# Sub-barrier fusion of $^{64}$Ni + $^{100}$Mo

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Abstract

Fusion cross sections, mean angular momenta and partial-wave cross sections for the $^{64}$Ni + $^{100}$Mo system have been determined within a multi-dimensional model and are compared with re-analyzed experimental data. The proposed multi-dimensional fusion model takes into account the ion deformations during the tunneling process through the fusion barrier and the energy dissipation. The sub-barrier fusion cross section, the mean angular momenta and the partial-wave cross sections are strongly enhanced. The agreement between the theoretical predictions and the experimental data is good.

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Tunneling processes play an important role in modern physics. In nuclear physics, barrier penetration occurs in fission, in alpha and light ion emission and in fusion of heavy ions at low energies. Since 1980 sub-barrier fusion has been extensively studied, both theoretically and experimentally (Vaz et al 1981, Beckerman 1985 and 1988, Permyakov and Shilov 1989, Satchler 1991 and Vandenbosch 1992). The main result is that the experimental cross section exceeds the theoretical predictions (within the one-dimension WKB approximation) by several orders of magnitude. In order to explain this discrepancy between the experimental data and the theoretical results, several models have been advanced. The observed enhancement has been connected with the low-energy surface vibrations, the nucleon transfer (Esbensen and Landowne 1987 and 1989), the neutron flow (Stelson 1988), the barrier distribution mechanism in entrance trajectory of fusion (Rowley 1992), the neck formation (Krappe et al 1983, Iwamoto and Harada 1987 and Aguiar et al 1989) and the multi-dimensional tunneling processes (Schneider and Wolter 1991, Denisov 1991 and 1993, Denisov and Royer 1993).

Coupled-channel (CC) models consider the influence of low-energy surface vibrations and of nucleon transfer. The CC models reproduce reasonably well the experimental data for light and /or very asymmetric systems ( $Z_1 Z_2 \leq 700$ ). For heavier and almost symmetric systems ( $Z_1 Z_2 \geq 1000$ ), the data ( especially for the mean angular momentum at sub-barrier energies ) are more poorly described (Halbert and Beene 1991, Halbert et al 1989, Vandenbosch 1992, Stefanini et al 1992). The authors of CC models note that the problem of convergency of next order of coupling arises for heavy systems. For small ion charges, the next order corrections in CC theories are negligible (Esbensen and Landowne 1989). In contrast, for heavy systems, the shape of ions changes strongly during the fusion process and the introduction of multi-dimensional theories which accurately describe the large amplitude motion is necessary.

Such multi-dimensional models take into account the dependence of the interaction energy both on the distance between the colliding ions and on the deformation degrees of freedom. They have previously been defined and considered in detail (Schneider and Wolter 1991, Denisov 1991). The lowering of the fusion barriers due to deformations of the ions has been evaluated recently (Royer and Piller 1992).

The most complete theory of fusion of heavy ions should include both CC model degrees of freedom ( excitations of ions and nucleon transfer ) and multi-dimensional model degrees of freedom ( such as shape evolution and neck formation). As we pointed before, the CC models are not sufficient in the case of very heavy systems, therefore our consideration is focused on the description of fusion reactions within a multi-dimensional model.

The energy dependence of the fusion cross section for the $^{58}$Ni + $^{58}$Ni system and the energy dependence of both fusion cross section and mean angular momenta for the $^{64}$Ni + $^{64}$Ni, $^{92,96}$Zr, $^{100}$Mo reactions have been studied in previous papers (Denisov 1991 and 1993, Denisov and Royer 1993). The $^{58}$Ni+$^{58}$Ni

fusion reaction has been analyzed by Schneider and Wolter (1991).

Here we extend the analysis of the reaction $^{64}$Ni + $^{100}$Mo in the framework of the multi-dimensional model to the partial-wave cross sections and compare calculations with re-analyzed experimental data (Halbert and Beene 1991).

The detail description of our model and discussion about other different approaches proposed for sub-barrier fusion may be found in previous papers. Therefore we will repeat below only the main features of our model.

The approximations, parameters, shape sequence and the hypothesis on the dissipation of the kinetic energy during the collision have already been given (Denisov and Royer 1993). The dissipation of energy during the fusion process is connected with the energy transfer to another degree of freedom. It plays a major role during the fusion of ions with large values of $Z_1 Z_2$ (Gross and Kalinowski 1978 and Birkelund and Huizenga 1983). The energy dissipation is taken into account before the outer turning point ( if the energy of the collision after dissipation is lower than the barrier ) and the energy of the collision is determined at distance given by one-dimensional spherical barriers ( two-sphere approximation ). Dissipation is neglected during the sub-barrier tunneling process and possible only before the outer turning point.

The calculations within this multi-dimensional model are performed for a quadratic type fusion trajectory. During the tunneling process the ions change from slightly oblate to well developed prolate or even one-body elongated shape with neck.

In Fig. 1 the fusion cross section calculated in our multi-dimensional model for the $^{64}$Ni + $^{100}$Mo system is compared with experimental data (Halbert et al 1989 and 1991). Results of calculations using a CC model and the simple one-dimensional WKB approach are also shown. The CC calculations have been performed with the program CCFUS (Dasso and and Landowne 1987) with the original parameters. The experimental features of low-energy levels have been taken from other works (Esbensen and Landowne 1989, Pignanelli et al 1984). $2^+$ and $3^-$ excitations in the $^{64}$Ni nucleus and the first five excitations in the $^{100}$Mo nucleus have been introduced. As assumed by Halbert et al 1989 and Stefanini et al 1992, the coupling to the nucleon-transfer channel has been neglected.

Our multi-dimensional model gives a good description of the experimental fusion cross section in the whole energy range. The reason of this good agreement below the Coulomb barrier is that our tunneling process occurs in a space with an extra dimension (the trajectory of minimal action is determined for each value of the energy and each partial wave on the potential surface). The good description of experimental data at high energy is connected with the dissipation, which modulates the process of removing the ions from the fusion channel. One of the reasons explaining that the mean angular momentum obtained in our calculations (see Figs. 2) is larger than the corresponding one given by the CC model at high energy is our taking into account of the ion finite size (the moment of inertia has larger value for finite ions than for point ones).

The values of $< L(E) >$ in the Coulomb barrier region deduced from the multi-dimensional approach are good compared with experimental data.

The comparison between theoretical and experimental partial wave cross sections for energies of 141.7 MeV, 135.5 MeV, 132.8 MeV, 130.1 MeV and 127.8 MeV is presented in Fig. 3. The amplitudes and widths of the partial wave cross sections calculated in the multi-dimensional model for energies 141.7 MeV and 135.5 MeV are slightly larger than the experimental data. The values of experimental and calculated amplitudes and widths of partial wave cross sections are practically the same for energies 132.8 MeV and 127.8 MeV.

The partial wave cross sections obtained in the framework of CC calculations for energies lower then 135.5 MeV are much smaller than the experimental ones and not displayed in Fig.3. The width of partial wave cross section calculated in CC theory sharply decreases with the collision energy. Therefore the values of the mean angular momentum for lower energies underestimates the experimental data.

In conclusion we have shown that the multi-dimensional description of the fusion process, which takes into account the ion deformation during the tunneling and the energy dissipation gives a strong enhancement of the sub-barrier fusion cross section, the mean angular momentum and the partial wave cross sections. This enhancement of the fusion cross section is in good agreement with experiment.

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## Figure Captions

**Figure 1.** Energy dependence of the fusion cross section for the reaction $^{64}$Ni + $^{100}$Mo. Experimental data are from Halbert et al 1989. The solid line shows the result of our multi-dimensional model calculation. The dot-dashed line corresponds to CC calculations in which the low-energy excitations of ions are taken into account, while the dashed line shows the result of one-dimensional WKB calculations.

**Figure 2.** Energy dependence of the mean angular momentum in $\hbar$ unit calculated in the multi-dimensional, CC and one-dimensional WKB models for the reaction $^{64}$Ni + $^{100}$Mo. Experimental data are from Halbert et al 1989. The type of lines is the same as in Fig. 1.

**Figure 3.** Partial cross sections calculated in the multi-dimensional and CC models for reaction $^{64}$Ni + $^{100}$Mo. The triangles correspond to multi-dimensional model calculations, squares - CC calculations, crosses - experimental data.

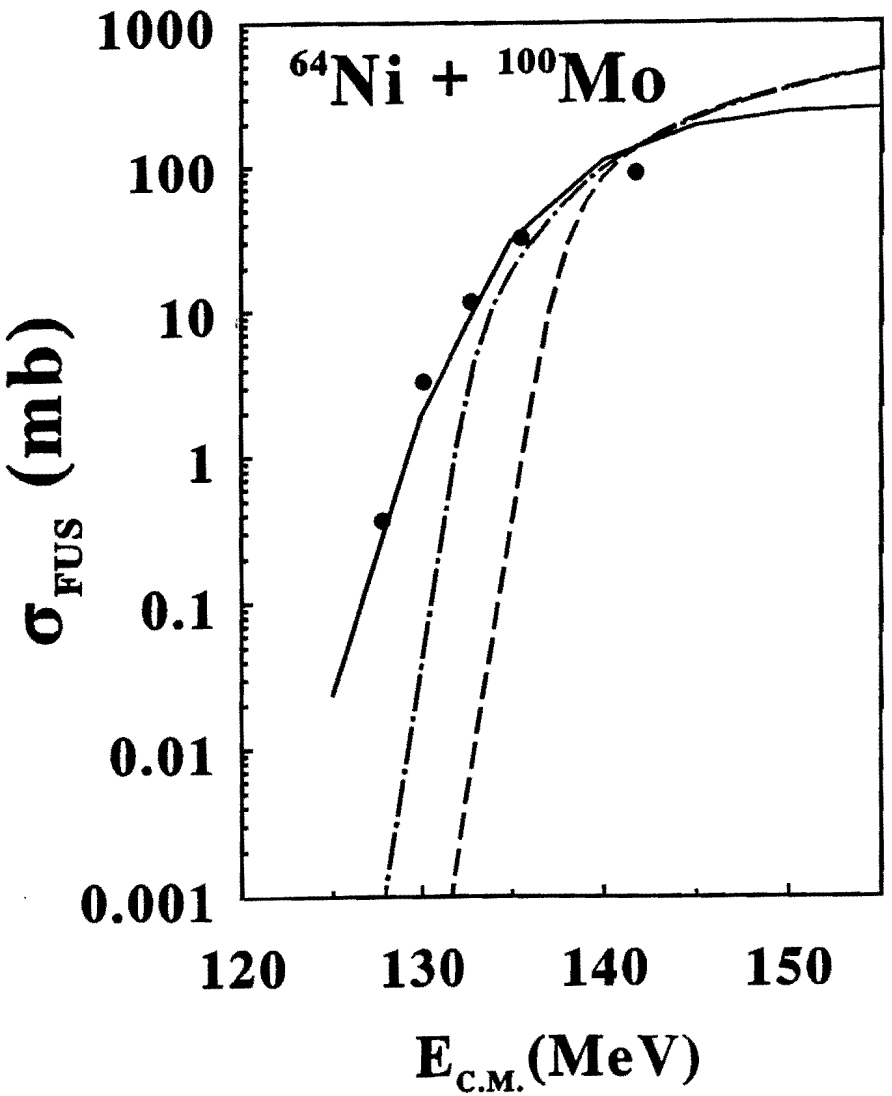


Figure 1

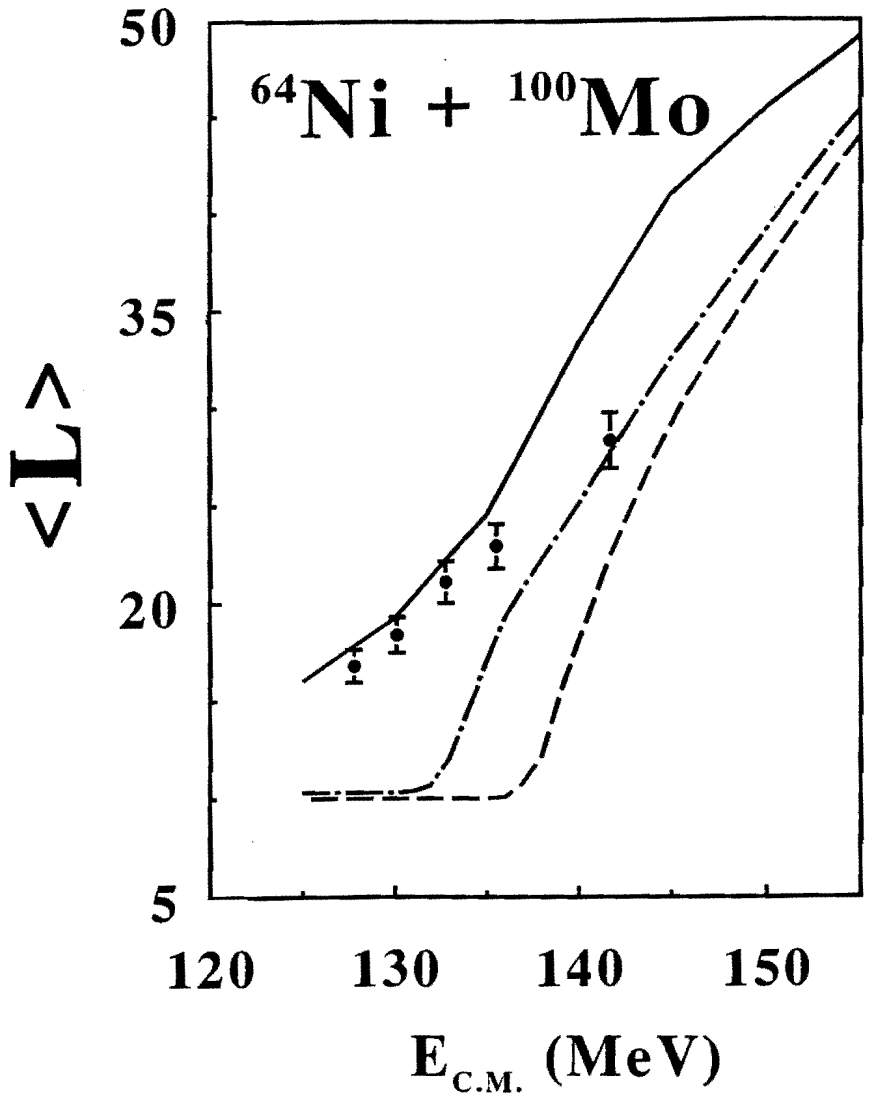


Figure 2

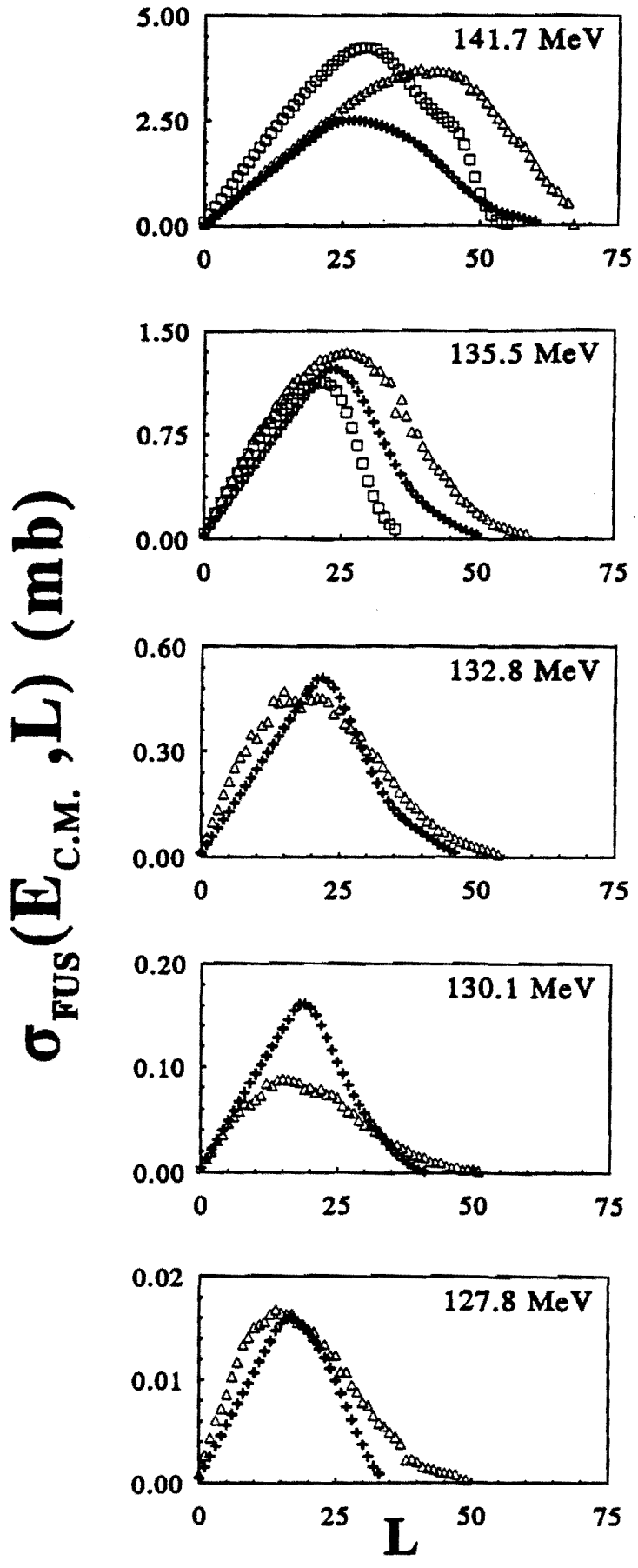
141.7 MeV
135.5 MeV
132.8 MeV
130.1 MeV
127.8 MeV
5.00
2.50
0.00
1.50
0.75
0.00
0.60
0.30
0.00
0.20
0.10
0.00
0.02
0.01
0.00
0
25
50
75
L
σFUS(EC.M. ,L) (mb)

Figure 3