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On Reducing Systematic Errors in The Determination of $\Gamma(Z^0 \to b\bar{b}, c\bar{c})$

J. Chrin

Department of Physics, University of Manchester, M13 9PL, U.K.

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Abstract. The role of fragmentation variables in unfolding the partial widths of Z^0 decays into heavy quarks from inclusive lepton analyses is studied. Investigations lead to a proposal for a new fragmentation variable which minimises the dependence of the results on the model used. A critical summary of measurements of the heavy quark semi-leptonic branching fractions from the e^+e^- continuum is presented.

Introduction

To date, measurements of the partial widths of Z^0 decays into the heavy b and c quarks, $\Gamma(Z^0 \to b\bar{b}, c\bar{c})$, at LEP have been made exclusively from analyses of inclusive leptons [1,2]. These event samples have been unfolded in terms of the product of the heavy quark fraction in multihadronic events and the heavy quark semi-leptonic branching ratios, $\Gamma\ (Z^0 \to q\bar{q})\ /\ \Gamma\ (Z^0 \to hadrons)\ .\ BR(q \to l\nu_l X)$, and the mean of the fragmentation variable $\langle x_E \rangle_q = E_{hadron}/E_{beam}$. In this paper the sensitivity of such measurements to the form of the fragmentation function used to generate the simulated data (Monte Carlo) is examined; investigations lead to a proposal for a new fragmentation variable which satisfies a number of criteria that minimise the dependence of the results on the model used. In addition, since knowledge of the semi-leptonic branching ratio of the b and c quarks is an important ingredient in obtaining $\Gamma(Z^0 \to q\bar{q})$, the latest measurements from the e^+e^- continuum are summarised.

Separation of Heavy Quark Flavours and their Fragmentation into Heavy Hadrons

The identification of leptons in final states containing hadrons remains one of the most effective methods of tagging heavy quarks and studying their subsequent fragmentation into heavy hadrons. However, any technique adopted by analysts to separate between the different quark flavours produced in e^+e^- annihilation depends heavily on the simulation, by means of a Monte Carlo program, of the processes involved. A commonly used Monte Carlo model is that of the Lund Parton Shower [3] with parameters slightly modified [4] from the default values. Whilst these parameters provide a good description of a number of global properties in the multihadronic data, they are known to give too hard a fragmentation function for b quarks [5]. This consequently leads to too hard a momentum spectrum for leptons in $b\bar{b}$ events, leading to an experimental detection efficiency for leptons which is unrealistically high. Partly to avoid this conflict, experiments revert to an alternative form of fragmentation function for b quarks, namely that of Peterson et al. [6], which has been well measured at lower energies [7]; [1] alternatively, or additionally, they choose to unfold the momentum spectra of prompt leptons in terms of the energy spectra of the primordial heavy hadrons.

To distinguish between the quark flavours, the lepton transverse momentum, $p_T(l)$, relative to some chosen axis is used. The transverse momentum measured with respect to the jet (or cluster) axis [10], as opposed to an event axis such as thrust or sphericity, has the advantage of avoiding long tails in the distribution which arise from jet broadening due to QCD, and therefore better reflects the dynamics of the mass of the heavy quark and leads to improved resolving capabilities between the quark flavour contributions to the inclusive lepton signal. Further improvements in flavour separation can be achieved by removing the lepton from its associated jet and then re-calculating the jet momenta (without the lepton), and measuring the p_T with respect to the new jet axis [2]. In the course of this analysis, all possibilities are investigated.

[1] It should be noted that the best fit parameters for the heavy quark fragmentation functions were obtained using the Matrix Element Monte Carlo and therefore require adjustment when used in the context of the parton shower [8,9]

The analyses proceed by deducing the $p(l)$ and $p_T(l)$ spectra as a function of some predefined fragmentation variable, using a sample of Monte Carlo events; then by fitting these distributions to the $p(l)$, $p_T(l)$ distribution of the data, experiments are able to measure the flavour composition of the lepton-inclusive sample and the mean value of the fragmentation variable used. Since current data are not sensitive to the detailed shape of the fragmentation function, the latter is parameterised by the form proposed by Peterson et al. which has the advantage of having just one free parameter, ϵ_q, for each heavy flavour, q:

$$f(z) \propto \frac{1}{z[1 - \frac{1}{z} - \frac{\epsilon_q}{(1-z)}]^2} \ . \tag{1}$$

The strict definition of the fragmentation variable, z, is the fractional energy-momentum taken by the primordial hadron from what is available. However, over the years, z has acquired a number of definitions, which have since come under scrutiny [7,11]. Since a careless choice of 'z' can lead to model-dependent results [12] it is as well to outline the reasons for the differences.

In the framework of the string fragmentation Monte Carlo model the fragmentation process occurs along the colour-flux lines which stretch between the partons. The energy available to the first rank hadron is the energy of the fragmenting system which is not a quantity that can be accessed from the generated four-vectors of the event. A common approximation, adopted in analyses employing the Matrix Element Monte Carlos, has been to define the reconstructed 'z' by:

$$z(rec) \ = \frac{(E + p_{||})_{hadron}}{(E + p)_{quark}} \tag{2}$$

where the energy and momentum components of the quark, and of the hadron (w.r.t. the quark direction), are reconstructed from the generated four-vectors. However, whilst this approximation may work well with Matrix Elements, in Parton Showers the available energy is often more than just that of the quark and there are many cases where the fraction z is greater than unity. In a Lund Jetset Monte Carlo sample [3] with parameters of ref. [4], about 20%(10%) of $c(b)$ events are found to have $z(rec)$ values exceeding unity at energies near the Z^0 resonance. Given that the choice of fragmentation function converges for $z \to 1$, it is not clear how best to classify those events with $z(rec) > 1$.

These problematic events can however be eliminated by using an alternative definition for the fragmentation variable used in (1), namely x_E, originally used in D^* analyses:

$$x_E = \frac{E_{hadron}}{E_{beam}} \ . \tag{3}$$

Since the energy of the primary hadron will always be less than that of the e^+ or e^- beam, the value of x_E will not exceed unity. Furthermore, the x_E spectrum of primary hadrons does, in itself, have fundamental meaning. Unfortunately, the $\langle p_T(l) \rangle$ distribution as a function of x_E is found to be sensitive to the effects of bremsstrahlung radiation, particularly that due to QCD as, by definition, two-jet (multi-jet) events mainly occupy high (low) bins of x_E. [2]

[2] This is important as leptons in multi-jet events tend to have a broader p_T spectra than in two-jet events. This difference is, however, minimised by measuring p_T w.r.t the jet axis rather than the thrust axis

Furthermore, the shapes of the distributions are dependent on the form (and parameters) of the fragmentation function used to generate the events. This is particulary significant for direct leptons from b decay as illustrated in Fig. 1.

The optimum choice of fragmentation variable is dictated by the requirements that the variable will rarely exceed values of unity, that it will as far as possible be insensitive to radiation effects, and that the resulting shape of the $\langle p_T(l)\rangle$ distribution as a function of the variable will be independent of the variable's functional form. These requirements are crucial to ensure the correct unfolding procedure necessary to get from the lepton spectrum to the heavy quark spectrum. In an effort to achieve this goal, a modified definition [13] is introduced which, in addition to the energy of the primary quark, takes into account the fact that energy from the neighbouring partons in the string will also contribute to the 'available energy':

$$z_\theta = E_{hadron} \;/\; (E_{quark} + \sum_i \frac{1}{(1+\theta_{qg_i})}.E_{g_i}(\theta_{qg_i} < \theta(max))) \;, \qquad (5)$$

where θ_{qg_i} is the angle between the directions of the primary quark and the neighbouring gluon, g_i, and $\theta(max)$ is some arbitrary cut-off. Gluons closer in θ to the quark direction therefore contribute a greater fraction of their energy to the 'available energy'. The variation of $\langle p_T(l)\rangle$ and $\langle p(l)\rangle$ as a function of z_θ is shown in Fig. 2 for $b \to \mu$ events. The figures illustrate that the requirement that the 'z' dependence of the flavour discriminator (i.e. $p_T(l)$) be relatively independent of the functional form of the primordial fragmentation variable, is better satisfied for z_θ than for x_E; the sensitivity to QCD radiation is likewise lesser for z_θ than for x_E as the multi-jet fraction is not biased towards a particular region of 'z'. The number of events having 'z' > 1 is also reduced to about 2(5)% for $c(b)$ events when using z_θ (cf. 20(10)% using $z(rec)$). These could be classed with events in the highest 'z' bin. Furthermore, by replacing the term $(1+\theta_{qg_i})$ in (5) by $(1+k.\theta^n_{qg_i})$ and adjusting the constants k and n, (and $\theta(max)$), the resulting fragmentation variable can be moulded to give a reasonable representation of the primordial fragmentation variable, $z(pri)$, as shown in Fig. 3.

The difference in variation between the lepton distributions as a function of z_θ and x_E is mainly attributed to the effects of gluon emission. With regards to the transverse lepton momentum distribution, this difference is partly minimised by measuring the momentum transverse to the jet (cluster) axis, as opposed to the thrust axis. Nevertheless, whatever the axis chosen, the use of the variable x_E does have a dependence on the model used (at least for $b \to l$ events) and therefore could act to cause interference between the separation of quark flavours. This dependence is largely removed with z_θ; in fact the most stable behaviour is seen when $p_T(l)$ is measured w.r.t. the thrust axis; and although this new variable is of limited relevance, having derived its mean value from a fit to the data, the Monte Carlo model can then be used to translate it to the more meaningful $\langle x_E\rangle$.

The Heavy Quark Semi-leptonic Branching Ratios

As already stated, the rate of inclusive leptons in multihadronic events is indicative of the product $\Gamma(Z^0 \to q\bar{q})/\Gamma(Z^0 \to hadrons)\,.\,BR(q \to l\nu_l X)$. Knowledge of the semi-leptonic branching fractions of heavy quarks is therefore a vital ingredient in obtaining a measurement for the partial width $\Gamma(Z^0 \to b\bar{b}, c\bar{c})$. The semi-leptonic branching fractions of heavy quarks in the e^+e^- continuum has been extensively measured at PEP and PETRA energies [12]. A prerequisite of these analyses is that $b\bar{b}$ ($c\bar{c}$) production in multihadronic events is dominated by single photon exchange and occurs with a probability of 0.09 (0.39), after correcting for initial state radiation. The latest results [14] are summarised in Table 1 (for both b and c quarks). The world averages are 0.117 ± 0.008 for b and 0.079 ± 0.009 for c. These measurements are averages over all heavy hadrons weighted by their relative production rates. Their inclusion in the product $\Gamma(Z^0 \to q\bar{q}).\,BR(q \to l\nu_l X)$ assumes that the mix of b- and c- flavoured hadrons on the Z^0 resonance is similar to that in the e^+e^- continuum. The world average for b compares with the $\Upsilon(4S)$ determination of 0.102 ± 0.007 [15]. Here the number of $B\overline{B}$ events is determined by subtracting the continuum contribution from the data collected at the resonance energy. The difference in results suggests that the b-flavoured hadrons produced in the continuum are of a different type and/or are produced in a different proportion as on the $\Upsilon(4S)$ resonance (where only B_u and B_d mesons are produced in a ratio of roughly 50:50). Therefore at LEP, use of the value 0.117 ± 0.008 should be preferred to 0.102 ± 0.007. Use of the inappropriate value leads to a systematic shift of +13% in measurements of $\Gamma(Z^0 \to b\bar{b})$.

Conclusion

The use of the fragmentation variable, x_E, in inclusive lepton analyses has been investigated. The p_T spectra of leptons from b decay as a function of x_E have been shown to depend on the form (and parameters) of the fragmentation function used to generate the Monte Carlo events. A new fragmentation variable has been introduced which minimises the model-dependence of the $p_T(l)$ spectra as a function of 'z', and consequently enables experiments to quote model-independent results. The latest experimental results on the semi-leptonic branching ratios of the heavy quarks from the e^+e^- continuum have been summarised.

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Table 1: Semi-leptonic branching ratios (PEP/PETRA). In calculating the world averages, electron-muon universality is assumed, and a common systematic error of $\pm 0.8\%$ ($\pm 0.6\%$) for $c(b)$ quarks is taken.

Expt	E_{CM}	l	$BR(c \to l\nu_l X)$ (%)	$BR(b \to l\nu_l X)$ (%)
MARK J	37	μ	$8.8 \pm 0.7 \pm 1.1$	$12.4 \pm 1.3 \pm 2.0$
TASSO	34.5	μ	$8.2 \pm 1.2 \pm^{2}_{1}$	$11.7 \pm 2.8 \pm 1$
TASSO	34.6	e	$9.2 \pm 2.2 \pm 4.0$	$11.1 \pm 3.4 \pm 4.0$
JADE	34.6	μ	$7.8 \pm 1.5 \pm 2$	$11.7 \pm 1.6 \pm 1.5$
CELLO	35	μ, e	$7.1 \pm 0.3 \pm 0.9$	$14.9 \pm 0.7 \pm 1.9$
CELLO	43	μ, e	$10.4 \pm 0.8 \pm 2.1$	$10.7 \pm 1.8 \pm 2.1$
MAC	29	μ	9 ± 3	$12.3 \pm 1.8 \pm^{1.7}_{1.3} \pm 0.8$
MAC	29	e	8 ± 3	$11.3 \pm 1.9 \pm 3.0$
DELCO*	29	e	$11.6 \pm^{1.1}_{0.9}$	$14.9 \pm^{2.2}_{1.9}$
TPC*	29	μ	$6.9 \pm 1.1 \pm 1.1$	$15.2 \pm 1.9 \pm 1.2$
TPC	35	e	$9.1 \pm 0.9 \pm 1.3$	$11.0 \pm 1.8 \pm 1.0$
MARK II	29	μ	$7.8 \pm 0.9 \pm 1.2$	$11.8 \pm 1.2 \pm 1.0$
MARK II	29	e	$9.6 \pm 0.7 \pm 1.5$	$11.2 \pm 0.9 \pm 1.1$
HRS	29	e	$7.6 \pm 1.1 \pm 0.7$	$10.8 \pm 0.8 \pm 1.3$
World Average	-	-	7.9 ± 0.9	11.7 ± 0.8

* Not included in world average calculation. See [12].

Fig. 1 $b \to \mu$. Plotted as a function of fragmentation variable x_E are: (a) $\langle p_T(\mu)\rangle$ w.r.t. the thrust axis, (b) $\langle p_T(\mu)\rangle$ w.r.t. the jet axis (without muon), (c) $\langle p_T(\mu)\rangle$ w.r.t. the jet axis (with muon), and (d) $\langle p(\mu)\rangle$. The filled points were generated using Jetset 7.2 with the Peterson fragmentation function ($\epsilon_b = 0.03$, see reference [8]). The open points were generated using Jetset 7.2 with the Lund fragmentation function ($a = 0.18$, $b = 0.34$).

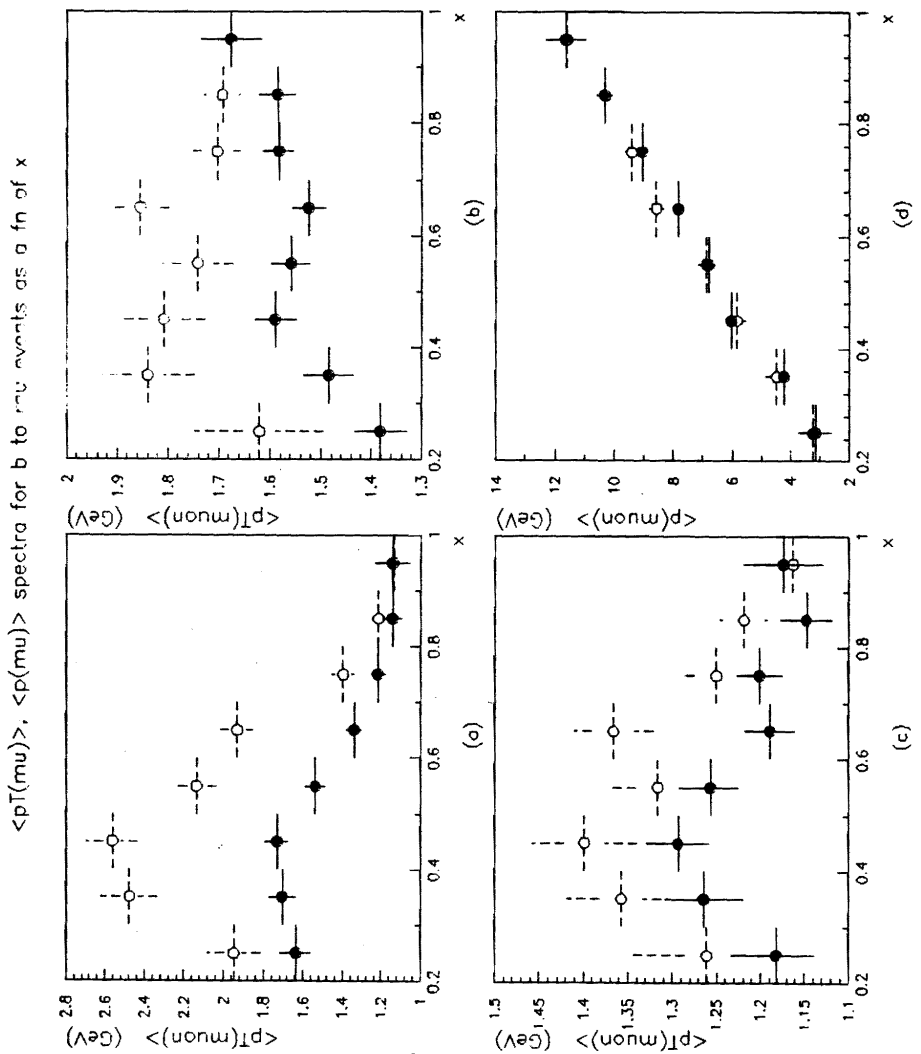

Fig. 2 $b \to \mu$. As Fig. 1 but as a function of fragmentation variable z_θ with $\theta(max) = 0.3$. Note the change in scales compared to Fig. 1.

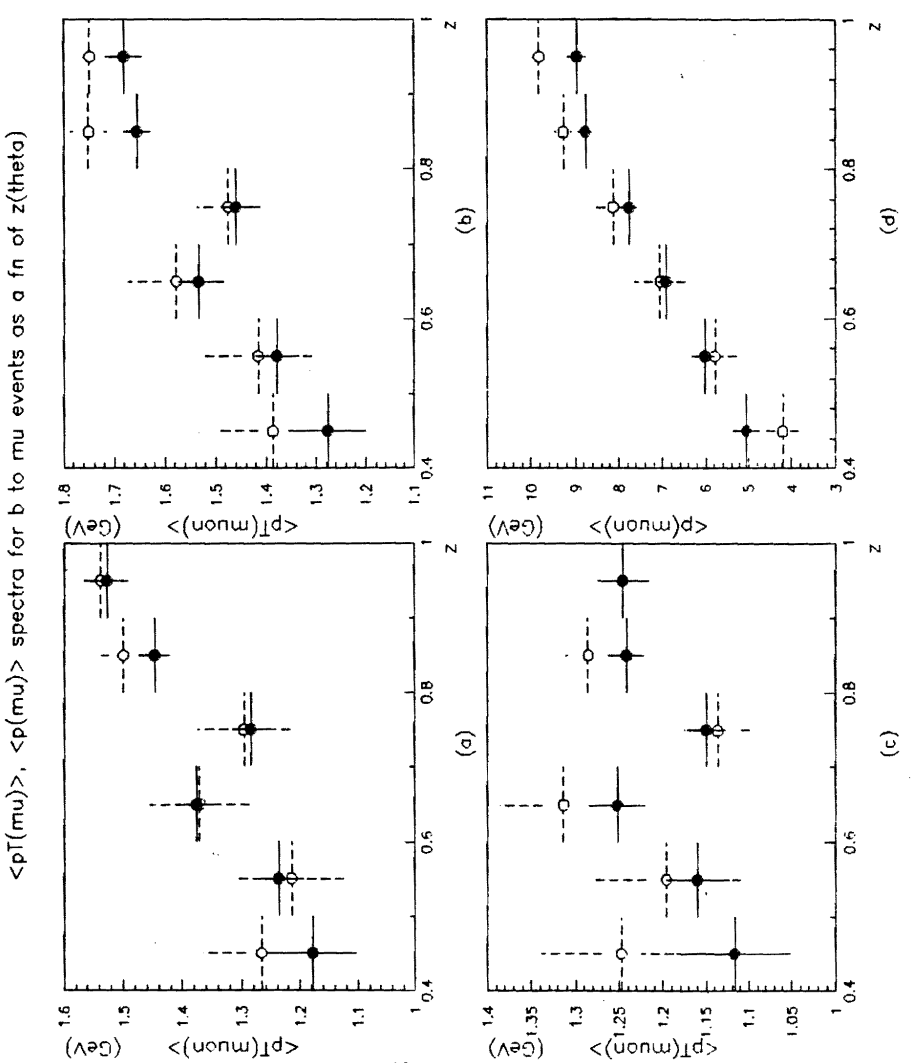
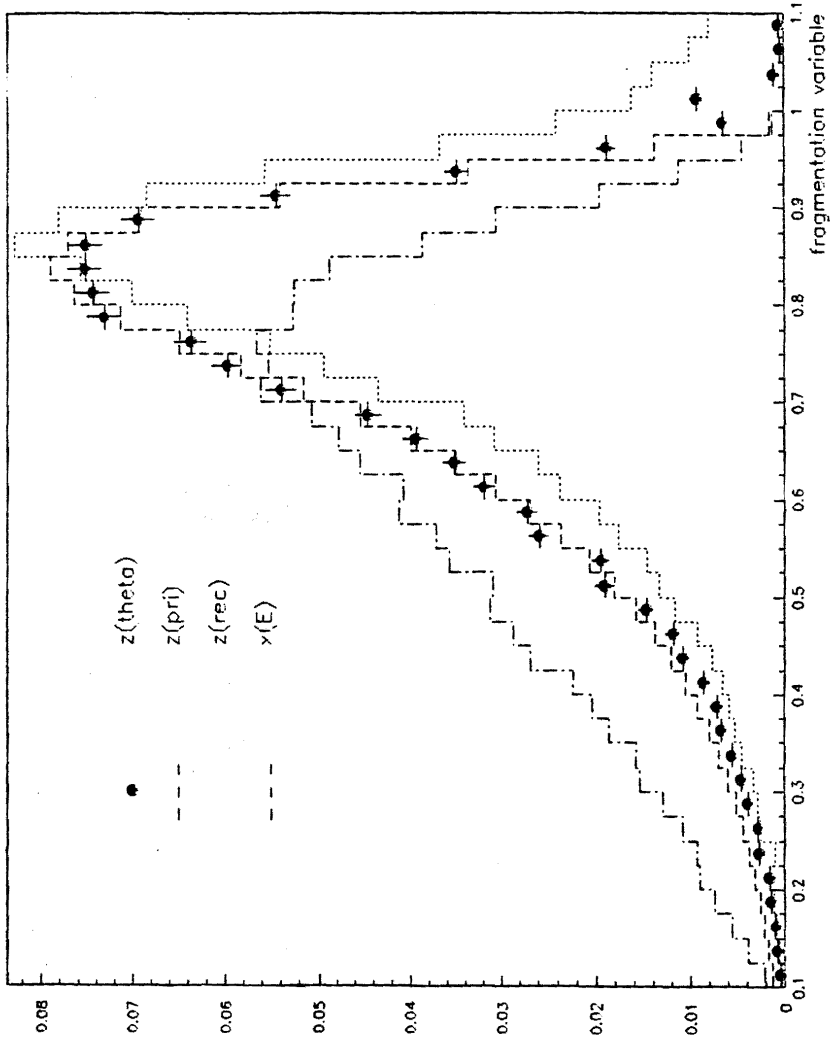
Fig. 3. The analytical fragmentation function of Peterson et al. with ϵ_b=0.03 (dashed line) compared with the reconstructed z_θ distribution (filled points) (for k=10, n=3 and $\theta(max)$=0.6 - constants only grossly tuned) from Lund Jetset 7.2 events generated using the Peterson fragmentation function with ϵ_b=0.03 . The short-dashed line shows the resulting $z(rec)$ spectrum and the long-short-dashed line the x_E spectrum.