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A pointing theorem for tracks in a cylindrically symmetric magnetic field

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Abstract

It is shown, (provided energy loss can be ignored), that the trajectories of charged particles, starting on the axis of rotational symmetry of a magnetic field (such as the solenoidal configurations used in several detectors at e^+e^- colliders) must point back at this axis when they emerge from the field. Thus the final direction of the track contains no momentum information.

Introduction

Many of the detectors [1] at e^+e^-, $\bar{p}p$, pp and ep colliders, employ a solenoidal magnetic field. Usually there is a central cylindrical volume centered on the collision point with an almost uniform longitudinal magnetic field which is used for tracking and this is surrounded by an iron return yoke which is used for hadron calorimetry and absorption. Muons are identified by their ability to penetrate the iron yoke and are detected outside in a field free region. Charged particles are bent in one direction in the central field and the other way in the return yoke. Measurements in the central region and before, inside and after the yoke are used to match track segments and to determine momentum. It is not immediately obvious whether or not the bend in the iron is significant; whether it can be ignored or whether it is as big as the bend in the central region.

In this paper we show that, except for the effects of energy loss, the bend due to the return flux exactly compensates for the bend in the central region in the sense that the emerging particles point back to the axis of symmetry. The conditions for this to apply are not as stringent as one might first expect. The magnetic field must have rotational symmetry, but it need not be forward-backward symmetric nor need the azimuthal component be zero. Furthermore the particle can start anywhere on the symmetry axis.

Proof of the Theorem

Firstly, we give a short proof for a simplified situation. Suppose the particle starts at the origin and moves in the $z = 0$ plane, where B_r and B_θ are zero. Let the magnetic field, B_z, depend on radius but not on θ. Beyond some radius, R, the field is zero, and the total flux inside including the return is zero, i.e.

$$\int_0^R B(r) r dr = 0. \tag{1}$$

In a practical situation, there might be a solenoidal field out to some radius with the magnetic field along z and beyond this a return yoke with the field along $-z$, all within R. As the particle moves, along its trajectory, from radius r to $r + dr$ (P to Q in figure 1) it is bent through an angle

$$d\phi = -A\ B(r)\ \frac{dr}{\cos\alpha}$$

where $dr/\cos\alpha$ is the path length in the field. α is the angle the trajectory makes with the radius vector i.e. $\alpha = \theta - \phi$. A is a constant that depends on the charge and momentum of the particle. The impact parameter, s, is given by

$$s = r\ \sin\alpha,$$

and the change in the impact parameter, as the particle moves from P to Q, is

$$ds = \sin\alpha\ dr + r\ \cos\alpha\ d\theta - r\ \cos\alpha\ d\phi.$$

The sum of the first two terms is zero and substituting for $d\phi$ gives

$$ds = A\ B(r) r dr.$$

The total change in s in going from the origin ($r = 0$) to the field free region outside the magnet ($r = R$) is

$$\Delta s_{(0\to R)} = A\int_0^R B(r)r\,dr$$

and this is zero because the total flux (equation 1) is zero. The impact parameter is zero at the origin and so it is again for $r \geq R$ i.e.

once outside the field the track points at the origin.

Secondly, we give a proof of the general case. Consider the trajectory $r(t), \theta(t), z(t)$ of a charged particle moving in a magnetic field. The angular momentum, L_z, about the z axis is $mr^2\dot{\theta}$ and

$$\frac{dL_z}{dt} = m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta}).$$

Here m is the relativistic mass, γm_0, which is constant because the magnetic forces do not alter the speed. The change in angular momentum in going from point C to point D is

$$\Delta L_{z(C\to D)} = m\int_C^D r(r\ddot{\theta} + 2\dot{r}\dot{\theta})dt$$

where the integration is along the trajectory. The term in brackets is the azimuthal acceleration and is given by the corresponding component of the Lorentz force, $\mathbf{F} = e\mathbf{v}\wedge\mathbf{B}$, so:

$$\Delta L_{z(C\to D)} = e\int_C^D r(\dot{z}B_r - \dot{r}B_z)dt,$$

which is proportional to

$$2\pi\int_C^D r(B_r dz - B_z dr). \qquad (2)$$

Let C be the start of the trajectory, any point on the z axis, i.e. $r = 0$, and let D be where the trajectory intercepts, S, a closed surface, enclosing the point C and on which and outside of which the field is zero.
The surface of revolution generated by rotating the trajectory around the z axis divides the volume defined by S into two parts. The total magnetic flux coming out of each of these two sub-volumes is zero, (unless of course there are magnetic monopoles lurking inside somewhere!). The part through S is by definition zero. Thus the flux through the surface of revolution of the trajectory is also zero. If $B_r(r,\theta,z)$ and $B_z(r,\theta,z)$ are independent of θ this is given by equation 2. Thus there is no change in angular momentum in going from C to D, and as it is zero at the start it must be zero also at D i.e. at the exit from the field $\dot{\theta}$ is zero and the track points at the origin in the r-θ plane.

A proof using the Lagrangian formalism is quite elegant and is left as an exercise.

Discussion

Figure 2 illustrates a trajectory (in the $z = 0$ plane) for the simple case of a uniform central field, followed by a field free region and surrounded by a return yoke again with uniform

field. An important consequence of the theorem is that the direction of the track as it emerges from the return yoke contains no momentum information. However, if two tracks of opposite sign start at the origin, back-to-back and with equal momenta, then when they emerge they are not back-to-back. The sign of the angle between them, (projected onto the z=0 plane) gives the charges of the tracks. This could be used, for instance, to determine the charges of the tracks in μ pair events.
In practice, the effects of energy loss are not always negligible. To give an example typical of the LEP detectors – a 45 GeV muon is bent by 13.3 mrads in 2 m of central field of 1 T. In the return yoke, at radii from 3.5 to 4.5 m, it is bent 3.3 mrad. If the muon loses say 2 GeV by ionisation loss in passing through a meter of iron, or equivalent, before entering the return yoke, the effect on the angle of emergence is only about 0.15 mrad which in most cases is much less than the measurement accuracy. However, a 5 GeV muon is bent 120 mrads and 30 mrad. The effect of a 2 GeV energy loss is to increase the bend in the yoke by the order of 20 mrad.

Summary

It has been proved that if a charged particle starts on the axis of rotational symmetry of any magnetic configuration, then when it emerges from the field it must necessarily point back to the axis. In the absence of energy loss this theorem is exact.

Acknowledgements

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References

1. Here reference is made only to the LEP detectors.

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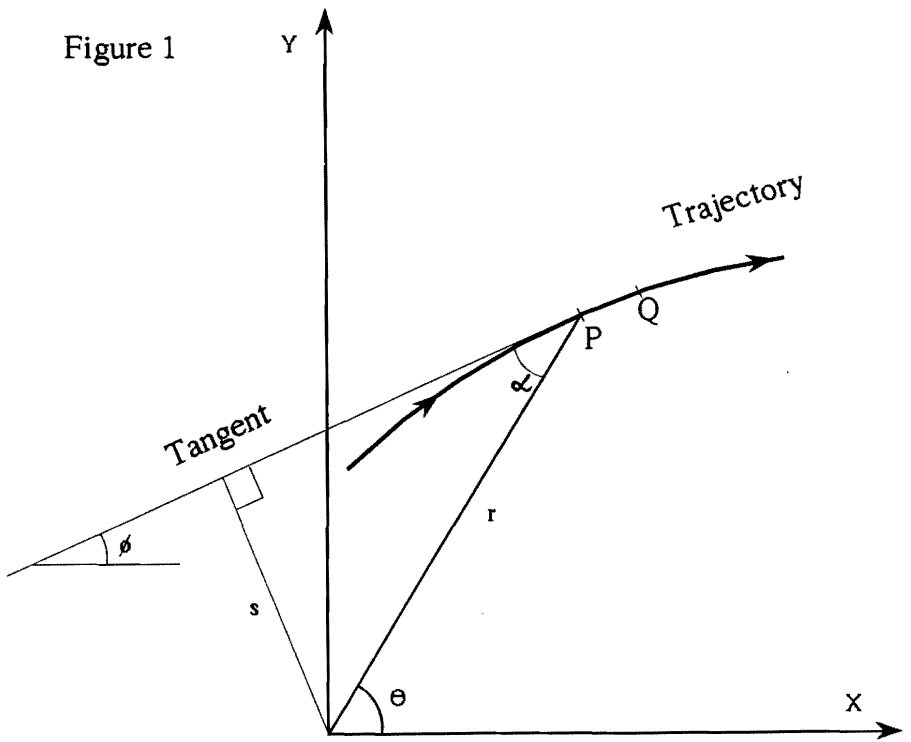

Figure 1. Geometrical parameters associated with the trajectory, in the $z = 0$ plane.

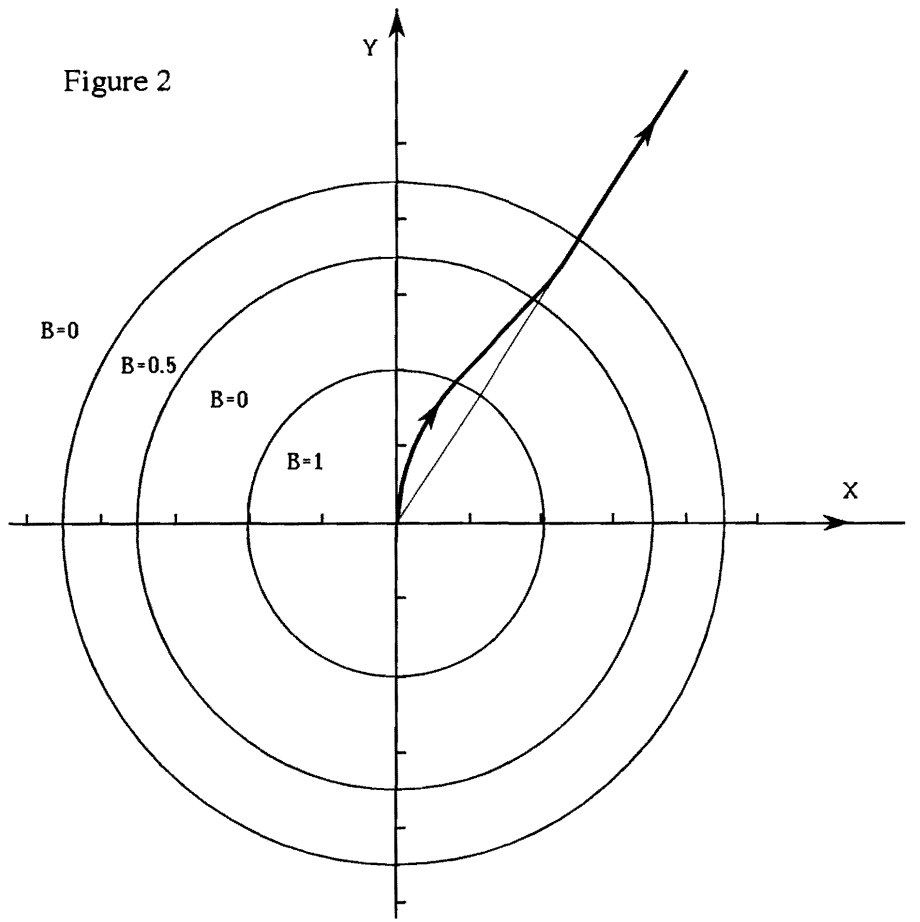

Figure 2. Illustrating the trajectory of a particle in a simple field; a uniform central field, followed by a field free region and surrounded by a return yoke again with uniform field.