table 5: Variation of the error on Γ_Z and other related quantities as a function of $\mathcal{L}^{fill}$, the average luminosity collected per fill. For Γ_Z the three errors given are, respectively, the total error, the LEP-combined statistical error and the error arising from the uncertainty in the LEP energy scale. (See text for the meaning of all the other symbols.) Results are given for scans at ±2GeV and ±3GeV. Energy calibration at the end of fill is assumed. All results correspond to a total cost $\mathcal{L}_{cost}^{on-peak}$=15pb^{-1}and a value of F^{choice}=0.0.

Off-peak energies	$\mathcal{L}_{cost}^{on-peak}$ (pb^{-1})	
	30	15
±2GeV	1.77	2.12
±3GeV	1.79	2.17

Table 6: The expected final error on Γ_Z (MeV) for the combined 1990-1995 dataset for various choices for the off-peak energy points and the total cost of the 1995 scan. The results correspond to the values F^{choice}=0.0 and $\mathcal{L}^{fill}$=0.8pb^{-1}.

the systematic error is uncorrelated with that expected for the 1995 scan. Table 6 gives the expected final error on Γ_Z (MeV) for the combined 1990-1995 dataset for various choices for the off-peak energy points and the total cost of the 1995 scan. It can be seen that the small differences in the expected errors for 1994–1995 between ±2GeV and ±3GeV are rendered even more negligible once the combination with the earlier data is made.

8 Conclusions

A more refined study of questions concerned with the LEP energy calibration has confirmed the result obtained in reference [1] that the accuracy of the final Γ_Z measurement is essentially independent of whether off-peak data are collected in 1995 at ±2GeV or ±3GeV. This conclusion is insensitive to the variations made in the assumptions about the 1995 energy calibration.

From the practical point of view of the energy calibration, collecting data at ±3GeV offers some advantages. The same Γ_Z error can be obtained with a smaller amount of off-peak luminosity and requires a smaller number of energy calibrations. The shorter the scanning period the easier it should be to guarantee the required stability of the machine; residual (i.e., not understood) fluctuations in the energy scale are also likely to be smaller. In addition, the scan at ±3GeV is less sensitive to the assumptions made about the success rate for calibrations and the residual fluctuations in the energy scale.

The physics advantages a scan at ±3GeV have been discussed in reference [1].

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Appendix

The contribution of uncalibrated fills to the uncertainty in the luminosity weighted average energy of the off-peak scan points is derived, taking into account the fact that all fills do not have equal integrated luminosity. (This result was quoted as equation 3 in section 4.)

Consider a collection of N fills with integrated luminosities $\mathcal{L}_i$ and energies E_i. The luminosity weighted average energy, $\overline{E}$ is given by:

$$\overline{E} = \frac{\sum_i \mathcal{L}_i E_i}{\sum_i \mathcal{L}_i}$$

If the N fills are uncalibrated and the energies E_i must be obtained from a model that predicts the energy with an accuracy σ_{scat} then the uncertainty on $\overline{E}$ is given by:

$$(\Delta\overline{E})^2 = \sigma^2_{\text{scat}} \sum_i \left(\frac{d\overline{E}}{dE_i}\right)^2 = \sigma^2_{\text{scat}} \frac{\sum_i \mathcal{L}_i^2}{(\sum_i \mathcal{L}_i)^2} = \frac{\sigma^2_{\text{scat}}}{\text{N}} \left\{1 + \left(\frac{\sigma_{\mathcal{L}}}{\overline{\mathcal{L}}}\right)^2\right\} \tag{5}$$

where the last step follows from the definitions:

$$\sigma^2_{\mathcal{L}} = \frac{\sum_i \mathcal{L}_i^2}{N} - \left(\frac{\sum_i \mathcal{L}_i}{\text{N}}\right)^2$$

$$\overline{\mathcal{L}} = \frac{\sum_i \mathcal{L}_i}{\text{N}}$$

The value of $(\Delta\overline{E})^2$ is therefore larger than the naive expectation by a factor:

$$K = 1 + \left(\frac{\sigma_{\mathcal{L}}}{\overline{\mathcal{L}}}\right)^2$$

For fills that reach the end of physics the value $\frac{\sigma_{\mathcal{L}}}{\overline{\mathcal{L}}}$=0.45 is assumed, which gives the value K^{eof}=1.20. It is assumed that the luminosity of fills lost before the end of physics follows a probability distribution that is flat between zero and the value expected if the fill had not been lost. Such fills have an average luminosity = $\overline{\mathcal{L}}/2$ and the variance recieves an extra contribution of $(\overline{\mathcal{L}}^2/12)/(\overline{\mathcal{L}}^2/2^2)$=1/3. For lost fills the relevant amplification factor for the scatter error is therefore given by:

$$K^{lost} = 1.33 + \left(\frac{\sigma_{\mathcal{L}}}{\overline{\mathcal{L}}}\right)^2$$

Combining all classes of fills at a particular off-peak energy point, the overall luminosity weighted average energy is given by:

$$\overline{E} = \frac{\overline{E}_{cal}\mathcal{L}_{cal} + \overline{E}_{eof}\mathcal{L}_{eof} + \overline{E}_{choice}\mathcal{L}_{choice} + \overline{E}_{lost}\mathcal{L}_{lost}}{\mathcal{L}_{cal} + \mathcal{L}_{eof} + \mathcal{L}_{choice} + \mathcal{L}_{lost}}$$

where for a particular class j of fills, $\overline{E}_j$ is the luminosity weighted average energy and $\mathcal{L}_j$ is the total integrated luminosity. The subscript refers to the following classes of fills: calibrated, lost during an energy calibration, by choice not calibrated, and lost during physics running.

The contribution of the uncalibrated fills to the uncertainty in the average energy is then given by:

$$(\Delta\overline{E})^2 = \frac{1}{\mathcal{L}_{tot}}\left(\Delta\overline{E}^2_{eof}\mathcal{L}^2_{eof} + \Delta\overline{E}^2_{choice}\mathcal{L}^2_{choice} + \Delta\overline{E}^2_{lost}\mathcal{L}^2_{lost}\right)$$

where

$$\mathcal{L}_{tot} = \mathcal{L}_{cal} + \mathcal{L}_{eof} + \mathcal{L}_{choice} + \mathcal{L}_{lost}$$

Using equation 5 this can be rewritten as:

$$(\Delta\overline{E})^2 = \frac{1}{\mathcal{L}_{tot}}\left(\frac{\sigma^2_{scat}}{N^{eof}}K^{eof}\mathcal{L}^2_{eof} + \frac{\sigma^2_{scat}}{N^{choice}}K^{eof}\mathcal{L}^2_{choice} + \frac{\sigma^2_{scat}}{N^{lost}}K^{lost}\mathcal{L}^2_{lost} + \frac{\sigma^2_{scat}}{N^{cal}}(\mathcal{L}_{eof} + \mathcal{L}_{choice} + \mathcal{L}_{lost})^2\right) \tag{6}$$

where, for a particular class j, N_j is the number of fills. The last term in the above expression takes into account the uncertainty with which the average energy scale is measured by means of the calibrated fills.

The number of fills in each class is given by:

$$N^{lost} = N^{fill}F^{lost}$$

$$N^{eof} = N^{fill}\left(1 - F^{lost}\right)F^{eof}$$

$$N^{choice} = N^{fill}\left(1 - F^{lost}\right)F^{choice}$$

$$N^{cal} = N^{fill}\left(1 - F^{lost}\right)\left(1 - F^{eof} - F^{choice}\right)$$

The luminosity weighted average energy for each class is given by:

$$\mathcal{L}_{lost} = N^{lost}\overline{\mathcal{L}}/2; \quad \mathcal{L}_{eof} = N^{eof}\overline{\mathcal{L}}; \quad \mathcal{L}_{choice} = N^{choice}\overline{\mathcal{L}}$$

which then gives:

$$\mathcal{L}_{tot} = N^{fill}\left(1 - F^{lost}/2\right)\overline{\mathcal{L}}$$

Substituting for these quantities into equation 6 then gives:

$$(\Delta\overline{E})^2 =$$

$$\frac{\sigma^2_{\mathrm{scat}}}{\mathrm{N^{fill}}\,(1-\mathrm{F^{lost}}/2)^2}\left\{\mathrm{K^{eof}}(\mathrm{F^{eof}}+\mathrm{F^{choice}})(1-\mathrm{F^{lost}})+\frac{\mathrm{K^{lost}F^{lost}}}{4}+\frac{\{(\mathrm{F^{eof}}+\mathrm{F^{choice}})(1-\mathrm{F^{lost}})+\mathrm{F^{lost}}/2\}^2}{(1-\mathrm{F^{eof}}-\mathrm{F^{choice}})(1-\mathrm{F^{lost}})}\right\}$$

which corresponds to equation 3 in section 4.