KAON PRODUCTION AT SUBTHRESHOLD ENERGIES

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ABSTRACT

We present a comprehensive investigation of subthreshold and threshold kaon production in the framework of the QMD model. We find a quite good agreement with experiment which demonstrates that the basic dynamics is well treated in the present transport theories. We discuss in detail the influence of the elementary kaon production cross section the contribution of different production channels and the properties of the nuclear environment of the place where the kaon is produced. The comparison with experiment allows the conclusion that most of the kaons are produced in Δ N reactions. The decrease of the kaon production per participant with decreasing participant number which is also observed experimentally has two sources: The non maxwellian momentum distribution for small participant numbers as well as a reduction of the $\Delta\Delta$ channel. This finding is supported by the observed decrease of high momentum pions.

1 Introduction

In recent years the production of subthreshold particles became one of the major research areas in relativistic heavy ion physics. In these reactions the creation of a particle is called subthreshold if the $\sqrt{s}$ in a collision of a nucleon at a beam energy with a target nucleon at rest is below the particle production threshold :

$$\sqrt{s_{nn}} = \sqrt{(\sqrt{p_{beam}^2 + m_N^2} + m_N)^2 - p_{beam}^2} < \sqrt{s_{threshold}} \qquad (1)$$

Thus at that energy a production in an elementary NN collision is impossible. In heavy ion reactions there are, however, three mechanisms which may increase the energy available in a single nucleon nucleon collision :

a) the intrinsic Fermi momentum of the nucleons increases the available energy if that momentum is parallel to the beam direction for the projectile or opposite for the target nucleon ;

b) in previous collisions the nucleons may have gained the additionally required momentum.

c) lower effective masses of the hadrons may lower the threshold and therefore yield higher production rates.

Therefore, the production of particles below threshold may reveal information about the dynamics of the reaction as well as about the momentum distribution of nucleons inside a nucleus. In addition, there may be also a coherent production processes.

Why is it interesting to study subthreshold particle production? The interest is threefold : firstly it offers the possibility to study properties of the produced particles in the nuclear environment. Here the topics of research include the study of in medium modification of the self energy of the particles as well as that of the production cross section. One example is the experimental determination of the mean free path as a function of the energy of the produced particle. Secondly, the particle may carry information about the nuclear environment itself for example about the compression which can be reached in heavy ion collisions because with increasing compression the mean free path decreases and more collisions take place in which particle can be produced. Thirdly, two step processes can occur, i.e.

$$N_1 + N_2 \rightarrow N_1 + \Delta \qquad \Delta + N_3 \rightarrow N_3 + \Lambda + K \quad or \rightarrow N_3 + 2N + \bar{N} \qquad (2)$$

which are not possible in elementary reactions and may lead to an enhancement of the production cross section with respect to the extrapolation from NN collisions. Here especially the Δ will play an effective role in subthreshold particle production[1].

Up to now the experiments have concentrated on the production of subthreshold pions. It has been found that pion production occurs at energies as low as 25 MeV/N. Most of the pions are created in individual nucleon nucleon collisions although recently also a collective production has been reported [2]. Using asymmetric projectile target combinations even the mean free path in a nuclear environment could be measured [3,4]. For a survey of subthreshold pion experiments we refer to ref. [5,6].

Far more interesting than pions, however, are subthreshold kaons because:

1) the threshold for kaons ($E_p = 1.58$ GeV) is much higher than that for pions ($E_p = 279$ MeV). The compression which can be reached in heavy ion collisions increases with increasing beam energy. Therefore subthreshold kaons can test nuclei at much higher compression than subthreshold pions. At energies above the pion threshold the pions are produced in elementary collisions independent of the impact parameter and therefore most of the pions come from peripheral reactions and are produced at the end of the heavy ion reaction, thus carrying little information about the high compression region. The subthreshold production of kaons requires central collisions in order to collect the energy required for the production.

2) If the two colliding nucleons have an additional intrinsic momentum due to the Fermi motion, which is close to the Fermi momentum and opposite in its direction, pions can be easily created at beam energies well below 60 MeV/N. Thus the production of pions is to a large extend determined by the Fermi momentum of the projectile and target nucleus. For the productions of kaons, however, an energy can be defined ($\sim$ 1 GeV/N) below that the Fermi momentum is not sufficient to provide the additional energy required for the production of a kaon. Thus kaons measure how energy is transferred between the nucleons in collisions prior to the production.

3) Kaons carry an $\bar{s}$ quark and therefore cannot be reabsorbed whereas most of the pions created in the central region of a heavy ion reaction get reabsorbed. Thus only kaons can give informations about the high compression/high density zone.

The first theoretical investigation of subthreshold kaons was performed in the framework of the Boltzmann-Uehling-Uhlenbeck (BUU) approach [7]. In this approach the time evolution of the one body phase space distribution is calculated

using the test particle method [8]. This calculation showed that the production of kaons in heavy ion collisions depends strongly on the nuclear equation of state (EOS). If the EOS is soft (compressibility K = 200 MeV) less energy is needed for the compression as compared to a hard equation of state (K = 380 MeV). Therefore more energy is available for the production of kaons and consequently the number of produced kaons is larger. It turned out that the production of kaons is concentrated at low impact parameters. In peripheral reactions not enough energy can be concentrated in a single nucleon nucleon collision. Futhermore, we observed that most of the kaons are produced via the two step process $N_1 + N_2 \rightarrow N_1 + \Delta; \Delta + N_3 \rightarrow N_3 + \Lambda + K$.

These results were confirmed in the framework of the n-body Quantum Molecular Dynamics (QMD) approach [9]. This approach allowed for the first time to include the momentum dependence of the optical potential [10]. The calculations showed that the kaon production yield is also quite sensitive to the momentum dependence of the interaction, a fact which can be understood quite easily. With increasing momentum the interaction becomes increasingly repulsive. Therefore, nucleons with large relative momentum decelerate most and thus, when colliding, do not have sufficient energy to produce a kaon. The momentum dependence of the optical potential can decrease the production cross section as much as an factor of 4. Later [11] it has been shown that measuring the excitation function of the kaon production for different projectile target combinations one may be able to disentangle the static EOS dependence of the kaon production yield from the momentum dependence due to the optical potential. Recently it has been demonstrated by Li et al. [12] that the uncertainty of the elementary kaon production process may cause an uncertainty of a factor of two of the kaon yield in heavy ion reactions. However one can overcome some of the problems by looking at ratios of kaon production cross sections [13]. The ratio of the kaon production yield in light and heavy systems seems to by little affected by the choice of the elementary cross section and still contains, as we will see, the important information about the nuclear equation of state.

Now, in 1993, the first experiments on subthreshold kaon production have been performed and analyzed by the KaoS collaboration [14,15,16], and therefore it is now possible for the first time to compare the theoretical calculations, which have been performed previous to the experiment [11] with data. This is the purpose of this article. In chapter 2, we discuss the details of the QMD calculations as far as the kaon production is concerned. In chapter 3 we give account of the potentials and of the NN cross sections as well as of the different parametrizations of the elementary kaon production cross section. Chapter 4 is devoted to the comparison with experiment. In chapter 5 we discuss the details of the kaon production and its mass and energy dependence. This is followed by a comparison of our predictive results with the other theoretical approach which was available

prior to the experiment [6,17] in chapter 6. There we also discuss the results obtained by other approaches [7,18], advanced after the experiment. In chapter 7 we compare our calculation with data above threshold. Finally, in chapter 8 we present our conclusions and discuss the future perspectives.

2 Details of the Calculations

The QMD model is a n body theory which simulates heavy ion reactions between 30 MeV/N and 1 GeV/N on a event by event basis. Each nucleon is represented by a coherent state of the form ($\hbar, c = 1$)

$$\phi_\alpha(x_1, t) = \left(\frac{2L}{\pi}\right)^{3/4} e^{-(x_1 - x_\alpha(t))^2 L} e^{ix_1 p_\alpha(t)} e^{-ip_\alpha^2(t)t/2m} \tag{3}$$

Thus the wave function has two parameters x_α, p_α, L is fixed. The total n body wave function we assume to be the direct product of coherent states.

$$\phi = \phi_\alpha(x_1, x_\alpha, p_\alpha, t)\phi_\beta(x_2, x_\beta, p_\beta, t)\cdots \tag{4}$$

thus antisymmetrization is neglected. The initial values of the parameters are chosen in that way that the ensemble of $A_T + A_P$ nucleons gives a proper density distribution as well as a proper momentum distribution of the projectile and target nucleus. The time evolution of the system is calculated by means of a generalized variational principle : we start out from the action

$$S = \int_{t_1}^{t_2} \mathcal{L}[\phi, \phi^*]dt \tag{5}$$

with the lagrange functional $\mathcal{L}$

$$\mathcal{L} = \left(\phi \left| i\hbar\frac{d}{dt} - H \right| \phi\right) \tag{6}$$

where the total time derivative includes the derivation with respect to the parameters. The evolution is obtained by the requirement that the action is stationary under the allowed variation of the wave function

$$\delta S = \delta \int_{t_1}^{t_2} \mathcal{L}[\phi, \phi^*]dt = 0 \tag{7}$$

If the true solution of the Schrödinger equation is contained in the restricted set of wave functions $\phi_\alpha(x_1, x_\alpha, p_\alpha)$ this variation of the action gives the exact solution of the Schrödinger equation. If the parameter space is too restricted, we obtain that wave function in the restricted parameter space which comes closest to the solution of the Schrödinger equation.

Performing the variation with the test wave function (4) we obtain for each parameter λ an Euler Lagrange equation.

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\lambda}} - \frac{\partial \mathcal{L}}{\partial \lambda} = 0. \tag{8}$$

For the coherent states and an Hamiltonian of the form $H = \sum_i T_i + \frac{1}{2}\sum_{ij} V_{ij}$ (T_i= kinetic energy, V_{ij} = potential energy) the Lagrangian and the Euler Lagrange function can be easily calculated :

$$\mathcal{L} = \sum_\alpha -\dot{x_\alpha} p_\alpha + \sum_\beta \langle V_{\alpha\beta}\rangle + \frac{3}{8Lm} \tag{9}$$

$$\dot{\vec{x}_\alpha} = \frac{p_\alpha}{m} + \nabla_{p_\alpha} \sum_\beta \langle V_{\alpha\beta}\rangle \tag{10}$$

$$\dot{p_\alpha} = -\nabla_{x_\alpha} \sum_\beta \langle V_{\alpha\beta}\rangle \tag{11}$$

with $\vec{x}_\alpha = x_\alpha + \frac{p_\alpha}{m}t$.

Thus the variational approach has reduced the n-body Schrödinger equation to a set of 6n differential equations for the parameters which can be solved on present day computes. Furthermore, the time evolution equations are very similar to the classical Hamilton equations. Especially this feature allows a very transparent interpretation of the results.

3 Interaction and Cross Section

3.1 Potential interaction

We have not yet specified the interaction we use. During the time evolution the nucleons interact via a Skyrme potential and a Coulomb interaction. We performed calculations with a static potential as well as with an additional momentum dependent interaction V_{MDI} which has been adjusted to the measured optical potential.

$$V_{ij} = V_{loc}^{(2)} + V_{loc}^{(3)} + V_{coul} + (V_{MDI}) \tag{12}$$

with

$$V_{loc}^{(2)} = t_0\delta(x_i - x_j) \tag{13}$$

$$V_{loc}^{(3)} = t_3\delta(x_i - x_j)\left(\frac{\rho(i)+\rho(j)}{2}\right)^\gamma \tag{14}$$

$$V_{MDI} = t_4 ln^2[t_5(\vec{p}_i - \vec{p}_j)^2 + 1]\delta(x_i - x_j) \tag{15}$$

The parameters t_4=1.57 MeV and t_3=5.10^{-4} MeV^{-2} are determined by a fit to the measured optical potential. The parameters t_0 and t_3 are determined by

the requirement that in infinite nuclear matter this potentials yield an equation of state which coincides with that frequently used in nuclear matter calculations. In nuclear matter the expectation value of the local interaction is a function of the density only:

$$t_0 \int \langle\phi(x_1)\phi(x_2)|\delta(r_1 - r_2)|\phi(x_1)\phi(x_2)\rangle dx_1 dx_2 \overset{\text{infinite matter}}{\Longrightarrow} t_0\rho \tag{16}$$

Thus in infinite nuclear matter the expectation value of the local potentials read as

$$\langle V\rangle = t_0\rho + t_3\rho^{(1+\gamma)} \tag{17}$$

The requirement that at normal nuclear matter density the binding energy per nucleon is - 16 MeV fixes two of the three parameters. The third one can be used to obtain the desired compressibility. The values for t_0,t_3 and γ as well as the compressibility K for our three choices, a soft and a hard static equation of state as well as a soft momentum dependent equation of state, are given in Table 1.

K [MeV]	$t_0[GeV]$	$t_3[GeV]$	γ	U_{opt}	EOS
200	-.356	.303	7/6	no	S
380	-.124	.0705	2	no	H
200	-.390	.320	1.14	yes	SM

Table 1: Parameters of the potentials

With this choice of potentials we obtain nuclei which are sufficiently stable and have the proper binding energy. As compared to ref. [9] we used a improved version of the QMD model which es explained in detail in ref.[19].

3.2 NN cross section

In addition to the interaction via potentials the particles interact as well by two body collisions. For the present calculation we employ a new parametrization of the free pp and np cross sections [20] which also differs somewhat from the isospin averaged cross section of ref.[9]. Two nucleons suffer a collision if the distance between the centroids of their wave function $|x_\alpha - x_\beta|$ becomes smaller

than $\sqrt{\sigma/\pi}$. The collision is blocked if there is no room for the scattered particles in phase space. The lower the energy, the more collisions are blocked. Thus for very low energy reaction the collisions do not play an important role.

In our approach there are no free pions due to the fact that Δ's can only be reabsorbed in $N\Delta \rightarrow$ NN collisions (or are destroyed in a kaon production collision). Thus the Δ has a infinite lifetime with respect to its decay. This approximation we compare with IQMD calculations [21] in which the Δ have a finite mass dependent lifetime with an average value of of 1.6 fm/c corresponding to the width of 120 MeV. In the IQMD approach the pions cannot produce a kaon, thus the IQMD results mark the lower bound for the production using a finite lifetime . As we will see later the difference of the kaon yield between finite and infinite lifetime of the Δ will be about 15%. Thus our approximation is well justified. We do not employ in medium cross sections because the ambiguities of how to calculate them are not settled yet. Instead we calculate the $\sqrt{s}$ of the collision with the free masses of the incoming particles even if we employ momentum dependent potentials.

In order to understand why particles interact in two ways, by potentials and by scattering one has to remember that the above mentioned potentials are not the NN potentials adjusted to scattering phase shifts but a parametrisation of the real part of the Brückner G-matrix. At sufficient energy, i.e. if energy conservation allows the population of states above the Fermi surface, the Brückner G-matrix is complex and the imaginary part can be formulated as a function of the cross section. Recently, we succeeded to describe the time evolution of the particles under the influence of a complex G-matrix consistently by a variational principle [22].

3.3 Kaon production cross section

The elementary production cross section is one of the most important ingredients for the calculation because the kaon cross section for the heavy ion reaction follows the elementary production cross section linearly . Unfortunatelly the knowledge of this cross section is all but satisfying, in view of the elementary production experiments as well as in view of the theoretical calculations.

From the calculation we find that at a beam energy of 1 GeV/n the average $\sqrt{s}$ of the elementary collisions, which produce kaons, is around 2.6 GeV, corresponding to a beam energy of 1.73 GeV in the elementary pp reaction. The maximal $\sqrt{s}$ is 3.2 GeV (equivalent to E_{kin} = 3.6 GeV in the elementary reaction).

Fig. 1 displays the world data as well as several parametrizations of the elementary production cross sections. We see that in the energy domain of interest there exists practically no experimental data point. Thus one has completely to rely on extrapolations from data points at higher energies or on calculations. We would

like to mention that at SATURNE p + C $\rightarrow K^+$ and p + Pb $\rightarrow K^+$ experiments have been performed which may soon improve the situation.

The theoretical efforts to calculate the cross section using experimentally known form factors are rather limited. We are aware of only two calculations: Laget[23]has calculated the cross section of the three elementary processes $pp \rightarrow K^+\Lambda p, K^+\Sigma^+ n$ and $K^+\Sigma^0 p$ in a partial wave analysis using effective form factors. The calculations underpredict the cross section close to the threshold, however the experimental errorbars are quite large and leave easily room for a factor of two. Xu and Ko [24] have calculated the production cross section for the channel $pp \rightarrow K^+\Lambda p$ in a one boson exchange model. They allow the exchange of pions only. Also these results are in reasonable agreement with data, despite of the fact that Laget claims that pions count only for 1/5 of the cross section whereas the other 4/5 are due to the exchange of a kaon.

Because of these ambigous theoretical results most of the calculations for heavy ion collisions make use of a parametrization of the experimental cross sections. Two parametrizations have been advanced. Randrup and Ko [25] parametrized the cross section for all kaon production channels in terms of coupling constants which have been adjusted to the few existing data. Using the isospin invariance of the strong interaction one obtains for isospin symmetric heavy ion collisions after isospin averaging over the nucleons in the entrance channel the relations:

$$\sigma_{NN\rightarrow\Lambda K^+} = 3/2\,\sigma_{pp\rightarrow K^+\Lambda p} \tag{18}$$

and

$$\sigma_{NN\rightarrow\Sigma K^+} = 3/2\,(\sigma_{pp\rightarrow K^+\Sigma^0 p} + \sigma_{pp\rightarrow K^0\Sigma^+ p}) \tag{19}$$

The latter expression accounts for the fact that for the Σ production two isospin channels (I=1/2 , 3/2) in the intermediate line contribute.

Assuming that the cross section is linear in p_{max} with p_{max} being

$$p_{max\,\Lambda} = \{[s - (M_K + M_N + M_\Lambda)^2][s - (M_K - M_N - M_\Lambda)^2]/(4s)\}^{1/2} \tag{20}$$

they found the following parametrization of the cross section

$$\sigma_{NN\rightarrow\Lambda K^+} = 36\,\frac{p_{max\,\Lambda}}{m_K c}[\mu b] \tag{21}$$

$$\sigma_{NN\rightarrow\Sigma K^+} = 36\,\frac{p_{max\,\Sigma}}{m_K c}[\mu b] \tag{22}$$

The latter expression is obtained using the experimental observation that the I= 1/2 and I = 3/2 cross sections are equal in between the errorbars.

The second parametrization has been advanced by Zwermann[13,26]. He parametrizes the total kaon production cross section as

$$\sigma_{NN\rightarrow K^+} = 800\,(p_{max}/(GeV/c))^4[\mu b] \tag{23}$$

where p_{max} is calculated from the channel with the lowest threshold ($NN \rightarrow K^+\Lambda p$). On the first view this parametrization seems to be closer to the only data point close to threshold (fig. 1 of [13]), however this it not really true. In this figure data points of two different channels ($pp \rightarrow K^+$ and $pp \rightarrow K^+\Lambda p$) are mixed. Furthermore in this curve it is assumed that the $pp \rightarrow K^+$ cross section is the same as the $NN \rightarrow K^+$ instead of $\sigma_{pp\rightarrow K^+} = 2/3\,\sigma_{NN\rightarrow K^+}$ obtained from the isospin invariance of the strong interaction.

Fig. 1 displays the different parametrization as compared to the world data. We see that the parametrization by Zwermann (corrected for the isospin invariance of the strong interaction) yields a quite low cross section close to the threshold and overpredicts the cross section for energies higher than 2.8 GeV. The consequences we will discuss later. A similar behaviour is observed for the Λ channel comparing the parametrization by Randrup and Ko with the calculation by Wu and Ko. The calculation yields a lower cross section at the threshold as compared to the parametrization but a larger value for intermediate beam energies.

In conclusion we observe that the parametrization by Randrup an Ko yield a quite reasonable agreement with experiment for all channels. However, it should be mentioned that the data leave easily room for a factor of two in the absolute value.

In the exploratory studies [7] as well as for the calculation presented in this paper we used - if not stated otherwise - a simplification of the Randrup and Ko parametrization:

$$\sigma_{NN\rightarrow K^+\Sigma N} = \sigma_{NN\rightarrow K^+\Lambda p} \tag{24}$$

thus neglecting the slightly higher threshold of the Σ channel. At 1 GeV, as we will see, this yields a 20% higher cross section than taking the Σ channel explicitely into account.

The differential cross section $\frac{d\sigma_{NN\rightarrow K^+}}{dp_K}$ is even less known than that of the total cross section. The few experiments performed show a quite peculiar behaviour of the differential cross section. At E_{kin} = 2.3 GeV Frascaria et al.[27] observe a quite strong enhancement of the high kaon momenta which Laget[23] explains in terms of the strong nucleon hyperon interaction in the final state. At E_{kin} = 2.54 GeV [28]the differential cross section resembles very much the expectation for a distribution according to the available phase space. At E_{kin} = 2.8 GeV[28] we see a strong suppression of the high kaon momenta. A part of this suppression is probably due to the fact that at that energy Δ's instead of the nucleons can be produced in the final state. This alone, however, cannot explain the spectra. Also on the theoretical side the situation is all but satisfying. The only calculation we are aware of is that of Laget who could reproduce the momentum distribution of the kaon at E_{kin} = 2.3 GeV. Thus certainly many efforts are necessary, theoretically as well as experimentally, before the kaon momentum distribution in heavy ion

collisions can be calculated more precise than with a factor of two uncertainty.

What worsens the situation is the fact that, as we will see, most of the kaons are not produced in the channel $NN \rightarrow K^+$ but in the channel $N\Delta \rightarrow K^+$. Nothing detailed is known about this channel. Randrup and Ko [25] as well as Wu and Ko [24] assumed that due to the different coupling constants the following relations hold:

$$\sigma_{N\Delta\rightarrow K^+} = 3/4\,\sigma_{NN\rightarrow K^+} \tag{25}$$

and

$$\sigma_{\Delta\Delta\rightarrow K^+} = 1/2\,\sigma_{NN\rightarrow K^+} \tag{26}$$

However, the experience with the calculation of the reaction $NN \rightarrow N\Delta$ [29]show that this can be considered only as a very crude approximation. Thus urgently reactions of the type

$$p + \text{light closed shell nuclei} \rightarrow K^+ \tag{27}$$

are required where reliable calculations for these channels seem to be possible.

Fig. 2 displays the influence of the different parametrizations of the elementary kaon production cross section on the kaon yield observed in heavy ion reactions for a soft equation of state at 1 GeV/N. For Au + Au we observe the ratios 3.4 : 2.75 : 2.35 : 1 for Randrup and Ko $\sigma_{tot} = 2\sigma_\Lambda, \sigma_{tot} = \sigma_\Lambda + \sigma_\Sigma$, Xu and Ko ($\sigma_{tot} = 2\sigma_\Lambda$) and Zwermann, respectively. This ratio is almost independent of the nuclear mass. This verifies the conjecture of Zwermann and Schürmann that ratios of the cross sections for different projectile/target combinations are not sensitive to the elementary production cross section and therefore much better suited to extract the physics than absolute cross sections. The total kaon yield is obtained by multiplying N(kaon) with the total reaction cross section. Thus in this presentation the trivial $A^{2/3}$ dependence of the cross section is taken out. For N(kaon) as a function of the mass number A we find for all parametrizations a straight line in the double logarithmic plot which can be parametrized as $N_A(kaon) = c * A^{1.38}$. Thus the A dependence is stronger than that of the average number of participants. We will come back to this point later.

The different parametrization have also an influence on the form of the momentum distribution of the kaons as can be seen from fig. 3. The full and dashed line give the result with the Randrup and Ko parametrization for $\sigma_{tot} = 2\sigma_\Lambda$ and $\sigma_{tot} = \sigma_\Lambda + \sigma_\Sigma$, respectively. We see, as expected, the influence of the different thresholds at kaon momenta around 300 MeV/c. The dashed-dotted line is the result obtained with the Zwermann parametrization. Since in this parametrization the increase of the cross section with increasing $\sqrt{s}$ is much stronger than in the aformentioned parametrizations it favours the production of high momentum kaons. The calculation with the dotted line assumed that also in the Zwer-

mann parametrization we have the following scaling: $\sigma_{N\Delta\to K^+} = 3/4\,\sigma_{NN\to K^+}$ and $\sigma_{\Delta\Delta\to K^+} = 1/2\,\sigma_{NN\to K^+}$.

As Eqs. (21-23) indicate, the kaon production cross section is only a small fraction of the nucleon-nucleon total cross section of $\sim$ 40mb, and therefore we treat the production of kaons perturbatively as in Ref.[7,25]. This means that we do not follow the trajectory of the kaon but only calculate the probability of its creation. Because of the conservation of strangeness, kaons will not be reabsorbed by the nucleons. Since, as we will see, the angular distribution of kaons is isotopic in the NN cm systems of the equal mass projectile target combinations, we shall not include the weak rescattering of the kaons after their productions. However, we would like to mention that recently it has been claimed that rescattering may change the spectra for light systems somewhat [30].

In the present version of the QMD program there are no free pions. Therefore the above mentioned processes are the only ones which can create a K^+. We will come back to this point later.

4 Kaon angular distributions and comparison with experiments

The angular distributions of kaons, produced in the collisions 1 GeV/N Ne + Ne and Au + Au, in the NN center-of-mass system are displayed in figs. 4 and 5. Fig. 4 displays the in plane distribution, fig. 5 the azimuthal distribution. We see in fig. 4 that the invariant cross section for Ne + Ne as well as of Au + Au at 1 GeV/N integrated over the azimuthal angle is isotropic, confirming the experimental findings at a higher energy for the light system Ne + Ne [44]. The same isotropy we observe for the azimuthal distribution at $\theta_{cm} = 90°$ as shown in fig. 5. These spectra coincide with those expected from a thermalized source at midrapidity, although the system as a whole does not completely equilibrate according to our calculation. However, since the system in central collisions, where - as we will see later - most of the kaons are produced, comes close to equilibrium we do not expect that rescattering with the surrounding nucleons changes the angular distribution or the slope of the spectra considerably. Therefore for heavy systems the isotropy of the primordial kaon spectra validates our approximation to describe the kaon production perturbatively even for the case that we would like to compare our results with the experimental data measured for a finite range of angles. For light systems rescattering is very unprobable since the mean free path of kaons in nuclear matter is of the order 6 - 8 fm and hence larger than the diameter of the system.

Fig. 6 presents the comparison of the KaoS data with QMD calculations, taken from ref. [16]. The QMD calculations have been performed for three different equations of state at 1 AGeV incident energy [31] employing an geometric acceptance cut of $40° < \theta_{lab} < 48°$ corresponding to setup of the KaoS experiment.

First of all we observe that the predicted cross section is very close to the experimental one. In view of the fact that this is the first experiment on subthreshold kaon production where the production yield is two orders of magnitude smaller than at the lowest energy measured so far [44] and in view of the fact that the only other approach advanced prior to experiment [6] yield an almost a factor of 5 lower cross section, as we will see later, this is a remarkable success of the QMD approach. It allows, as will be discussed in chapter 5, to draw firm conclusions on the production process.

We observe that for the Ne + Ne collision the static soft and hard equation of state gives almost identical results. This is not unexpected because this light system gets little compressed during the collision and therefore the effect of different compressibilities is little. This is not true for the heavy gold system. Here we observe that a soft equation of state allows the production of twice as many kaons as a hard equation of state, a result which has been anticipated in [7]. For a

soft equation of state the compressional energy is lower than for a hard equation of state and therefore the particles have a higher kinetic energy which yields an enhancementof the kaon production.

If one includes the momentum dependence of the optical potential without changing the compressibility (soft+mdi) the kaon production decreases by almost a factor of 3. A reasonable simulation should include the momentum dependence of the optical potential even if there are large uncertainties about its functional form at finite temperatures and at densities larger than normal nuclear matter density. Hence the most reasonable calculation underpredicts the experimental data by a factor of 2-3. A hard equation of state with the proper optical potential would give still a lower kaon production yield. The reason for this discrepancy between experiment and theory is not understood yet. Possible reasons will be discussed in chapter 8. It is, however, remarkable that this discrepancy of a factor of two is almost the same for Ne and Au despite of the fact that the total kaon yield differs by more than a factor 100. Thus the A dependence of the kaon production cross section is quite well reproduced.

Because experiments are planned for Au + Au at 800 GeV/n as well as for Ni + Ni at .650, .8, 1., and 1.8 GeV/n we display the prediction of our calculation for these future experiments as well. The spectra displayed are for the angles $40° < \theta_{Lab} < 48°$

In fig. 7, the invariant cross section for Au + Au is plotted for the 3 different equations of state. As compared to 1 GeV we expect a reduction of the kaon production by a factor of three as well as a steeper slope of the spectra. In fig. 8 we display $d\sigma_{Kaon}/dp_{Lab}d\Omega$ for the system Ni + Ni. We observe that above threshold we do not find a dependence of the kaon yield on the static equations of state. This finding has also been observed for the pion production above threshold. Above threshold kaons are produced dominantely in first chance collisions. Since peripheral reaction are more frequent, most of the kaons come from large impact parameters where no considerable compression is obtained. The lower the energy the more the kaon production is concentrated at central collisions. Therefore with decreasing beam energy the equation of state dependence becomes more pronounced.

5 Details of the production process

The dependence of the kaon yield on the projectile mass (equal to the target mass) for impact parameter averaged and central (averaged over the most central 100 mb) collisions is displayed in fig. 9. In the top row we see the calculated probability that a kaon is produced in a heavy ion collision for E_{beam} = 600 MeV/N, in the bottom row that for E_{beam} = 1 GeV/N. The cross section can be obtained by multiplying N(kaon) with the reaction cross section (resp. 100mb) for inclusive (resp. central) events. In all cases we observe that for static potentials the cross section for the light Ne system is independent of the nuclear equation of state, a result which is quite important if one would like to disentangle the different dependences (like on the optical potential, on the compressibility etc) of the kaon production yield. With increasing mass number the difference between the static equations of state increases. This goes along with the higher central densities for heavier systems. At higher densities the compressional energy for the different equations of states becomes significantly different and therefore the kinetic energy which is available for kaon production [7] as well. This effect is more pronounced in central collisions where a higher density is reached and naturally more important at lower energies. Therefore with decreasing beam energy the equation of state dependence of the cross section increases. Thus the 600 MeV/N collisions are certainly the proper choice if one wants to disentangle the influence of the compression from other dependences.

The momentum dependent interaction suppresses the kaon production by factor of 7 for Ne + Ne at 600 MeV/N. Thus this reaction would be a very good test for the optical potential. For all the other cases the suppression factor due to the momentum dependence of the optical potential is around a factor of 3.

Remarkable is the mass dependence of the kaon production probability. At 1 GeV/N we observe a straight line in the double logarithmic plot corresponding to a power law $N(\text{kaon}) \sim A^{\tau}$. We obtain for central collisions τ= 1.62, 1.38 and 1.48 for the hard, soft and soft+mdi equation of state, respectively. This has to be compared with an $A^{1/3}$ dependence which is expected if one assumes that the incoming nucleon can scatter with all the target nucleons which it can meet traversing the target nucleus on a straight line. Since the calculated A dependence has been confirmed by experiment, kaon production is clearly a very collective process. The calculations at 600 MeV show a power law dependence as well.We obtain for inclusive collisions τ= .94, 1.30 and 1.43 for the hard, soft and soft+mdi equation of state, respectively.

At energies below 1 GeV/N kaons can almost never be created in the first collision between a projectile and a target nucleon even if one assumes that the

Fermi motion can give rise to a larger available center of mass energy than calculated from the beam momentum only. Hence there are two sources of the kaon production. Either a nucleon gains sufficient momentum in collisions previous to the kaon production collision or the additional energy is provided by the mass of the Δ in the entrance channel. In Fig. 10 we have plotted the fraction of the different types of collisions which contribute to the production of kaons. For each collision, in which a kaon is produced, we have recorded the type of the baryons which enter the collision. We find that, independent of the mass of the system, at 1 GeV/N about 60% of the kaon production probability is due to NΔ interaction and about 25% due to $\Delta\Delta$ interactions. Only a small fraction of the kaons, around 15% are produced in NN collisions. At lower energies the influence of ΔN collision increases even because in NN collisions it is very unlikely that sufficient energy is available to create a kaon.

The finding, that about 85% of the kaons are produced in collisions in which Δ's are involved, together with the agreement between theory and experiment allows as a first conclusion that the production via the Δ channel plays an essential role in the kaon production. The importance of this two step mechanism $N_1 + N_2 \rightarrow \Delta + N_1, N_3 + \Delta \rightarrow N_3 + \Lambda + K$, which is due to kinematical reasons virtually absent in p nucleus collisions, can therefore be established due to this subthreshold kaon experiment for the first time. It explains not only the large experimental enhancement factor as compared to an extrapolation from p-nucleus data [32] but it is also expected that this mechanism plays an essential role in the production of other hadrons like $\bar{N}$ or η, where the extrapolation of p-nucleus data underpredict the observed cross section by a large factor [33] as well.

The influence of the different production mechanisms on the momentum distribution of the kaons is shown in fig. 11. We see that the slope of that kaons which are produced in NN collisions is steeper than that of those kaons which are produced in collisions which involve a Δ. This indicates that it is difficult for nucleons to gain in previous collisions sufficient energy to overcome the threshold.

This fact is eluciated from another point of view in Fig. 12. There we display for Au + Au, b < 3 fm, soft equation of state, the invariant cross section for the produced kaons as a function of the number of collisions N_C the baryons in the entrance channel of the kaon production process have suffered before. We observe that even with the Fermi momentum the available energy in the first collisions ($N_C(1) = 1, N_C(2) = 1$) is hardly sufficient to produce a kaon. Furthermore, due to the limitation of the available energy the momenta of the kaons are very low and can in no way generate the experimental spectrum. It should also be mentioned that the number of 'true' first collisions ($N_C(1) = 1, N_C(2) = 1$) is very small, since during the collision a region of 'stopped' nucleons is built up rapidly, so that most of the incoming nucleons will do their first collision with a partner that has already been stopped [34]. Only if we include those reactions

where least 1 nucleon has suffered a collision before ($N_C(1) = 1, N_C(2) > 1$) we get a substantial kaon production yield and the proper momentum distribution. The largest contribution to the kaon spectra is, however, made in collisions of those nucleons which had at least 3 collisions before ($N_C(1) > 3, N_C(2) > 3$). Also this finding points towards a collective process. The many collisions fill the available phase space and hence we find nucleons with a high momentum component according to a Maxwell momentum distribution even if these high momenta have not been available initially.

The same information from another point of view is shown in fig. 13 where we display the invariant kaon cross section as a function of the time at which it is formed. The early produced kaons, show a steeper spectrum because the thermalization is still in progress. Especially the high momentum component is suppressed. The later produced kaons show the expected exponential slope.

During the heavy ion collision Δ's are frequently created and destroyed as long as the system is dense. The finally observed Δ's (or being more exact the pions from the Δ decay), however, are produced during the expansion state and therefore do not carry information about the high density/high compression phase[35,36,37]. Nevertheless they may serve as a test whether the basic production mechanism for the Δ's is correct, a prerequisit for studying the kaon production in these heavy ion collisions. An useful number for checking the dynamics of the Δ's is the number of finally observed pions as a function of the participants. Assuming that the number of participants can be related to the observed charge multiplicity close to beam rapidity, Harris et al. [38] have found that number of pions per participant is constant independent of the number of participants and the mass of the projectile/target system. It only depends on the beam energy. This experimental observation has been verified in QMD calculations by Rosenhauer [39]. He assumes in the numerical simulations that all geometrically overlapping nucleons are participants whereas the others are spectators. For the present reactions at 1 GeV/n we observe an average number of pions, the sum of π^+, π° and π^-, of 0.22 per participants independent of the number of participants. The same independence has been observed for a limited acceptance in the experiment [14]. Applying the isospin model, i.e. assumption that the production of Δ's and their subsequent decay is determined by the isospin Clebsch Gordon coefficients only, we expect that in Au + Au reactions 21%, 41%, 38%, of the produced pions are π^+, π° and π^-, respectively. However, if one takes the different inelastic cross section for pp and pn reactions into account the relative contribution of π^+, π° and π^- should be 24%, 33%,43%. The absolute value of π^-'s per participant reported by Harris et al. [38] for La + La at 990 MeV/N is around 35% smaller than that obtained in our calculation. A part of this difference, however, can be easily attributed to the different methods to determine the number of participants as well as to the experimental cuts which are severe for the low energy pions.

The kaons, on the other hand, show a completely different dependence on the number of participants. In fig. 14 we observe a decrease of the number of kaons per participant with a decreasing number of participants for two parametrizations of the elementary production cross section. The dots, triangles and squares mark Au+Au, Nb+Nb and Ne+Ne collisions, respectively. The probability per participant that a kaon is created in peripheral Au + Au collisions is only 60% of that in central collisions. We observe that the kaon production probability as a function of the participants is independent from the projectile target combination. Therefore, peripheral Au + Au reactions and central Ne + Ne collisions with the same number of participants yield the same number of kaons. This finding, if confirmed by experiments, reduces the task to find the projectile/target mass dependence to a study of the multiplicity dependence of the kaon yield for a heavy system, an encouraging perspective in view of the large cross section for peripheral reactions of large systems as compared to that of small systems. Of course this dependence will be different for different equations of state because in peripheral reactions the influence of the equation of state is little whereas for central collisions we expect differences of the order of a factor of two. We observe furthermore a strong reduction of the kaons per participant employing the Zwermann cross section as compared with the calculation using the Randrup and Ko cross section with $\sigma_{NN\to\Lambda} = \sigma_{NN\to\Sigma}$, as anticipated by the total production cross section of fig. 1.

It is interesting to investigate in detail the reason for the decreasing number of kaons per participant. In fig. 15 we display this quantity as a function of the different production channels. About 50% of that reduction is due to the decrease of the $\Delta\Delta$ collisions. This decrease is expected since the pions/participant, i.e. the Δs per participant, are constant. Thus for small systems the $\Delta\Delta$ collisions become less frequent. However, also the NΔ and even more the NN channels shows a similar behaviour. A hint on the physical reason is given by the observation that also the average available $\sqrt{s}$ decreases with decreasing participant number in these channels.

As we have seen the system comes close to equilibrium. We therefore expect a spectrum of the nucleons close to a Maxwell spectrum. Deviations become only important if the particle energy is of the order of the total available energy. In heavy ion reactions the total available energy is proportional to the number of participants. For large systems the total energy is much larger than the single particle energy relevant for the kaon production. For small systems, however, this is not the case and at the relevant single particle energies the spectra deviates already substantially from a Maxwell spectrum, i.e. it is steeper. Therefore the kaon yield is reduced.

Of course this reduction should be also present for other particles which are produced in collisions with a similar $\sqrt{s}$. Indeed the KaoS collaboration has found

that high energetic pions are suppressed as well [40]. However, very small in number, they do not influence the total number of pions per participant in a noticable way.

Thus the decrease of the kaons per participant with decreasing participant number has two independent reasons: on the one hand this is a consequence of the finite available energy in a system which contains only few participants. On the other hand it follows from the smaller probabilities of $\Delta - \Delta$ interactions in small systems.

It is difficult to compare the calculated number of kaons per participant with experiment. The statistics does not allow the search for this quantity in the limited acceptance region. Since in central collisions the system comes close to equilibrium we do not expect that this ratio depends on the angle. It can therefore be compared with the calculated value of $3.3 \cdot 10^{-3}$ in 4π. The values agree quite well. The difference of 50% can easily be caused by the ambiguities of the extrapolation from the small solid angle where the particles have been observed to 4π. This confirms another time that the pion dynamics is reasonable reproduced in our approach.

Finally we investigate whether the conjecture that the kaons test the high density region can be substantiated. As a function of the beam energy in fig. 16 we display the average density at that space point at which kaons (squares) and pions (triangles) have been created. For the case of kaons we display the maximal density observed at the creation point as well. The open symbols refer to QMD calculations the closed symbols to VUU/BUU calculations [8].We observe that the kaons come indeed from the high compression zone in contradistinction to the pions. This explains the much stronger equation of state dependence of the kaons as compared to the pions.

6 Comparison with other models

Since it was pointed out, that the kaons may provide a good tool to measure the nuclear equation of state [7,10] other groups have joint the effort to predict the different dependences of the kaon yield.

Cassing et al. [6,17] have advanced calculations based - as those of ref. [7] - on the BUU approach. They claim that Δ's are not important for the production of the kaons but only their decay products, the pions, assuming a zero life time of the Δ's which are produced in the elementary reaction N + N $\to$ Δ +N. Thus kaons can only be created in NN and πN collision. As mentioned, in our approach as well as in ref. [7] the Δ's have an infinite lifetime with respect to the decay into pions but they are frequently destroyed by N + Δ $\to$N + N collisions on a time scale which is comparable with the decay time. The reaction cross section for this process is determined from detailed balance.

The BUU/VUU calculations use a mean field whereas in the QMD approach explicit two body interaction are employed. We have investigated whether these differences of calculating the propagation of the particle can have a influence on the kaon production. The result of this study is displayed in fig. 17. There we compare the kaon production obtained by three completely independent numerical programs VUU[8], IQMD [21,35] and QMD. The VUU employs mean field dynamics as the BUU calculations. In the IQMD as well as in the QMD model the particles interact via two body forces. In distinction to QMD the IQMD (as well as the VUU) model allows the decay of Δ's, employing a mass dependent decay width. $\pi N \rightarrow K^+ + X$ collisions are neglected. As we can see the differences in the three programs are on the 25% level. Half of the difference between IQMD and QMD is due to the different treatment of the Δ's. The remaining difference is due to the completely different numerical methods employed for the solution of the time evolution equations. Thus we expect no big difference between the prediction of the kaon production between the QMD and BUU/VUU models.

In order to compare our calculation with that of the Giessen group we have divided the published data (Fig. 7.4 of ref [6]) by the total reaction cross section which is assumed to be 3342 mb, 1905 mb and 854 mb for the Nb, Ca and C reactions.

The upper part of fig. 18 presents the target/projectile mass dependence at 1 GeV/N for both approaches. We observe a quite different A dependence. For large target/projectile masses the neglect of the Δ channel reduces the kaon production probability by almost one order of magnitude.

In view of the fact that even QMD produces fewer kaons than are seen in the experiment, the approach by of ref. [3,17]is clearly far off the experimental results. Thus in heavy ion reactions the Δ is much more effective in creating a kaon as compared to its decay product, the pion. This is a consequence of the kinematics. Due to its mass the Δ may contribute about 300 MeV to the threshold energy for kaon production, the pion only 140 MeV. This finding is verified quantitatively by Xiong et al. [41], who showed that at 800 MeV/N for Ca+Ca the πN channel contributes only 20% to the total kaon yield, whereas the rest is due to baryon baryon collisions. Indeed, if one multiplies the production probability used in ref.[6,17] by a factor of 5 one finds a reasonable agreement with the QMD results.

The beam energy dependence of the two approaches, ref.[6,17] and the work presented here, however, is very similar. This can be seen in the lower part of fig.18. Here the kaon production probability is plotted for different system obtained in both approaches as a function of the beam energy. We would like to mention that this earlier result [6,17] is superseeded by the recent calculation which includes the Δ channel [42].

Two calculations have been advanced after the experiment has been performed. They use the relativistic extension of the BUU model, dubbed RVUU. Lang at al.

[42] use a modified Walecka model (by including nonlinear σ terms) whereas Ko [18] uses the original linear Walecka model.

Both calculations differ in the parametrization of the elementary kaon production process. In ref. [18] a one boson exchange model is used for obtaining the $NN \rightarrow K^+\Lambda N$ cross section. Since the Σ channel is not taken into account, the parametrization underpredicts the kaon yield by a factor of about two. Ref. [42] uses the Zwermann cross section with the modification that $\sigma_{\Delta\Delta} = .5\sigma_{NN}$ and $\sigma_{N\Delta} = .75\sigma_{NN}$. The Zwermann cross section itself yields a factor of three reduction of the kaon yield as compared to the standard Randrup and Ko parametrization employed in our calculation (see fig. 1 and 6). The modification suppresses the elemenatry cross section to about one forth of the standard parametrization.

The optical potential in the relativistic mean field models raises linearly with the beam energy whereas that measured in the p-A reactions saturates for energies larger than 200 MeV. In ref [42] a linear approximation to the optical potential in between 0 and 200 MeV and 0 and 1000 GeV, respectively. has been made,using two different forms for the sigma terms. The measured optical potential values are therefore in between these two boundaries.

The momentum dependent interaction relected in the optical potential reduces the kaon yield by a factor of three as compared to a static potential with the same compressibility [10]. This has originally been shown for the system La + La , but fig. 6 shows that the same strong supression by a factor of 2-3 is also observed for Au + Au and Ne + Ne. Since in ref [42] a similar form of the optical potential is used a similar reduction should occur in these calculations. Combining both sources of reduction we expect a total reduction factor of 12 as compared to a static equation of state with the standard kaon production parametrization. However, in ref. [42] no reduction is found at all but complete agreement with data. This is shown in fig. 19. This discrepancy remains unclear.

The Walecka model yields a optical potential which is too strong at high momenta as compared to the experimental value. It is obvious that therefore the production of the kaons is suppressed more strongly than with a realistic parametrization. This additional suppression as compared to a realistic optical potential can easily give an additional suppression factor of two. Adding all factors together the calculations of Ko should give a factor of 8 - 12 lower kaon probabilities as compared to a static soft equation of state. Indeed a suppression of this order of magnitude is reported in ref. [18] and can be seen from fig. 19

Thus we can conclude that the differences between calculation of Ko et al. and the present approach are due to different optical potentials and different parametrizations of the elementary cross section. The calculations of ref. [42] differ by a factor of 10 from the extrapolation of our calculations.

We would like to mention that calculations using the original program [7] have been also performed by the Tübingen group [43] .

7 Kaon production in heavy ion collisions above threshold

In this section we want to give a short account of the extension of our calculation to energies above threshold. At energies above threshold we expect that the production in the first collision between a projectile and a target nucleon becomes the dominant production process for kaons. Thus at this energy the physical process is completely different from that for subthreshold production. Experimental data are available at 2.1 GeV for the reaction Ne + Ne [44]. These data show an isotropic distribution of the kaons in the NN center of mass system and a very large slope of T = 122 MeV if one assume a maxwellian spectrum. This slope is determined mostly by the data at $\theta_{lab} = 80°$. Early calculations in the cascade model [25] failed to reproduce the slope of the data. This finding has been confirmed by analytical calculations which required either unreasonable large Fermi momenta, i.e. an unlimited exponential fall off of the Fermi momentum or collective production mechanisms, i.e. scattering on a cluster of nucleons, to reproduce the experimental slope. For details we refer to ref. [44]. Since the influence of the potential interaction is low for this light system at this high energy and relativistic corrections [13] can at most count for a small increase of the slope we do not expect a large difference between our calculation and the cascade approach, because the applied elementary cross sections is the same. Indeed we find about the same slope, as displayed in fig. 20 where we compare our results with the experimental data [44]. This result has been veryfied in an relativistic RQMD approach [12] if one corrects the misprint of the experimental high energy data point at at $\theta_{lab} = 80°$ in fig. 20.

There may be two reasons for this discrepancy, which however cannot be explored presently due to the perturbative approach to the kaon production or due to the lack of knowledge. The first reason may be the neglect of rescattering effects. At 2.1 GeV the kaons have still a much lower momentum than the surrounding nucleons and therefore can increase their momenta in K^+N collisions. A second reason has been pointed out recently by Laget [23] who showed that the measured momentum of the kaon [27] in elementary pp collisions at 2.3 GeV is larger than expected from phase space considerations due to the strong final state interaction between the nucleon and the Λ. No experiments exist, however, which allow to extract the beam energy dependence of that effects. As mentioned, at the next higher energy where data exist [28], at 2.53 GeV, the kaon momentum distribution seems to be in perfect agreement with a decay according to the available phase space.

We would like to mention that our integrated cross section for kaon production for Ne + Ne 2.1 GeV/n of 20 mb agrees well with the experimental value of 23 ± 8 mb [44].

Recent preliminary results, however, may also point towards an error in the experimental data set. The KaoS collaboration finds slope values of around 84 MeV[16], much closer to the value of 70 MeV we obtain in our calculation as the value of 122 MeV from the data of Schnetzer et al [44].

8 Conclusions and Perspectives

We have presented microscopic calculations of the production of kaons and pions in relativistic heavy ion collisions. These calculations, performed before the subthreshold kaon production was studied experimentally, agree quite well with recent experimental findings. The calculated absolute cross section reproduces the data in between of factor of two and the slope of the kaon spectra is reproduced. In view of the fact that the kaon yield at the lowest energy measured before is two orders of magnitude larger and that the production mechanism is quite different below subthreshold, this theoretical prediction proves that the very complicated dynamics of heavy ion collisions is quite well described in the present microscopic approaches.

In agreement with experiment we have found that the kaon production probability per participant as a function of the number of participants decrease with decreasing number of participants. This points towards a highly collective process, which yields not only a very strong A dependence of the production probability but also requires many collisions before a nucleon can produce a kaon. This dependence reflects the limited available energy as well as the prodaction via the $\Delta\Delta$ channel. With an increasing number of participants the available energy increases and therefore the probability that nucleons can acquire sufficient energy to create a kaon as well. The dominant production process at subthreshold energies is the two step process $N_1 + N_2 \rightarrow N_1 + \Delta$; $\Delta + N_3 \rightarrow N_3 + \Lambda + K$ because the Δ has a higher mass which can be used to create a kaon.

One of the aims in studying subthreshold particle production is the extraction of the nuclear equation of state. This goal has not been achieved yet although we could show that the kaons come really from the high density region.

On the experimental side further experiments which explore the excitation function of the kaon production are necessary. Also the theoretical model has to be improved to allow the accuracy of prediction with is necessary to predict a factor of two effect. These improvements include a more detailed treatment of the delta and the pion, studies of the in medium properties of the kaon production cross section,

which seem to be present but not very large [24], and a proper parametrisation of the momentum distribution of the kaons in the elementary production process. Furthermore it may be possible, that simultaneous collisions among many nucleons may contribute to the kaon production, an idea which was recently advanced by Batko et al. [45]. However if one invokes this type of collisions one has to explain why it is obviously not present in p - nucleus collisions and in subthreshold π production. Both processes have been well described by models restricted to two body collisions.

However, a prerequisite for a better understanding of the kaon production in heavy ion reactions are experimental studies of the elementary production process. The predictive power of the microscopic theories can not be more accurate than the input. Presently, the kaon production cross section close to the threshold has uncertainties of at least or factor of two. This is about the same value as the present discrepancies between theory and experiment. This also coincides with the difference of the kaon yield expected for the two different static equations of state.

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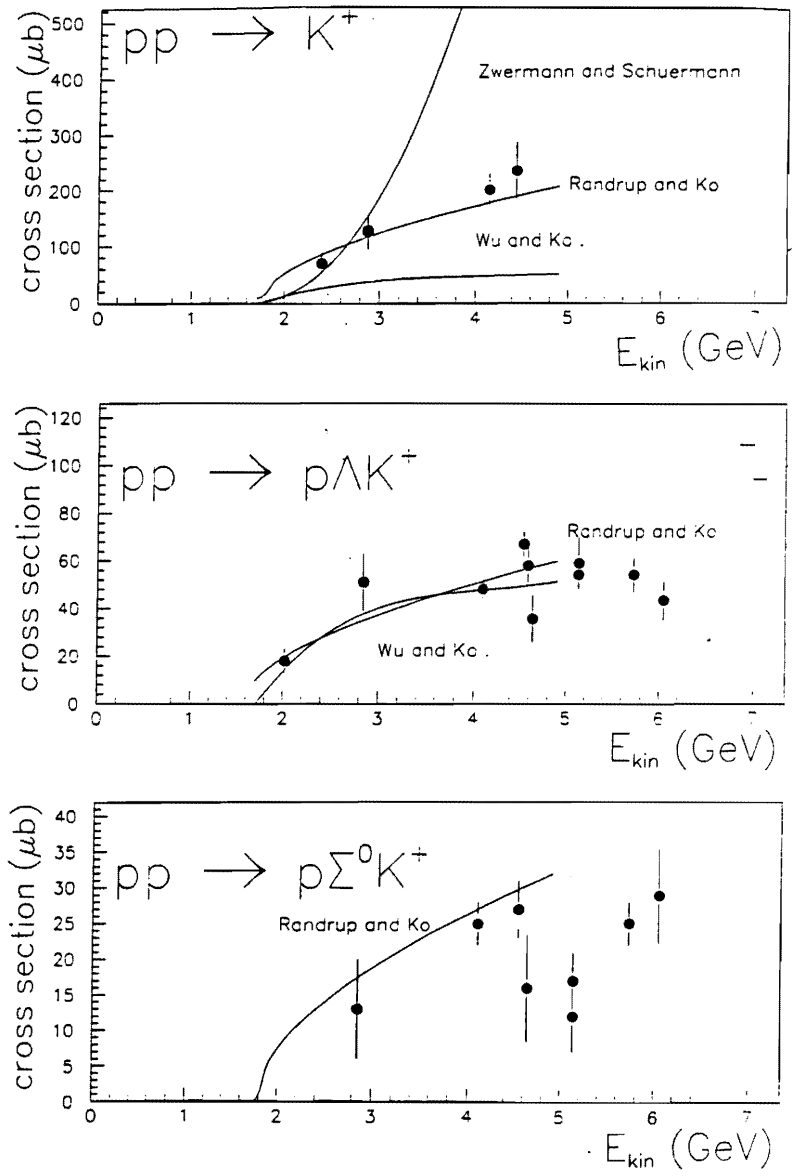

Fig. 1: World data of the elementary production of kaons close to the threshold as compared with different parametrizations.

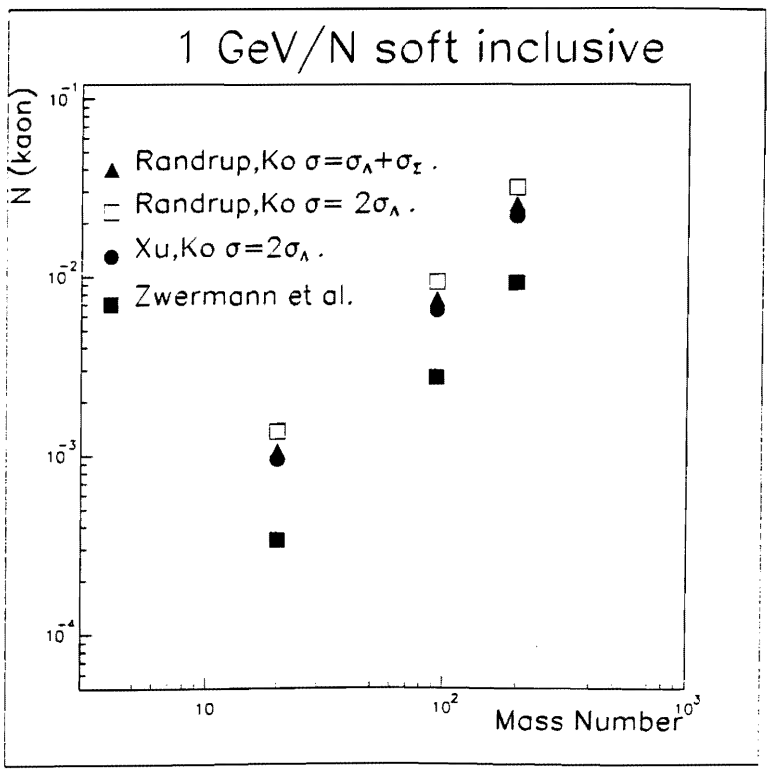

Fig. 2: Average kaon production probability as a function of the projectile (equal target) mass number for the reaction 1 GeV/N, soft equation of state for different parametrization of the elementary production process.

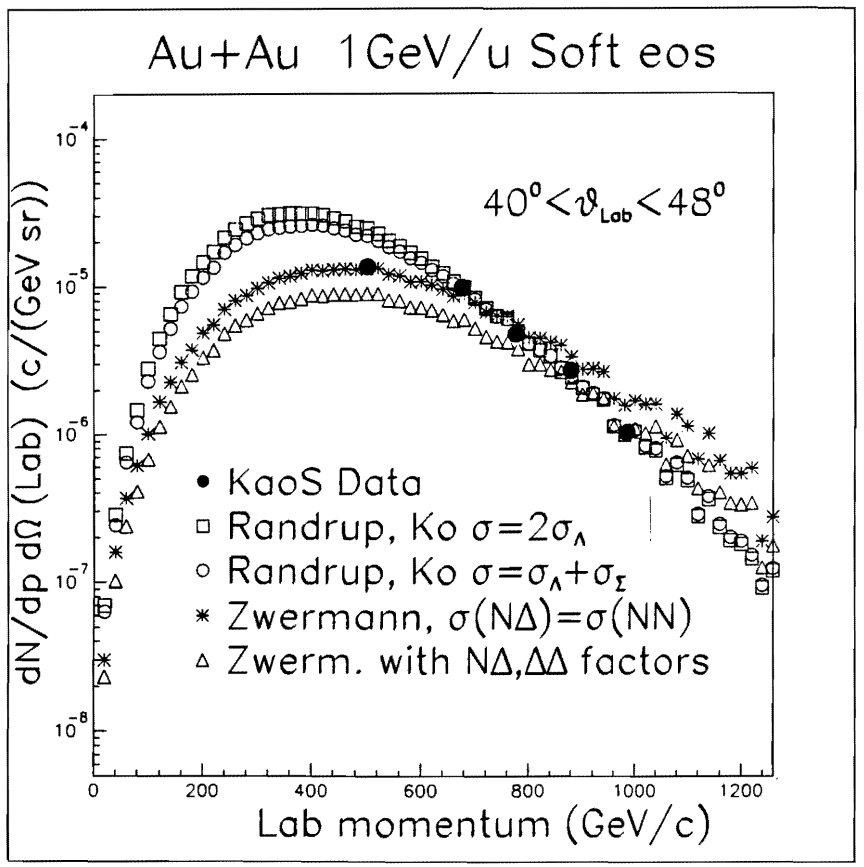

Fig. 3: Momentum distribution of the produced kaons at 1 GeV/n Au + Au, soft equation of state for the different parametrizations of the elementary production process as compared to data.

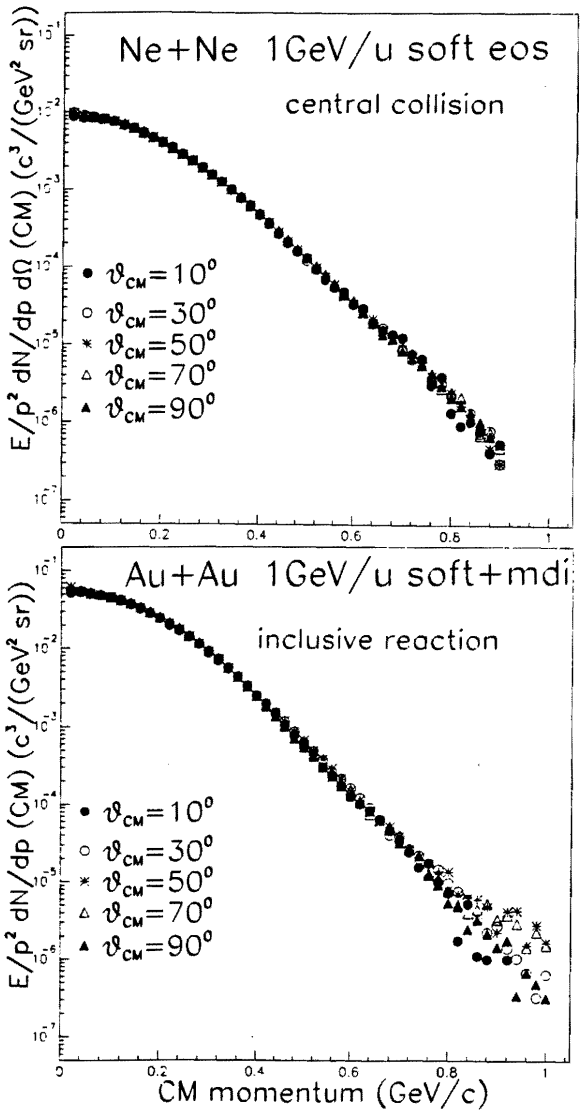

Fig. 4: Angular distribution in the NN center of mass system of the produced kaons for the reactions Ne+Ne (S) and Au + Au (SM) at 1 GeV/N.

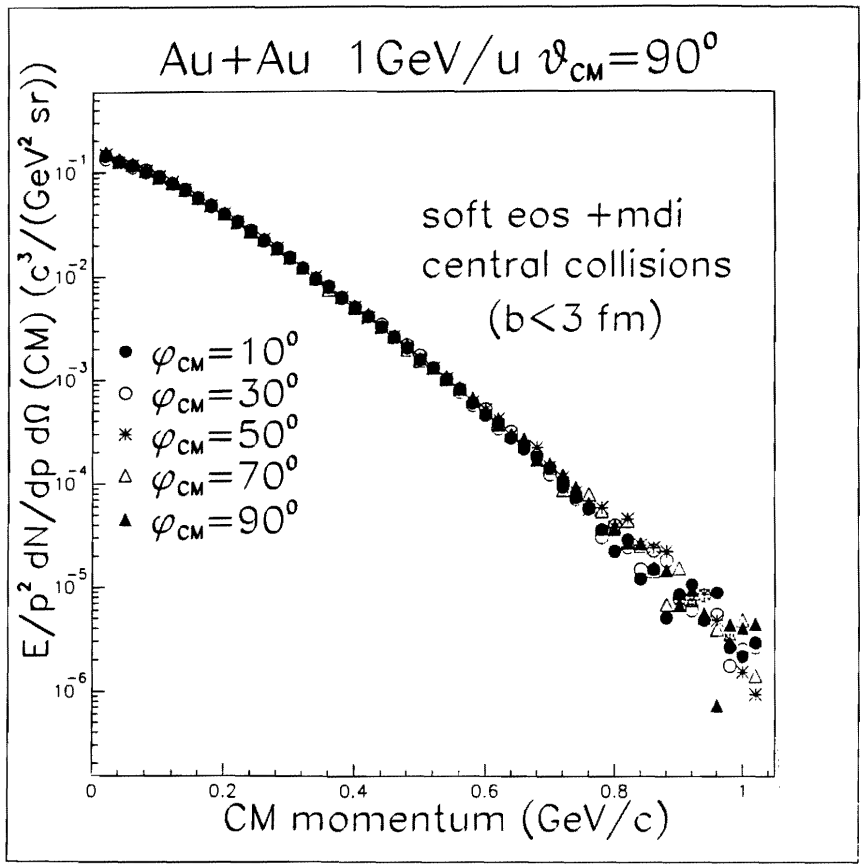

Fig. 5: Azimuthal distribution in the NN center of mass system of the produced kaons for the reaction Au + Au at 1 GeV/N at θ_{cm} = 90°, calculated with an equation of state with included momentum dependence.

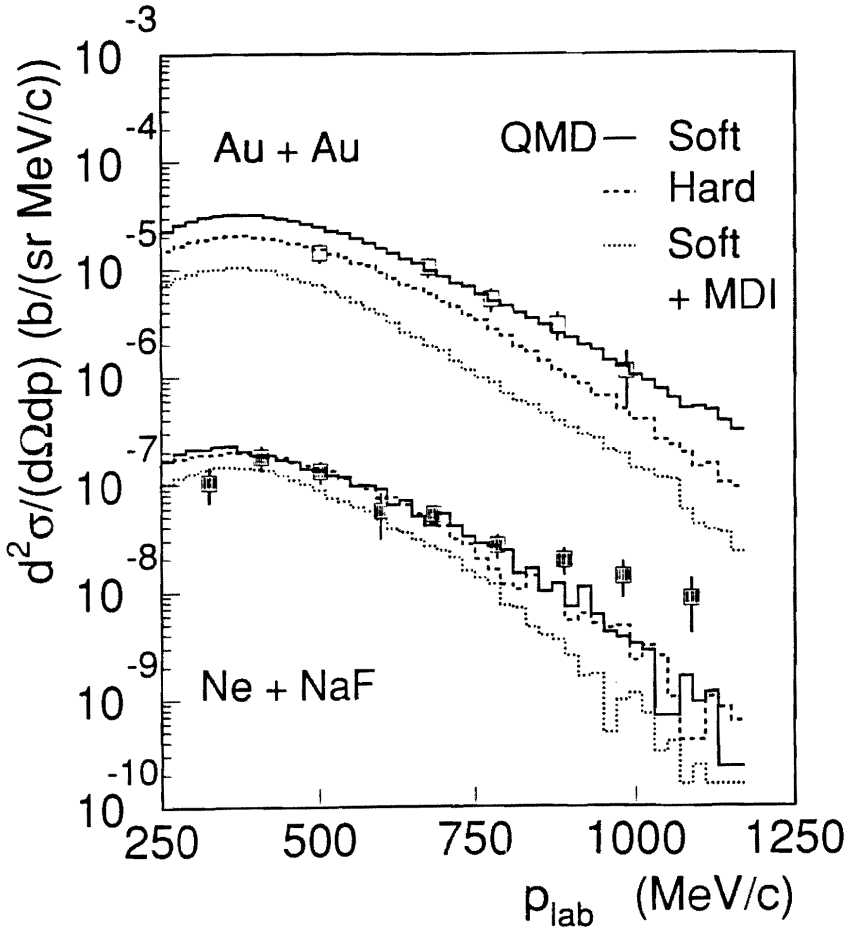

Fig. 6: Comparison of the experimental kaon momentum spectra with QMD calculations for Ne + Ne and Au + Au at 1 GeV/N. The calculations have been performed for the three different equations of state as explained in the text.

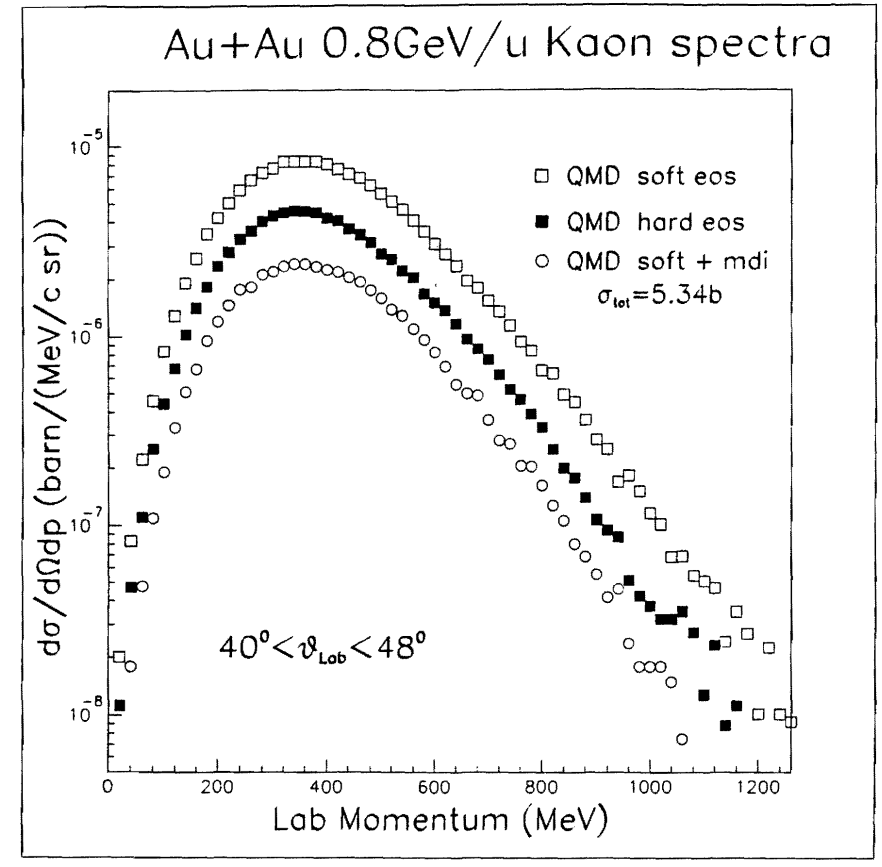

Fig. 7: Prediction of the kaon momentum spectra for the reaction Au + Au at .8 GeV/N. We performed the calculation for the three different equations of state explained in the text.

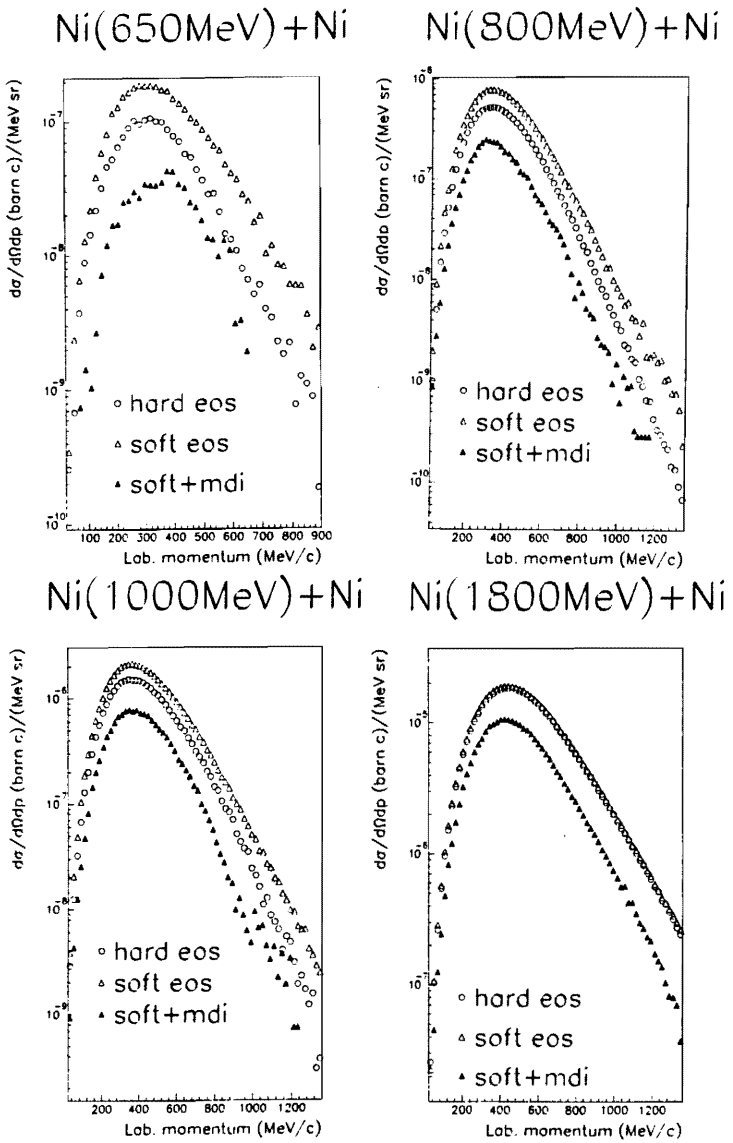

Fig. 8: Prediction of the kaon momentum spectra for the reaction Ni + Ni at .650, .8 ,1. and 1.8 GeV/N. We performed the calculation for the three different equations of state explained in the text.

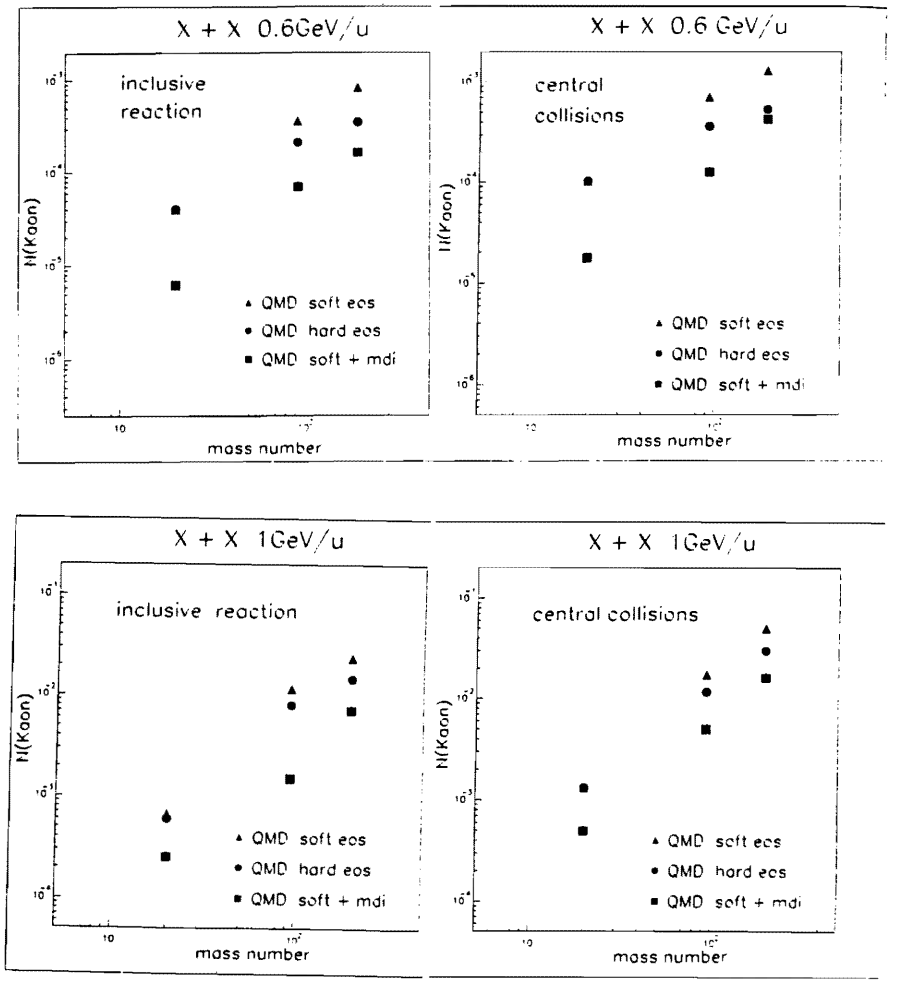

Fig. 9: Calculated target/projectile mass dependence of the kaon production probability for inclusive (left) and central (right) reactions. At the top row we present our calculations for .6 GeV/N at the bottom row for 1 GeV/N.

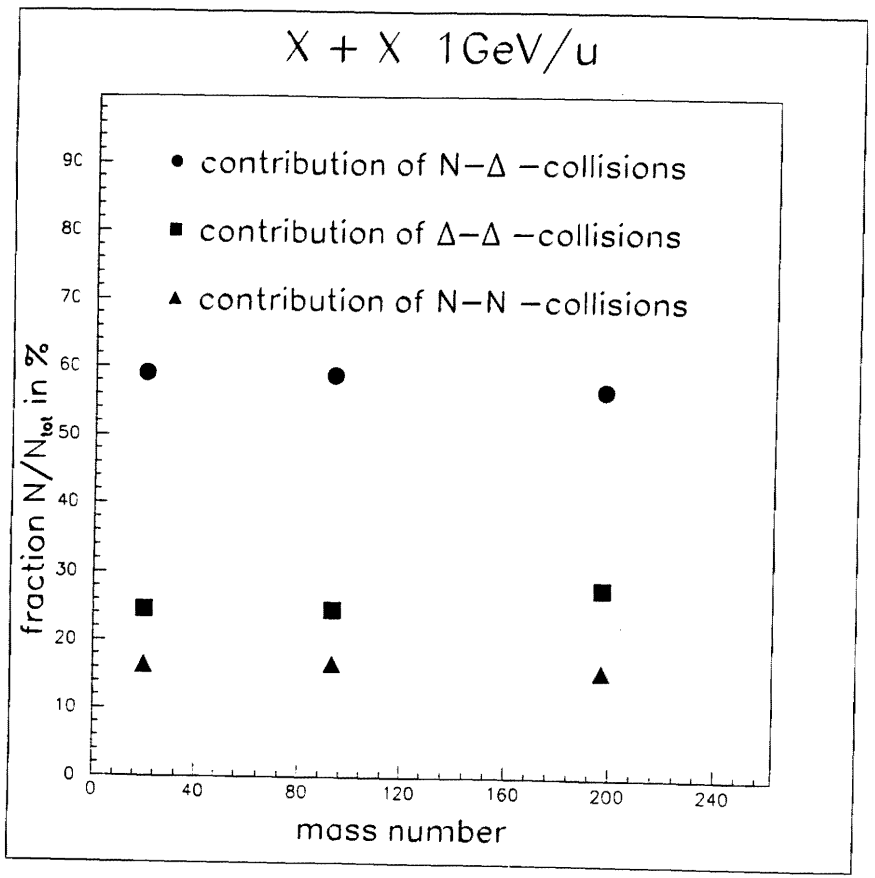

Fig. 10: Contribution in % of the different production channels to the kaon production probability.

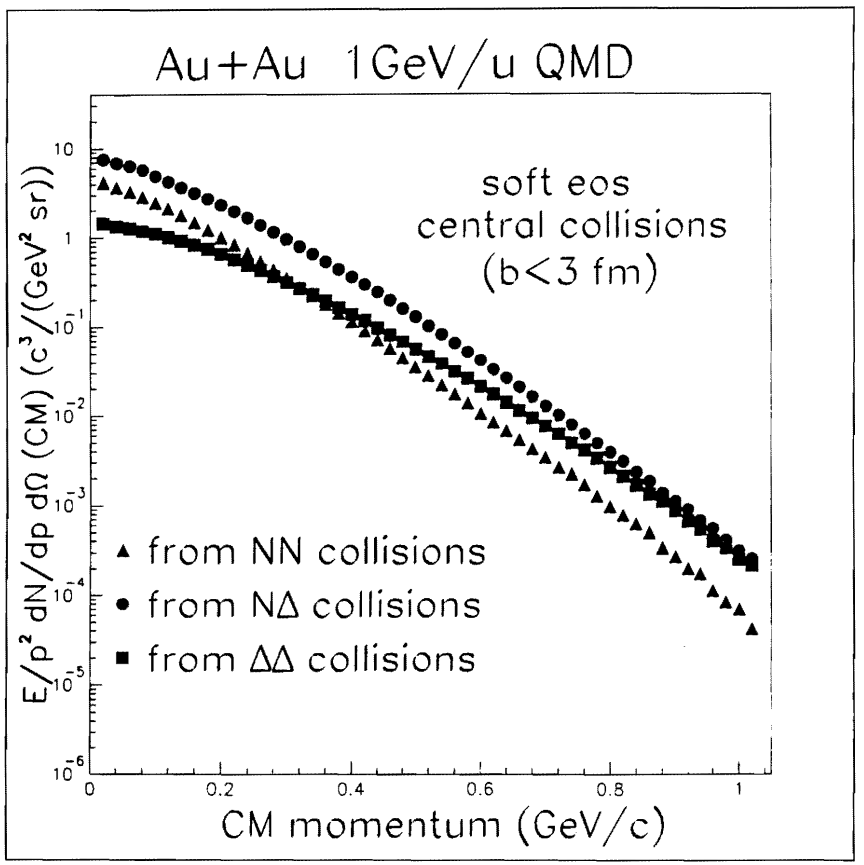

Fig. 11: Invariant cross section of the kaons produced in $\Delta\Delta, \Delta N$ and NN collisions at 1 GeV/N Au + Au, soft equation of state.

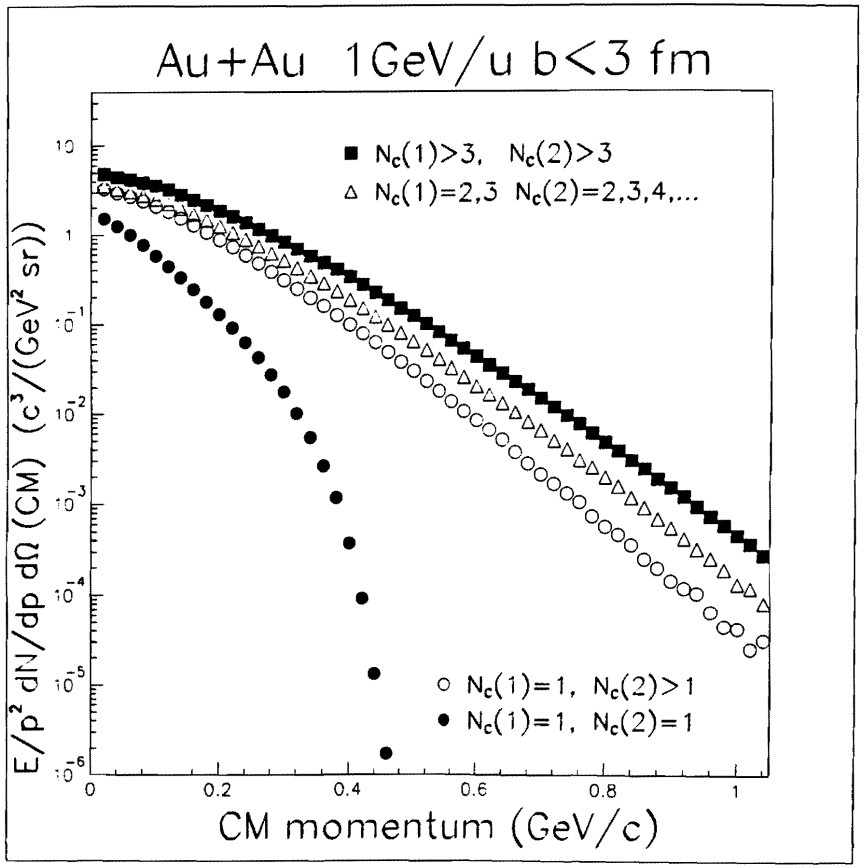

Fig. 12: The invariant cross section of the produced kaons (Au + Au , b < 3fm) as a function of the number of collisions the nucleons in the entrance channel of the kaon production process have suffered before.

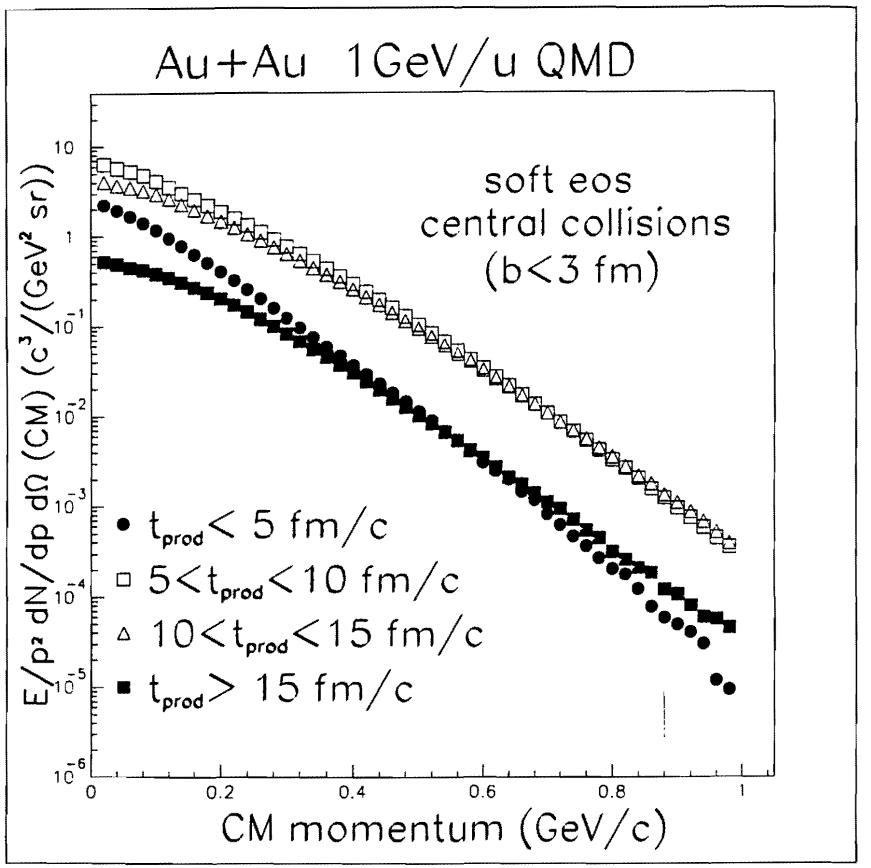

Fig. 13: The invariant cross section of the produced kaons as a function of time. The reaction investigated is Au + Au 1 GeV/N b < 3fm, soft equation of state.

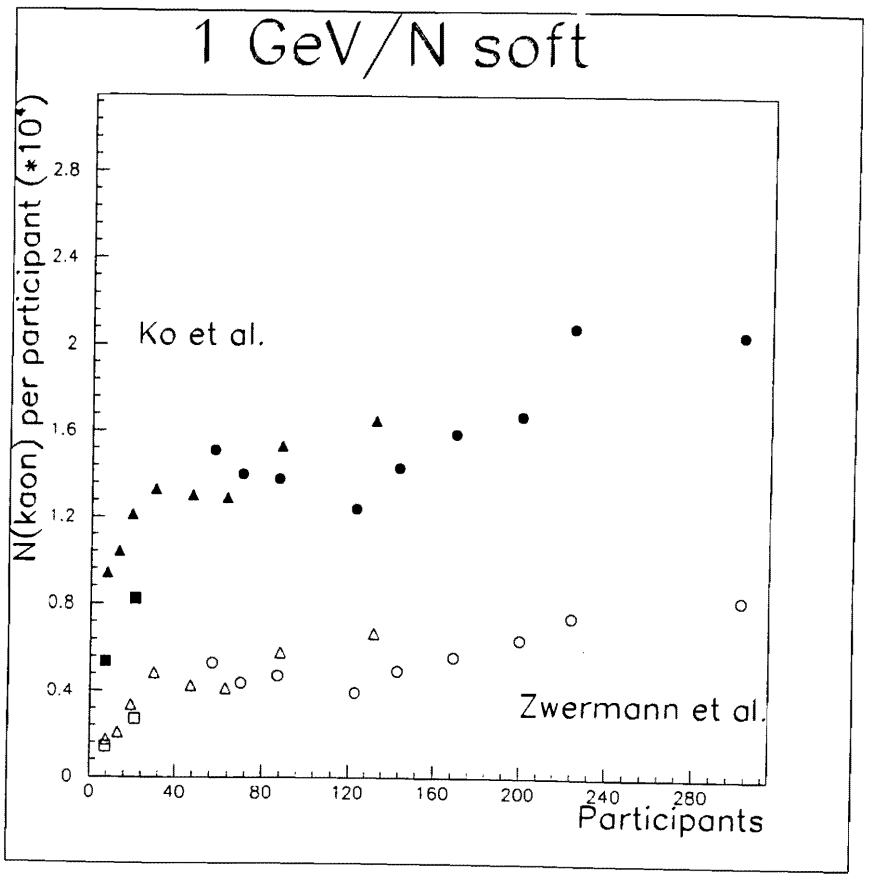

Fig. 14: Kaon production probability per participant as a function of the participant number at 1 GeV/N. We employed two parametrizations of the elementary production process and a soft equation of state. The different symbols refer to different projectile target combinations explained in the text

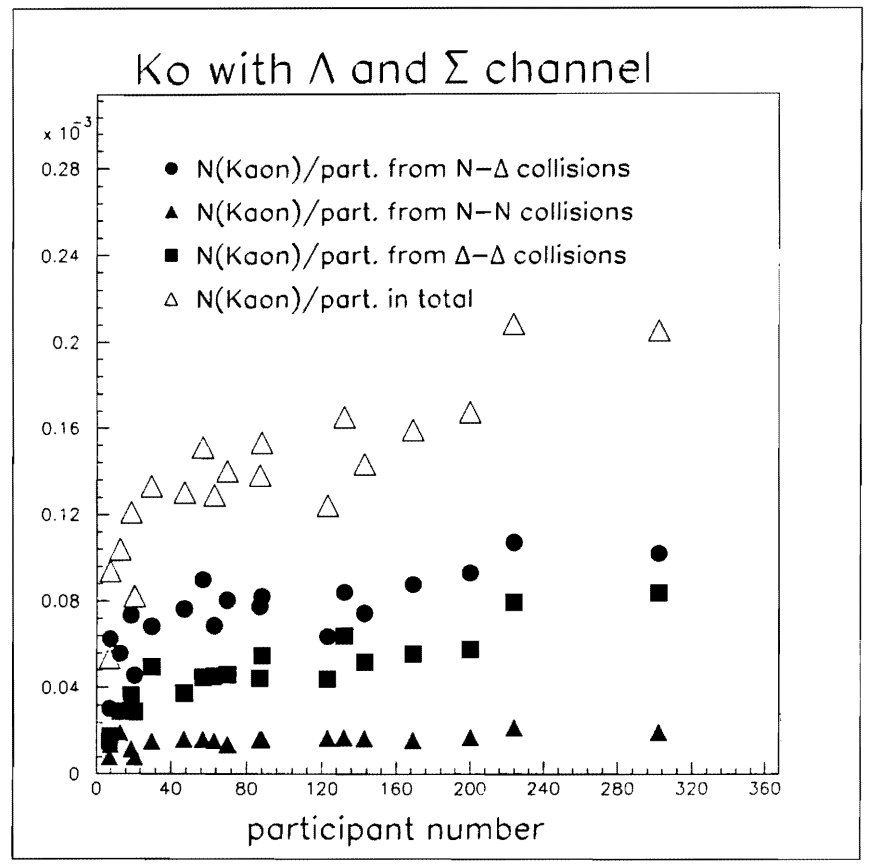

Fig. 15: Kaon production probability per participant for the different production channels as a function of the participant number at 1 GeV/N. We employed the Ko parametrizations including the Σ.

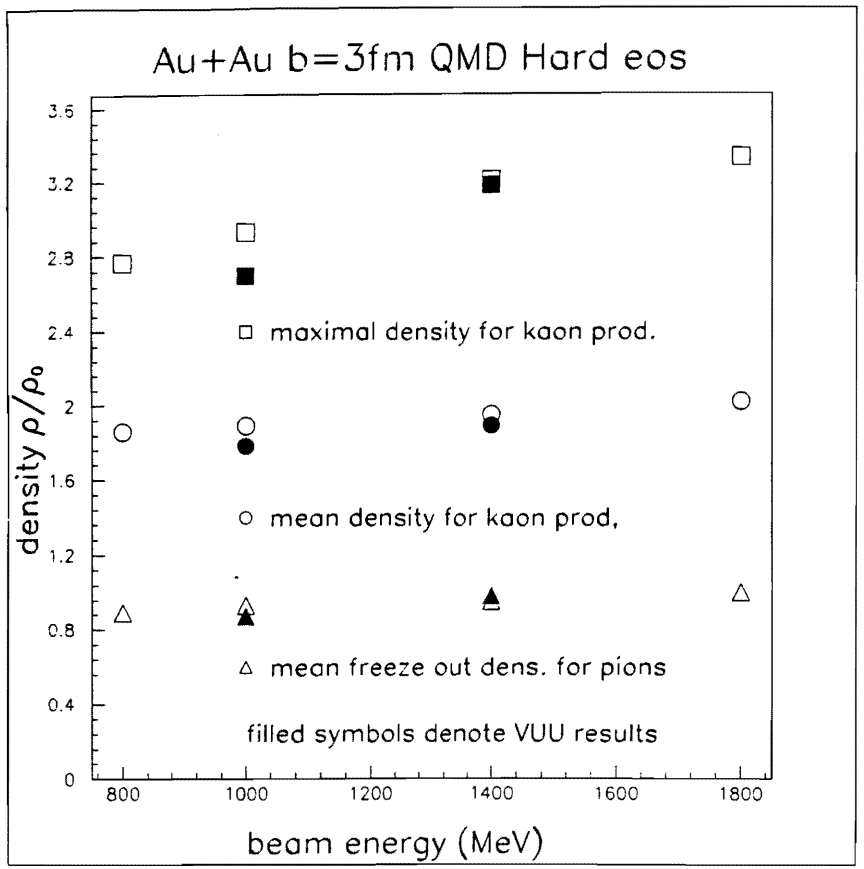

Fig. 16: Average nuclear density at the space points where Kaons (squares) and pions (triangles) are created. For kaons we display also the maximal observed density (circles). Open symbols refer to QMD calculations, close symbols to BUU/Vuu calculations. The reaction is Au + Au b = 3fm, hard equation of state

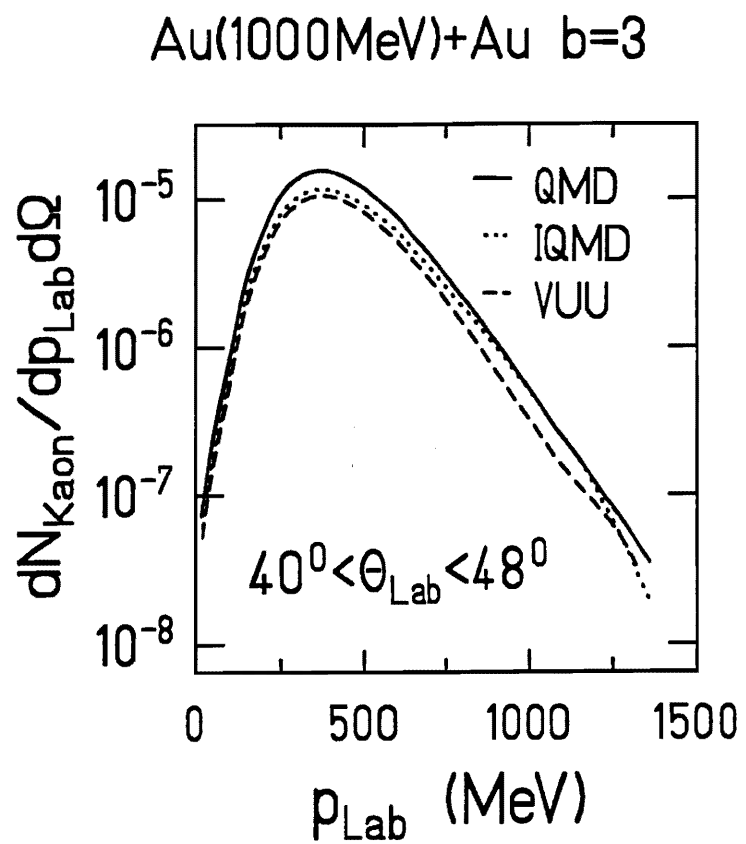

Fig. 17: Momentum distribution of the kaons for the experimental acceptance obtained with three different programms : QMD, IQMD and VUU, all calculated at a fixed impact parameter of b = 3fm.

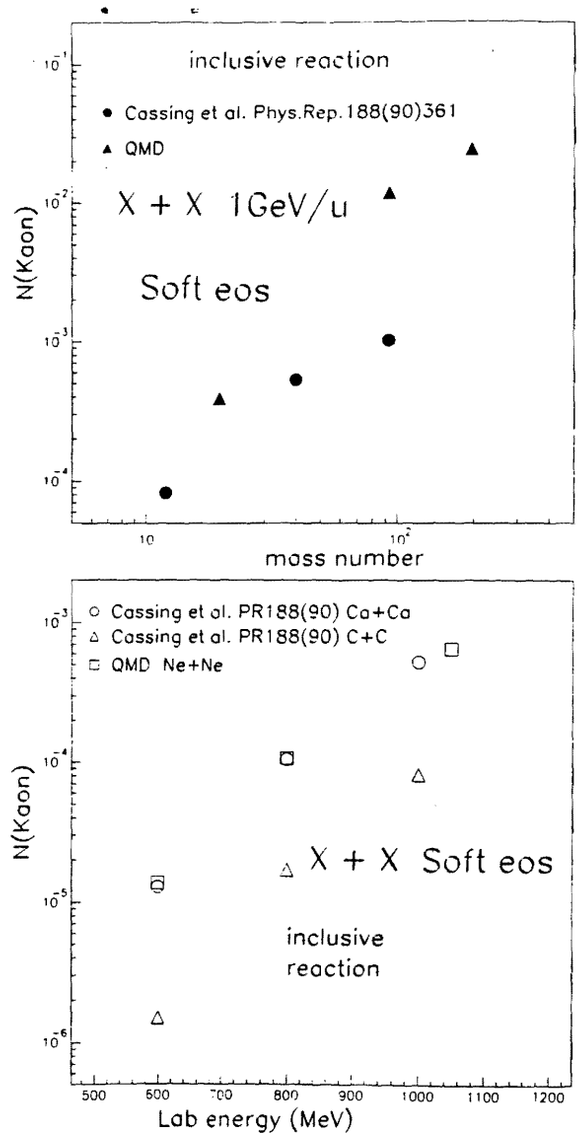

Fig. 18: Top: Comparison of the kaon production probability as a function of the projectile/target mass in QMD with the model of Cassing et al.
Bottom: Comparison of the kaon production probability as a function of beam energy in QMD with the model of Cassing et al. The main difference is the infinitely short life time of the Δ in the model of Cassing et al.

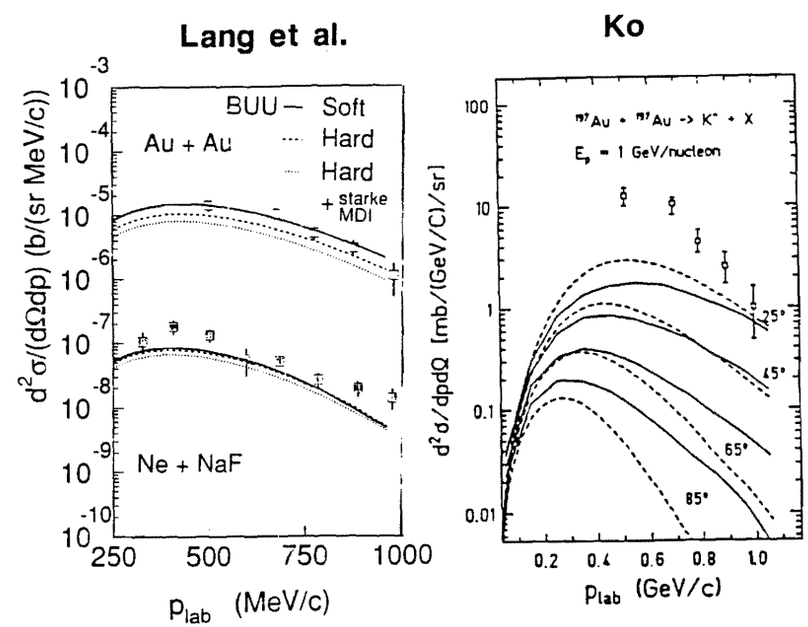

Fig. 19: Invariant cross section as compared to the experimental data for the two available RBUU calculations.

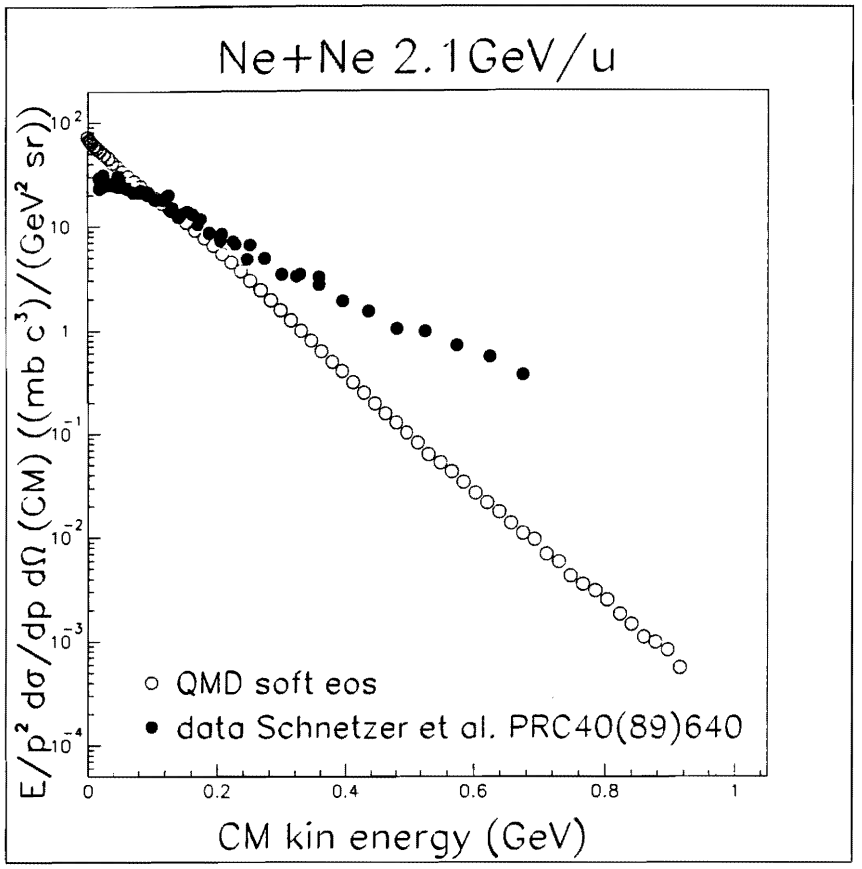

Fig. 20: Comparison of the kaon energy distribution in the NN center of mass system as compared with experiment for the reaction 2.1 GeV/N Ne + Ne