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RESUMMATION OF LARGE PERTURBATIVE CONTRIBUTIONS : THEORETICAL OVERVIEW*, **

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ABSTRACT

The problem of summing large perturbative contributions is considered. A theoretical review of the main results is given. Recent developments and their impact on the LEP physics are also briefly discussed.

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A particularly interesting class of processes in perturbative QCD is represented by the semi-inclusive ones. These can be defined with a sum over the final states taken with a given energy or angle resolution. This limitation corresponds to introducing, in addition to the hard scale Q^2, of a second much smaller scale Q_1^2, $\Lambda^2 << Q_1 << Q^2$. The restriction of the allowed phase space that follows spoils the inclusive nature of the reactions and has the important effect that the Kinoshita-Lee-Nauenberg Theorem[1], that assures the order-by-order cancellation of the divergences associated with the real and virtual contributions, is no longer applicable. The main consequence of this is that within the perturbative expansion do appear contributions that are now unprotected from the cancellation of the real virtual singularities under the form of logarithms of the ratio of the two, very different, scales. These logarithms are therefore large. They originate from soft and collinear configurations, and order-by-order make larger the effective coupling constant. For each power of α_s, depending on the particular kinematical configuration, a maximum of two logarithms are present. Therefore one has a class of dominant double logarithmic corrections. Less leading configurations do also appear as single logarithmic correction as well as further less dominant constant corrections. This peculiar behaviour is not new in gauge theories. It is named after Sudakov since was found by Sudakov[2] for the electron form factor and a similar form was also found in the quark form factor[3].

In order to get reliable theoretical predictions these perturbative contributions need to be resummed. For the process of the electron-positron annihilation this is, for example, the case of the initial state electromagnetic QED radiative corrections, around the peak of the Z^o resonance. The main part of the radiation emission is represented by photons that are soft with respect to a given resolution scale ΔE within the process. The shape and the size of the resonance peak is modified by these large double logarithmic corrections that have to be resummed. The particularly effective formalism of the electron and photon structure functions can be introduced to sum these corrections[4]. The "bare" Born $e^+e^- \rightarrow Z^o$ cross-section is modified by a "radiatior" that represents the initial state radiation so that the corrected cross-section may be written as

$$\sigma(s) = \int dx\, H(x, s)\, \sigma_o\left((1 - x)s\right)$$

whose x represents the ratio of the emitted photon energy with the total energy 2E and the radiatior H(x, s), given by the convolution of the electron and positron structure functions, can be written to order $O(\alpha^2)$ as $H_e(x, s) = \int_{1-x}^{1} D(z, s)\, D\left(\frac{1 - x}{z}, s\right)\frac{dz}{z}$

$$H_e(x, s) = \Delta(s)\, \beta\, x^{\beta-1} - \frac{1}{2}\beta(2-x) + \frac{1}{8}\beta^2 \left[(2-x)\left[(3\,\ell n(1-x) - \Delta\,\ell n\, x\right] - \frac{\Delta\,\ell n(1-x)}{x} - 6 + x\right]$$

where $\beta = \frac{2\alpha}{\pi}(\ell n \frac{s}{m_e^2} - 1)$ and the first term represents the exponentiated soft photon radiation contribution with the virtual correction factor $\Delta(s)$. The obtained expression can be extended to include also next-to-leading contributions and lepton-pair production to all orders[5].

The features described above are shown by a large class of processes when the semi-inclusive limit is enforced. If z defines the semi-inclusive region of the phase space. The limit $z \to 1$ represents the boundary where parton radiation is strongly inhibited.

Examples are :

- Transverse momentum distributions of the lepton pair in the Drell-Yan process as well as the $W^{\pm}$ and $Z^o p_t$ distributions in $p\bar{p}$ collisions for small transverse momenta i.e. when $\Lambda^2 << p_t^2 << Q^2$ with Q^2 the total $(\text{mass})^2$ $z = 1 - p_t^2/Q^2$,

- Higgs production at small transverse momenta by the gluon fusion mechanism,

- Drell-Yan cross-section at large values of the $z = Q^2/s \to 1$ with s total center of mass energy squared,

- Heavy quark production near threshold when $z = \rho = \frac{4m^2}{s} \to 1$,

- Deep inelastic Scattering for large values of the Bjorken variable $z = x_{Bj} \to 1$,

- e^+e^- annihilation. Energy-Energy correlation in back-to-back configurations, Thrust distributions for large Thrust values, jet distributions for small jet masses, etc.

From the point of view of the perturbative treatment of these processes. Several new questions have to be considered :

- The standard Renormalization Group approach is no longer sufficient. The additional soft logarithms need a separate treatment. Therefore new algorithms have to be developed in order to recover the perturbative expansion.

- A richer structure of the perturbative series emerges given by leading double logarithmic and non-leading single logarithmic contributions. It is therefore natural to consider classes of contributions according to their place in the logarithmic chain. Therefore the various terms are more easily classified according to the power of the logarithm $\left(\frac{\alpha_s}{\bar{\pi}}\right)^n L^m$ where m = 2n gives the leading contribution for each n, m = n the next-to-leading and m < n the entire class of non-dominant terms.

The perturbative evaluation of semi-inclusive quantities requires the summation of these large logarithmic contributions[6,7,8]. The first successful attempt of Parisi and Petronzio[9,10] to take into account the correct leading behaviour of the quark form factor, has been followed by a systematic study of the collinear combined with the infrared[11,12,13] singularity structure.

Together with the factorization of the soft radiation[14] and the infrared logarithmic structure[15]. A systematic program to calculate down to next-to-leading accuracy semi-inclusive quantities was started.

The particular case of a semi-inclusive distribution as the energy-energy correlation for hadrons near the back-to-back configuration or, equivalently, at small relative transverse momentum $Q_T^2 << Q^2$ showed that a next-to-leading evaluation of the quark form factor, which controls such configurations, is possible[16]. The final distributions in terms of the collinearity angle $\theta \leqslant 180°$ takes the form, by transforming to the impact parameter space,

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{d\cos\theta} = \frac{Q^2}{8} \int b\, db\, J_o\,(bQ \sin\theta/2) \exp\,[T(Qb)] + F(Q, \theta) \qquad (1)$$

where $F(Q, \theta)$ contains the $O(\alpha_s)$ contribution not taken into account in the exponentiated first term.

$$T(Qb) = - C_F \int_{(b_o/b)^2}^{Q^2} \frac{dq^2}{q^2} \left(A^{(1)}\, \alpha_s(q^2)\, \ell n \frac{Q^2}{q^2} + B^{(1)}\, \alpha_s(q^2) + A^{(2)} \left(\frac{\alpha_s(q^2)}{\pi}\right)^2 \ell n \frac{Q^2}{q^2} \right) \qquad (2)$$

with $b_o = 2\, e^{-\gamma E}$ and

$$A^{(1)} = 1 \quad A^{(2)} = C_A \left[\frac{67}{18} - \frac{\pi^2}{6}\right] + \frac{N_F}{2}\left(-\frac{10}{9}\right)$$

$$\text{and } B^{(1)} = -\frac{3}{2} \ . \qquad (3)$$

Here N_f is the number of the flavours and the $A^{(i)}$ and $B^{(i)}$ coefficients correspond to a perturbative expansion of the form $A = \sum_{n=0}^{\infty} A^n \left(\frac{\alpha_s}{\pi}\right)^n$ and $B = \sum_{n=0}^{\infty} B^n \left(\frac{\alpha}{\pi}\right)^n$ of the exponent. The two-loop coefficient $A^{(2)}$ allows the sum up to the next-to-leading $(\alpha_s L)^n$ logarithms. The knowledge of other coefficients $A^{(n)}$ $(n > 2)$ and $B^{(n)}$ $(n > 1)$ would permit the inclusion of further non-dominant logarithms of the form $\alpha^n\, \ell n^{n-k}$ $(k > 0)$. More generally the distribution in eq. (1) can be written as

$$\frac{\sigma(\cos\theta, Q^2)}{\sigma_o} = C(\alpha_s(Q^2))\, \Sigma(\alpha_s(Q^2), \cos\theta) + F(\cos\theta, \alpha_s(Q^2)) \qquad (4)$$

where σ_o is the Born cross-section and $C = 1 + \sum_{n=1}^{\infty} \alpha_s^n C_n$ takes into account the constant terms in the development. $\Sigma = \exp\{\sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \alpha_s^n C_n L^m\}$ gives the form factor of eq. (1) and the additional term $F = \sum_{n=1}^{\infty} \alpha_s^n f_n$ takes into account contributions that vanish for $Q_T \to 0$ i.e. in the semi-inclusive region. The universal structure of the logarithmic corrections in eq. (2) and eq. (4), is confirmed by the evaluation of the Drell-Yan p_T distribution as for example for the $W^{\pm}Z^o$ production at the $p\bar{p}$ colliders[17] with the explicit appearance of the leading and next-to-leading coefficients of the logarithmic terms $A^{(i)}$ and $B^{(i)}$ in the final distributions.

A similar structure at the next-to-leading accuracy can be also found in the gluon form factor[18], which, apart from the different spin structure dependent $B^{(1)}$ term can be obtained with the change $C_F \to C_A$ from the quark form factor.

The study of the leading and next-to-leading singularities in the large x ($x \simeq 1$) region of the deep inelastic and Drell-Yan distributions has further confirmed this universal behaviour. After a first approach[19] a complete evaluation of the exponentiated next-to-leading logarithmic structure has been carried[20].

In ref. (20) the followings exponentiated structure has been demonstrated to take place for deep inelastic and Drell-Yan processes :

. Deep Inelastic Scattering

For large values of the Bjorken x variable the Mellin transform of the structure functions $F(x, Q^2)$, $F_N(Q^2) = \int_o^1 dx\, x^{N-1} F(x, Q^2)$ are given by the expression

$$F_N(Q^2) = \exp\left\{ \int_o^1 dz\, \frac{z^{N-1} - 1}{1 - z} \left[\int_{Q_o^2}^{(1-z)/Q^2} \frac{dq^2}{q^2} A(\alpha_s(q^2)) + \frac{1}{2} B(\alpha_s(q^2)) \right] \right\} \qquad (5)$$

with A and B given by the same eq. (3).

. Drell-Yan

According to the mass singularity factorization theorem[20] the DY cross-section is

$$\frac{Q^2}{\sigma_o}\frac{d\sigma}{dQ^2} = W(Q^2,\tau) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} F(x_1, Q^2)\, F(x_2, Q^2)\, \Delta\left(\frac{\tau}{x_1 x_2}, Q^2\right) \qquad (6)$$

(for large values of $\tau = \frac{Q^2}{s}$ variable) in terms of the moments $W_N(Q^2) = \int_0^1 d\tau\, \tau^{N-1}\, W(\tau, Q^2)$.

It can be shown[21] that the coefficient function $\Delta_N(Q^2) = \frac{W_N(Q^2)}{F_N^2(Q^2)}$ has in the $\tau \to 1$ $(N \to \infty)$ limit the following expression

$$\Delta_N(Q^2) \underset{N\to\infty}{\simeq} 1 + \sum_{N=1}^{\infty} \sum_{m=0}^{2n} a_{nm}\, \alpha_s^n(Q^2)\, \ell n^m N \qquad (7)$$

that at $O(\alpha_s^2)$ takes the explicit form :

$$\ell n\, \Delta_n(Q') = \frac{\alpha_s(Q^2)}{\pi} C_F\left[\ell n^2 N - \left(\frac{3}{2} - 2\,\gamma_E\right)\ell n\, N\right] + \left(\frac{\alpha_s(Q^2)}{\pi}\right) C_F$$
$$\left[\pi\,\beta_o\, \ell n^3 N + \left(\frac{1}{2}K - \frac{3}{4}\pi\,\beta_o + 3\,\gamma_E\,\pi\,\beta_o\right)\ell n^2 N + O(\ell n\, N)\right] + O(\alpha_s^3) \qquad (8)$$

with $\beta_o = \frac{33 - 2N_f}{12\pi}$ and γ_E the Euler gamma constant.

The coefficients in eq. (8) coincide with the corresponding ones to $O(\alpha)$ and $O(\alpha_s^2)$ with the finite order expression in ref. (22) and ref. (23).

The exponentiated two-loop expressions in eq. (5) and eq. (7) constitute the all-order extension of the two-loop evaluation for the structure and coefficient function in deep inelastic and Drell-Yan processes[24] needed to describe the boundary region of phase space for these processes. Recently the problem of the resummation of leading and next-to-leading contributions to event shape variables has been also considered.

In particular the case of the Thrust distribution and of the heavy Jet Mass have been considered[25]. For large values of the Thrust the appearance of large corrections of the type $\alpha_s\, \ell n^2(1 - T)$ and $\alpha_s\, \ell n\,(1 - T)$ in the large Thrust region $T \simeq 1$ corresponding to the two jet

configurations can spoil the perturbative expansion until these are not taken into account and resummed. Also for this case as for the[9,15,16] case and the deep inelastic case[21] a suitable integral transform given by the Laplace transform considerably simplifies the problem[25]. In fact for final states corresponding to a large value of the Thrust it can be shown that, with next-to-leading accuracy in this region the Thrust cross-section $\sigma(\tau) = \int_{1-\tau}^{1} dT \frac{d\sigma}{dT}$ can be written as

$$\sigma(\tau) = \sigma_{tot}\, R(\tau, \alpha_s(Q^2) = C(\alpha_s(Q^2)\, \Sigma(\tau, \alpha_s(Q^2)) + F(\tau, \alpha_s(Q^2)) \qquad (9)$$

where, as in eq. (4), the C, Σ and F contributions have a series exponents in terms of $\alpha_s(Q^2)$. The exponentiated Σ factor can be written by taking the inverse Laplace transform as

$$\Sigma\left(\tau, \alpha_s(\mu^2), \frac{Q^2}{\mu^2}\right) = \frac{\exp\,[G(-\ell n\, \tau, \alpha_s(\mu^2), Q^2/\mu^2)]}{\Gamma(\varphi(\ell n\, \tau, \alpha_s(\mu^2)))}$$

with

$$\varphi(\ell n\, \tau, \alpha_s(\mu^2)) = 1 + \frac{2A^{(1)}}{\pi\beta_o}\left[\ell n\,(1 + \beta_o\, \alpha_s\, \ell n\, \tau) - \ell n\,(1 + 2\beta_o\, \alpha_s\, \ell n\, \tau)\right]$$

and

$$G\left(\ell n\, N, \alpha_s(\mu^2), Q^2/\mu^2\right) = -\, C_F \frac{A^{(1)}}{\pi\beta_o}$$

$$\left(\frac{1 - 2\beta_o\, \alpha_s L}{\beta_o\, \alpha_s}\, \ell n\,(1 - 2\beta_o\, \alpha_s L) - 2\, \frac{1 - \beta_o\, \alpha_s L}{\beta_o\, \alpha_s}\, \ell n\,(1 - \beta_o\, \alpha_s L)\right)$$

$$+ \frac{1}{\beta_o^2\, \pi^2}\left[C_F\, A^{(1)}\, \pi\, \beta_o\, \ell n\, Q^2/\mu^2 - A^{(2)}\right].$$

$$.\left[2\ell n(1 - \beta_o\, \alpha_s L) - \ell n\,(1 - 2\beta_o\, \alpha_s L)\right] + C_F \frac{B^{(1)}}{\pi\beta_o}\, \ell n\,(1 - \beta_o\, \alpha_s L) - 2C_F \frac{A^{(1)}\, \gamma_E}{\pi\beta_o}$$

$$\left[\ell n\,(1 - \beta_o\, \alpha_s L) - \ell n\,(1 - 2\beta_o\, \alpha_s L)\right] - C_F \frac{A^{(1)}\, \beta_1}{\pi\beta_o^3}$$

$$\left[\ell n\,(1 - 2\beta_o\, \alpha_s L) - 2\ell n\,(1 - \beta_o\, \alpha_s L) + \frac{1}{2}\ell n^2\,(1 - 2\beta_o\, \alpha_s L) - \ell n^2(1 - \beta_o\, \alpha_s L)\right]$$

$$+ O(\alpha_s(\alpha_s L)^s)$$

with $L = \ell n\, N$, $\alpha_s = \alpha_s(\mu^2)$, $\beta_1 = \frac{153 - 19N_f}{24\pi^2}$ and $A^{(1)} = 1$, $B^{(1)} = -\frac{3}{2}$ and $A^{(2)}$ as given by the eq. (3), thus confirming the universal behaviour of these coefficients. By comparing with the known finite $O(\alpha_s)$ analytical results[26] also the finite C_1 and $F_1(\tau)$ coefficients can be determined and a numerical estimate can be obtained of the C_2 and $F_2(\tau)$ coefficients by comparing with the numerical two-loop result[27].

The obtained distribution in eq. (9) does therefore contain the finite order one and two-loop results and, resumming the large contributions at large Thrust values is also finite at the border of the phase space. A comparison with the experimental data particularly also shows that a good description of the data can be obtained by using the value of the renormalization scale $\mu^2 = Q^2$ [25,28]. This same behaviours for this renormalization scale choice is shown by the resummed "$k_\perp$" jet algorithm[29] all over the jet resolution region. This also shows that the contrary to what happens with the "Jade" algorithm[30] where the resummation of large logarithms is not feasible[31] the dynamical content of the soft and collinear configurations sizeably improves the over-all perturbative description of the results.

The value of $\alpha_s(Q^2)$ that can be extracted from the data is affected by this analysis[32] even if there are the large non-perturbative hadronization effects that still play a relevant role all over.

The non-leading accuracy obtained and the ability to resum to all-orders next-to-leading contributions have put the perturbative description of these processes on a firmer basis.

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REFERENCES

1) T. Kinoshita, Journal of Mathematical Physics 3, 650 (1962) ; T. D. Lee, M. Nauenberg, Physical Review 133, 1549 (1964).

2) V. Sudakov, Soviet Physics JETP 3, 65 (1956). See also Abrisokov, JEPT 30, 96, 386, 544 (1956).

3) J. Cornwall, G. Tiktopoulos, Physical Review Letters 35, 338 (1975). See also ref. (6) and references therein.

4) E. A. Kuraev, V. S. Fadin, Yad. Fiz. 41, 753 (1985) ; (Soviet Journal of Nuclear Physics 41 (3), 466 (1985)). G. Altarelli, G. Martinelli in "Physics at LEP", CERN-Yellow Report, 86-02, J. Ellis, R. Peccei editors, Geneva, February 1986. O.Nicrosini, L. Trentadue, Physics Letters 196B, 551 (1987). See also volumes 1 and 3 of the CERN-Yellow Report 89-08 "Z Physics at LEP 1", G. Altarelli, R. Kleiss, C. Verzeguassi editors, Geneva, September 1989.

5) S. Catani, L. Trentadue, Pis'ma, Zh. Eksp. Teor. Fiz. 51, 72 (1990) (JETP Lett. 51, 2, 83 (1990)).

6) Yu. L. Dokshitzer, D. I. Dyakonov, S. Troyan, Proceedings XIIIth Winter School of LNPI (Leningrad, 1978), SLAC-TRANS-183 (1978) ; Physics Letters 79B, 269 (1978); Physics Reports 58, 269 (1980).

7) G. Parisi, Physics Letters B 90, 295 (1980).

8) G. Curci, M. Greco, Physics Letters B 92, 175 (1980) ; B 102, 280 (1981).

9) G. Parisi, R. Petronzio, Nuclear Physics B 154, 427 (1979).

10) G. Curci, M. Greco, Y. Srivastava, Physical Review Letters 43, 834 (1979). DOT, Physics Letters B 78, 290 (1978) ; 79, 269 (1978).

11) M. Ciafaloni, G. Curci, Physics Letters B 102, 352 (1981).

12) S. Catani, M. Ciafaloni, Nuclear Physics B 236, 61 (1984) ; B 249, 301 (1985).

13) J. Frenkel, J. G. M. Gatherall, J. C. Taylor, Nuclear Physics B 233, 1307 (1984).

14) J. Collins, G. Sterman, Nuclear Physics B 185, 172 (1981).

15) J. Collins, D. Soper, Nuclear Physics B 193, 381 (1981) ; Nuclear Physics B 197, 446 (1982).

16) J. Kodaira, L. Trentadue, Physics Letters B 112, 66 (1982), B123, 335 (1983) ; preprint SLAC-PUB-2934 (1982).

17) G. Altarelli, R. K. Ellis, M. Greco, G. Martinelli, Nuclear Physics B 246, 12 (1984). J. C. Collins, D. E. Soper, G. Sterman, Nuclear Physics B 250, 199 (1985). C. T. H. Davies, J. Stirling, B. K. Webber, Nuclear Physics B 256, 413 (1985).

18) S. Catani, E. d'Emilio, L. Trentadue, Physics Letters B 211, 335 (1988).

19) G. Sterman, Nuclear Physics B 281, 310 (1987).

20) J. C. Collins, D. Soper, G. Sterman, Nuclear Physics B 308, 833 (1988).

21) S. Catani, L. Trentadue, Physics Letters B 217, 539 (1989) ; Nuclear Physics B 327, 323 (1989) ; ibid. 353, 183 (1991).

22) J. Kubar-André, F. E. Paige, Physical Review D 19, 221 (1978). G. Altarelli, R. K. Ellis, G. Martinelli, Nuclear Physics B 157, 461 (1979).

23) W. L. van Neerven, Physics Letters B 147, 175 (1984). T. Matsuura, W. L. van Neerven, Zeitschrift für Physics C 38, 623 (1988). T. Matsuura, S. C. van der Marck, N. L. van Neerven, Physics Letters B 211, 171 (1988).

24) See W. L. van Neerven, talk at this conference.

25) S. Catani, L. Trentadue, G. Turnock, B. Webber, Physics Letters B 263, 491 (1991) and CERN-TH-preprint (1992) in preparation.

26) A. De Rujula, J. Ellis, E. G. Floratos, M. K. Gaillard, Nuclear Physics B 138, 387 (1978).

27) R. K. Ellis, D. A. Ross, A. E. Terrano, Nuclear Physics B 178, 421 (1981) as in Z. Kunszt, P. Nason, G. Marchesini, B. R. Webber in : Zeitschrift für Physics at LEP 1, CERN 89-08, vol. 1, p. 373. R. Fiore, A. Quartarolo, L. Trentadue, in preparation.

28) R. Fiore, A. Quartarolo, L. Trentadue, in preparation ; see the results presented by the ALEPH, DELPHI, L3 and OPAL collaboration at this conference.

29) S. Catani, Yu. L. Dokshitzer, G. Turnock, M. Olsson, B. Webber, Physics Letters B (1991). S. Catani, CERN-TH 6281 (1991).

30) Jade collaboration : W. Bartel et al., Physical Review C 33, 23 (1986).

31) N. Brown, N. J. Stirling, Physics Letters 252B, 657 (1990).

32) S. Bethke, S. Catani, contribution to this conference.