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On the electromagnetic field generation in the universe

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Abstract

We show that Dolgov's result on electromagnetic fields amplification by the cosmical gravitational field in an inflationary phase can be equivalently obtained by the direct coupling of the electromagnetic field to the inflaton. The two processes can be competitive in some particular cases.

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1 INTRODUCTION

In a recent paper [1] A.D.Dolgov has considered the electromagnetic amplification in an inflationary stage of the evolution of the Universe as a consequence of the breaking of conformal invariance. He uses the quantum conformal anomaly modifications of Maxwell 's equations which leads to the following:

$$F^{\mu\nu}_{,\mu} + \alpha F^{\mu\nu}\frac{A_{,\mu}}{A} = 0. \tag{1}$$

where α is a constant, $A = A(\eta)$ is the scale factor and η the conformal time.

In the spatially homogeneous and isotropic FRW background, the metric takes the form $ds^2 = A^2(\eta)(d\eta^2 - d\sigma^2)$. Choosing the Lorentz gauge $W^{\mu}_{,\mu} = 0$ and $W^0 = 0$ and making a standard Fourier decomposition of the vector potential he gets the fundamental driving equation

$$W'' + k^2 W + \alpha\frac{A'}{A}W' = 0. \tag{2}$$

where the symbol $'$ means derivative with respect to the conformal time η. In the case of inflationary Universe studied by Dolgov, the scale logarithmic derivative takes the value

$$\frac{A'}{A} = -\frac{1}{\eta}.$$

As pointed out by Dolgov himself, a non-minimal coupling of the curvature scalar to the electromagnetic field of the type $\epsilon R W^{\mu} W_{\mu}$ can also be used to provide for an alternative mechanism of breaking conformal invariance and consequently as a competitive procedure.

Here we show that a much simpler program can be used to get exactly the same result, without the intermediate inflationary state.

For that purpose let us introduce a direct coupling of the inflaton $\Phi(x)$ field with the W^{μ} electromagnetic field in a **flat** spacetime background. The density Lagrangian takes the form:

$$L = \frac{1}{2}(\Phi_{,\mu})^2 - V(\Phi) - \frac{1}{4}F^2 + \frac{\lambda}{4}\Phi F^2. \tag{3}$$

For simplicity sake we consider here the $V(\Phi) = 0$ case. The Euler-Lagrange equations of the motion are

$$\eta^{\mu\nu}\Phi_{,\mu,\nu} - \frac{\lambda}{4}F^{\mu\nu}F_{\mu\nu} = 0. \tag{4}$$

$$F^{\mu\nu}_{,\nu} - \frac{\lambda}{1-\lambda\Phi}\Phi_{,\nu}F^{\mu\nu} = 0. \tag{5}$$

Let us follow the Dolgov 's choice for the gauge and use $W^{\mu}_{,\mu} = 0$ and $W^0 = 0$. We make the standard expansion of the vector potential:

$$W_i(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int d_3k[\omega_i(k^j)W(t)exp(-ik.x)] + h.c. \tag{6}$$

We now use the first excited homogeneous state of the scalar field by setting $\Phi = \gamma + \beta t$, with β and γ constants. This means that we take the electromagnetic field as a test field in the same way as the electromagnetic field is a test field for gravity in Dolgov 's paper. This yields the following equation for $W(t)$:

$$\ddot{W} + k^2W - \frac{\lambda\beta}{1-\lambda\gamma-\lambda\beta t}\dot{W} = 0. \tag{7}$$

Introducing a new definition of time by setting $\eta = 1-\lambda\gamma-\lambda\beta t$ we arrive at the following expression for the previous equation

$$W'' + \bar{k}^2W - \frac{1}{\eta}W' = 0 \tag{8}$$

where $\bar{k}^2 = (\frac{k}{\lambda\beta})^2$.

It is exactly the Dolgov equation for the evolution of the electromagnetic field, although obtained in a different context. Thus Dolgov 's conclusions remain valid. This means that the non-minimal coupling of the Φ and W_μ fields may produce an enhancement of the electromagnetic field even when the curvature of the background geometry can be neglected or when the inflationary scenario is absent. As a consequence, it follows that a non-minimal coupling of Φ and W_μ even in the ordinary terrestrial laboratory can provide a procedure to yield an enhancement of the electromagnetic field as well as amplification of its quantum fluctuation. It is not difficult to

imagine that such a process may also occur in the interior of crystal structures submited to external electromagnetic field, the role of Φ being played by the crystal environment.

These considerations suggest that a third mechanism, e.g. the non-minimal coupling of electromagnetic field with gravity can also successfuly operates. Let us examine it.

The most general interacting Lagrangian of electromagnetic and gravitational fields compatible with charge and parity conservation and which provides the observed behavior of photons in gravitational field (e.g. null geodesics and conservation of the polarization through the optical path) is given by [2], [3]:

$$L = \sqrt{-g}[R - \frac{1}{4}F^2 + \chi RF^2]. \tag{9}$$

Since we are limiting our analysis to the electromagnetic field as a test-field, neglecting the back reaction, we must only consider the equation

$$D_\nu F^{\mu\nu} = \frac{\lambda}{1-\lambda R} R_{,\nu} F^{\mu\nu} \tag{10}$$

in which the curvature of the background geometry is provided by the matter and D_ν means covariant derivative. Just in order to exhibit an example of our previous claims let us take the simple example of a massive star, the geometry of which is given by the standard static metric

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2 \tag{11}$$

Outside the star the ν and λ coefficients are given by the Schwarzschild expressions for a mass M spherically symmetric structure; and inside the star ($r < r_0$) we take the simplest model of a constant density [4]

$$\rho = constant = \frac{3M}{4\pi r_0^3} \tag{12}$$

while the pressure is given by

$$p(r) = \lambda(r)\rho \tag{13}$$

where $\lambda(r) = \frac{U}{Q}$ is given by

$$U = -\sqrt{1 - \frac{2M}{r_0}} + \sqrt{1 - \frac{2M}{r_0}(\frac{r}{r_0})^2} \tag{14}$$

and

$$Q = 3\sqrt{1 - \frac{2M}{r_0}} - \sqrt{1 - \frac{2M}{r_0}(\frac{r}{r_0})^2} \tag{15}$$

The internal scalar curvature is given by $R_{int} = \rho - 3p$. Using these results in the non-minimal coupling equation (10) it follows that a pure static electric field $F^{01} = E(r)$ takes the form

$$E = \frac{E_0}{r^2} \frac{e^{-\frac{\nu+\lambda}{2}}}{(1 - \chi R)} \tag{16}$$

It shows that in the external region the behaviour of the electric field is the usual one (i.e, without gravity effect) $\frac{E_0}{r^2}$; while, inside the star, an enhancement of the electric field may appear depending on the three parameters: χ (the strength of the coupling), the mass M of the star and its associated radius r_0. It then follows that such an enhancement may be very important near a critical radius r_c corresponding to a scalar curvature of the order of $\frac{1}{\chi}$. The condition that r_c must be in the interior region of the star acts as a kind of selection process among all possible stars, once only for certain values of the star characteristics M and r_0 such an internal critical value occurs.

Let us point out that such a mechanism may also occur in a non-static case, which is obtained by just considering the same procedure as above and making the metric coefficients depend on time.

The same phenomenon of enhancement of an electromagnetic field occurs also in the case of a non-static expanding Universe, in a competitive way to Dolgov's mechanism. Finally one should ask if such a phenomenon was not at the origin of a preliminary phase of some cosmic objects which had at a certain phase of their history a huge electromagnetic field.

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