M/C-TH 94/01

Total Cross Sections in Electron-Positron Annihilation

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Abstract

Results from a previous analysis of e^+e^- annihilation data are combined to produce total cross sections for the ρ-like, ω-like and ϕ-like channels. It appears that duality is remarkably well satisfied on average, and that by an energy of 1.8 to 2.0 GeV the cross section is "asymptotic" i.e. it is given simply by $e^+e^- \to q\bar{q}$.

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1 Introduction

The data on e^+e^- annihilation have been analysed up to energies of 2.0 GeV, with the aim of identifying the $J^P = 1^-$ meson states produced and studying their properties. A summary and update of these analyses can be found in [1].

It is instructive to combine the analyses of these various channels to produce total cross sections for the ρ-like, the ω-like and ϕ-like channels, and to present these in terms of the ratio, R, to the cross section for $e^+e^- \to \mu^+\mu^-$ at the same energy. Naively the simple quark-parton model of e^+e^- annihilation in which a quark-antiquark pair is produced predicts values of R:

selection	R expected
ρ-like	1.50
ω-like	0.17
ϕ-like	0.33

Here "ρ-like" has been defined formally as the $q\bar{q}$ combination $(u\bar{u} - d\bar{d})/\sqrt{2}$, "$\omega$-like" as the $q\bar{q}$ combination $(u\bar{u} + d\bar{d})/\sqrt{2}$ and "ϕ-like" as simply $s\bar{s}$. In practice, ρ-like events are selected as $G = +1$ states of non-strange mesons; ω-like events are selected as $G = -1$ states of non-strange mesons; and ϕ-like events are selected as those containing strange mesons. As there is a finite probability of the ρ-like and ω-like states picking up an $s\bar{s}$ pair from the vacuum there will be some leakage from the ρ-like and ω-like events into the ϕ-like events, and hence the apparent ϕ-like cross section will be somewhat larger than anticipated. This point is returned to below. It should also be noted that higher order QCD corrections increase the predicted values of R (by about 14% at 2.0 GeV), but the results presented here are not of such an accuracy as to make this significant.

2 ρ-like states

In analysing the measurements of the decays of ρ-like states it is necessary to use the classification developed by Pais [2] in terms of the permutation symmetries of the isotopic spin wave function. This classifies $n\pi$ states in terms of partitions of

the integer n into three, or fewer, integers. For 4π states with I = 1 there are two states corresponding to the partitions [31] and [211]. For 6π states there are four states corresponding to the partitions [51], [411], [33], and [321]. Wave functions corresponding to different partitions are orthogonal, so results must be analysed in terms of the intensities of these states.

The explicit channels considered for the ρ-like cross section are $e^+e^- \to \pi^+\pi^-$; $\omega\pi$; 4π in the [31] isospin state; 4π in the [211] isospin state, other than $\omega\pi$; $\eta\pi^+\pi^-$ and 6π. There is considerable uncertainty attached to the interpretation of the 6π system as data are available only for production of the $\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$ states, and nothing is known about production of the $\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$ state [3]. The dominant contribution to the 6π final state is known to be either in the isotopic spin state [411], when the total 6π cross section would be 2.5 times that observed for production of $\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$ state, or in the isotopic spin state [321] for which the total 6π cross section would, in fact, be equal to that for $\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$ [1]. In general there will be some incoherent mixture of these two possibilities, so that the 6π cross section lies between 1 and 2.5 times that for $\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$. Although grounds have been presented [3] that the [411] state dominates at lower 6π masses, where the interest of the present work lies, we consider the range of possibilities between these two extremes.

Figure 1 shows the resulting values of R separately for production of the six different states discussed. For clarity on this figure the larger estimate for production of 6π states is shown i.e. it is assumed to be dominated by the [411] state. A lower limit on the 6π contribution is obtained by multiplying this curve by a factor of 0.4.

Figure 2 shows the results of combining all the individual contributions to produce the total ratio for ρ-like states, R_ρ, for energies greater than 1 GeV. The two curves in Fig.2 are obtained on the basis of two extreme assumptions about the cross section for production of 6π states:

(A) The 6π contribution is obtained from the observed cross sections by assuming that the [411] state dominates.

(B) The 6π contribution is obtained from the observed cross sections by assuming that the [321] state dominates.

The cross section should therefore lie between the limits indicated by A and B.

As already indicated, the simple quark-parton model predicts a value of 1.5 for R_ρ for ρ-like states, which is close to the value found experimentally for masses greater than $\sim$ 1.4 GeV, further the average values of R_ρ, calculated over various ranges, are impressively close to the prediction of 1.5:

Average		Data set	
From	to	A	B
0.28 GeV	1.8 GeV	1.605	1.547
0.28 GeV	1.4 GeV	1.620	1.618
1.4 GeV	1.8 GeV	1.540	1.331

The lower mass range in these averages is completely dominated by the ρ. Thus duality [4] "on average" is remarkably well satisfied, and it is tempting to assert that by $\sim$ 1.8 to 2.0 GeV the cross section is "asymptotic" i.e. it is given simply by $e^+e^- \to q\bar{q}$.

3 ω-like states

The only available data for production of ω-like states are $\pi^+\pi^-\pi^0$, which is almost entirely $\rho\pi$, and $\omega\pi^+\pi^-$. The former is in the [111] isotopic spin state and the latter is in the [311] isotopic spin state which has the combination of charge states:

$$\frac{2}{3}\omega\pi^+\pi^- + \frac{1}{3}\omega\pi^0\pi^0.$$

The separate values of R for production of $\pi^+\pi^-\pi^0$ and for production of $\omega\pi\pi$, together with their sum, R_ω, are presented in Fig.3. The total R_ω shows a behaviour similar to that for the ρ-like states, rising steeply with energy and reaching the quark-parton model prediction at an energy of about 1.5-1.6 GeV. However, the experimental value of R_ω for the ω-like states does not stay at the predicted value but falls again with increasing energy. This is not surprising when one recalls that for ρ-like states the value of R_ρ is maintained at an approximately constant level by the contribution from the 6π channels. No data exist for the corresponding reaction $e^+e^- \to 5\pi$ and it seems reasonable to conclude that these data, if available, could make up the deficit.

4 ϕ-like states

The selection of these states is somewhat less certain, as the only identifier is to require the final state to include two mesons of opposite strangeness. There is then the presumption that such states produced in e^+e^- annihilation will have $I = 0$. However, non-strange $q\bar{q}$ states can produce two mesons of opposite strangeness with a finite probability, introducing an admixture of $I = 1$. For example, it has been shown in [1] that there is a definite $I = 1$ contribution to $e^+e^- \to K\bar{K}$, demonstrating explicitly a leakage of ρ-like events into ϕ-like events.

In addition to the $K\bar{K}^* + \bar{K}K^*$ and $K\bar{K}$ channels discussed in detail in [1], data are available for $e^+e^- \to K^+K^-\pi^+\pi^-$ [5]. However as this is the only $K\bar{K}\pi\pi$ charge state for which data exist there is a very severe problem of determining the factor by which one should multiply the cross section found for this particular charge state to give the total cross section summed over all charge states. Plotting a $K\pi$ mass spectrum [5] for cases where K and π have opposite charge shows that approximately one such $K\pi$ pair per event results from $K^*(892)$ decay. Hence this pair must have $I = \frac{1}{2}$. The other neutral $K\pi$ pair is presumably in an s-wave state and should also have $I = \frac{1}{2}$ (excluding exotic contributions) and thus the total $K\bar{K}\pi\pi$ system can be in either $I = 0$ or $I = 1$ isospin states. If the $K^+K^-\pi^+\pi^-$ system is in a pure $I = 0$ state then the factor by which the $K^+K^-\pi^+\pi^-$ cross section must be multiplied to give the ϕ-like $K\bar{K}\pi\pi$ contribution is 4.5. If the $K^+K^-\pi^+\pi^-$ system is in a pure $I = 1$ state then the factor by which the cross section must be multiplied is again 4.5, but in this case it must be transferred to the ρ-like cross section and there is no contribution to the ϕ-like cross section. However if the $K\bar{K}\pi\pi$ system is in both isospin states and there is interference between them then the multiplicative factor can be anything. Clarification can only come from other data, particularly on $K^0\bar{K}^0\pi^+\pi^-$. Without this there is complete uncertainty on the $K\bar{K}\pi\pi$ contribution to the ϕ-like (and to the ρ-like) cross section. The separate and summed values of R for strange final states are shown in Fig. 4, where we have taken a multiplicative factor of 4.5 as an illustration for the $K\bar{K}\pi\pi$ channel.

When the asymptotic regime is reached, and it has been suggested above that this is probably by 2.0 GeV, then states formed initially as $u\bar{u}$ or $d\bar{d}$ can hadronise by picking up an $s\bar{s}$ pair from the vacuum, with a relative probability with respect to the latter of some 30%. Thus there will be a leakage of events from what has been defined as the ρ-like and ω-like samples, primarily from the former, into the ϕ-like

sample, resulting in an almost equal mix of $I = 1$ and $I = 0$ final states with strange mesons. (This is much less likely at the lower energies which are resonance dominated, as there the overlap of the initial state and final state meson wave functions is the crucial ingredient.) If there is indeed an approximately equal mixture of $I = 1$ and $I = 0$ isospin final states in the $K\bar{K}\pi\pi$ channel then the $K\bar{K}\pi\pi$ contribution to the ϕ-like cross section, and hence the total ϕ-like cross section, will be appreciably less than that shown in Fig.4. There will also be a relatively small addition to the ρ-like cross sections of Fig.2.

5 Summary

The data are consistent with the value of R reaching its asymptotic quark-parton value somewhat below 1.8 to 2.0 GeV. The results for the ρ-like events demonstrate this most clearly, despite the uncertainty in 6π production and some leakage of events into the ϕ-like sector. The ω-like cross section provides reasonable support for this hypothesis, but has the uncertainty of the lack of data on 5π production. The ϕ-like cross section is much less well defined because of the problem associated with $K\bar{K}\pi\pi$ production but is not inconsistent with this conclusion. If the data do reach the quark-parton value at about 1.8 to 2.0 GeV, then above this energy individual channels will be dominated by the onset of successive thresholds masking any underlying resonance structure.

References

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Figure Captions

1. The ratio of the cross sections for individual channels in the ρ-like sector of e^+e^- annihilation to the cross section for $e^+e^- \to \mu^+\mu^-$. The 6π contribution shown represents the upper limit, as explained in the text. A lower limit for this contribution is obtained by multiplying the curve shown by 0.4.

2. The total ratio, R_ρ, of the summed cross sections of Fig.1 to the cross section for $e^+e^- \to \mu^+\mu^-$. The upper and lower curves indicated by A and B correspond respectively to including the upper and lower limits of the 6π contribution as explained in the text.

3. The ratio of the cross sections for individual channels in the ω-like sector of e^+e^- annihilation to the cross section for $e^+e^- \to \mu^+\mu^-$ and the total ratio, R_ω, summed over the individual channels.

4. The ratio of the cross sections for individual channels in the ϕ-like sector of e^+e^- annihilation to the cross section for $e^+e^- \to \mu^+\mu^-$ and the total ratio, R_ϕ, summed over the individual channels. The $K\bar{K}\pi\pi$ contribution and the total are indicative values only as explained in the text.

Fig.1

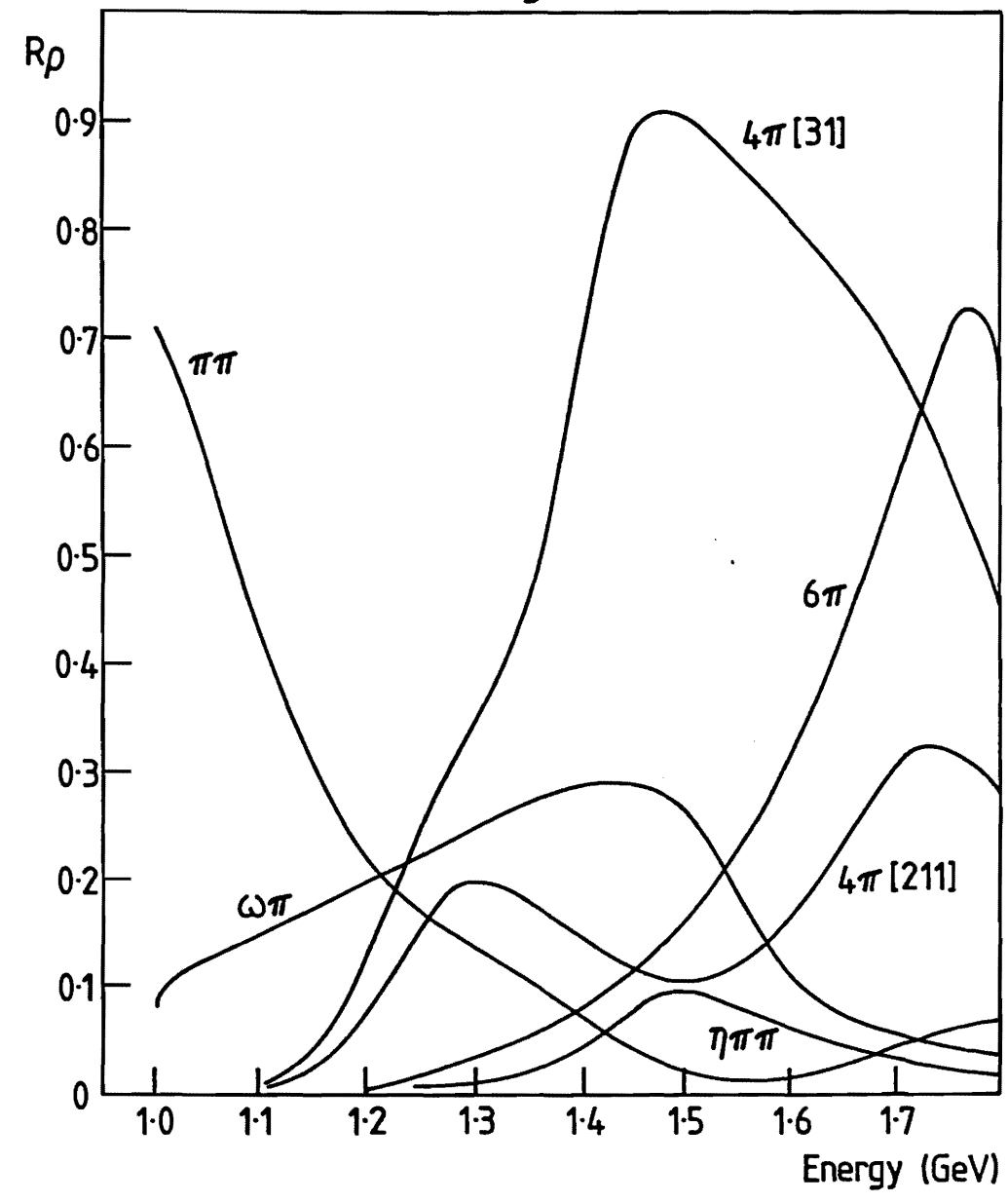

Fig. 2

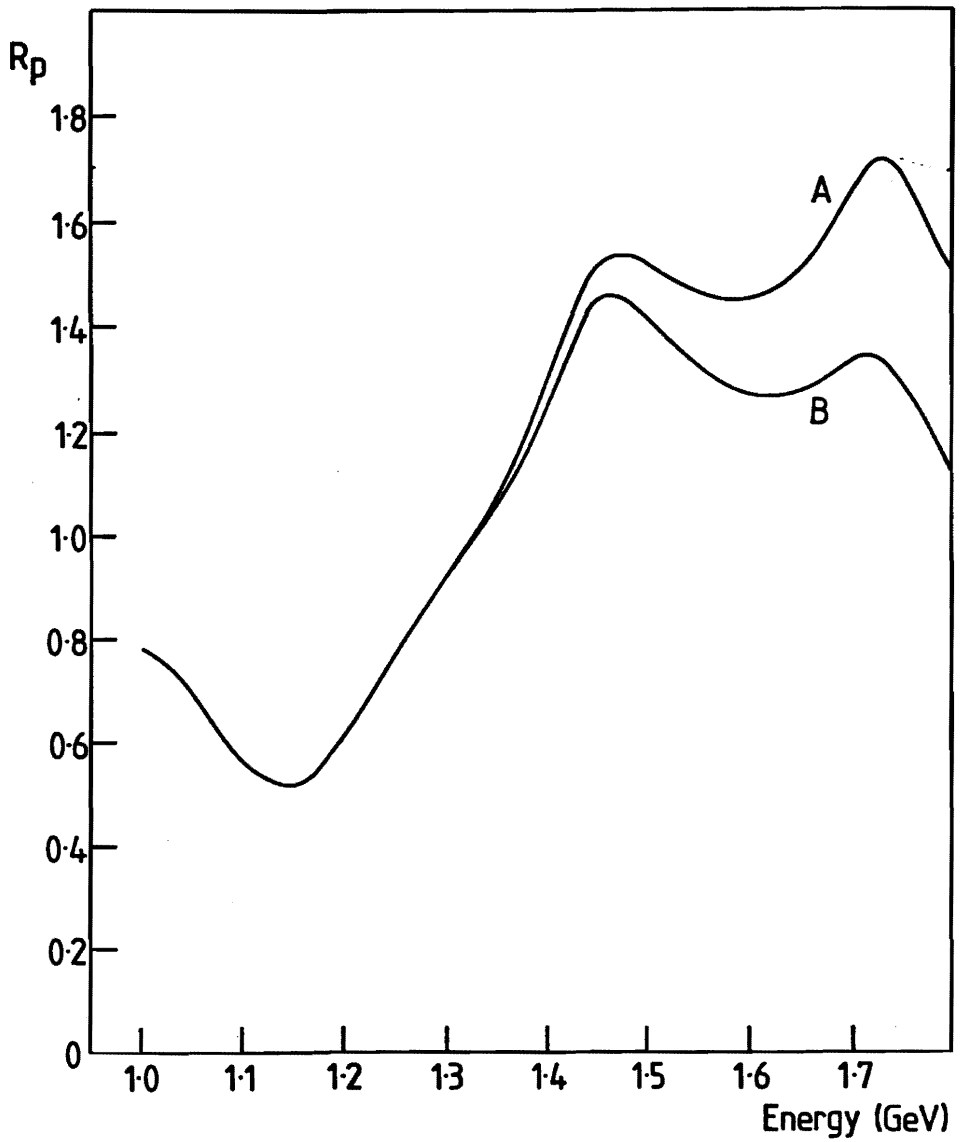

Fig. 3

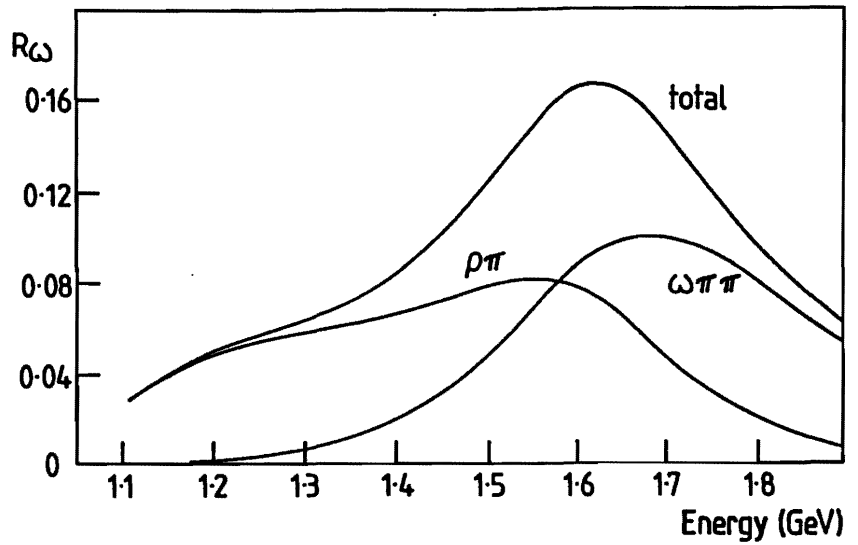
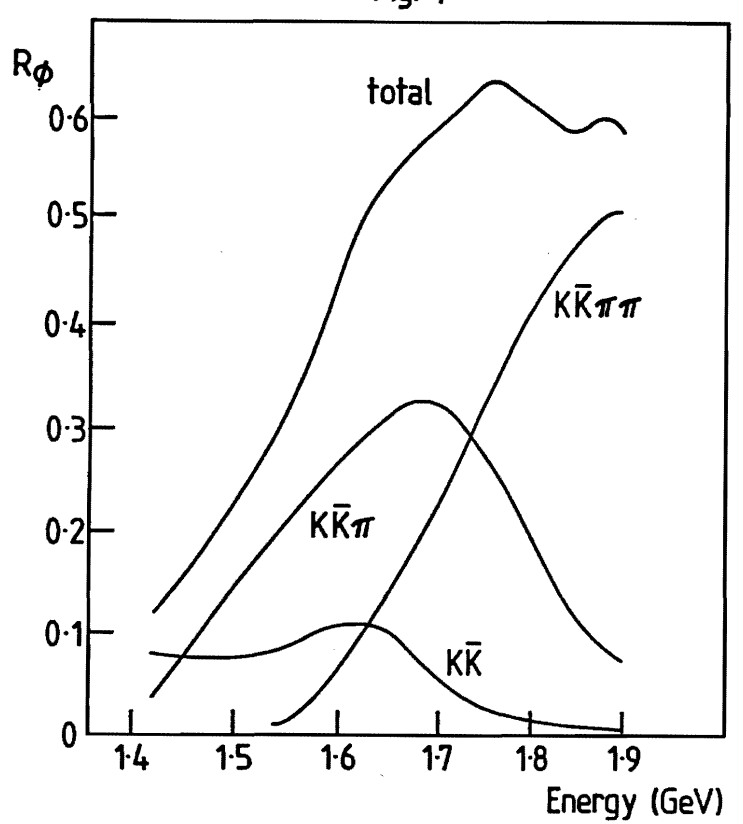
Fig. 4