Multiple Scattering Effect on Radiation of High Energy Electrons in a Thin Layer of Substance.

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Abstract

The influence of the multiple scattering on radiation of ultrarelativistic electrons in a thin layer of substance has been considered. Quantitative results concerning the coherent radiation suppression in a thin crystal, analogous to Landau-Pomeranchuk effect of bremsstrahlung suppression in amorphous medium, have been obtained by computer simulation. They indicate that the effect could already be observed for electron energies of 10 GeV. Thus, an experimental verification can be undertaken at the CERN.

1. Introduction

The radiation process induced by a relativistic electron penetrating the matter take place in a large spatial region along the particle trajectory. The extension of this region is rapidly growing with the energy of the particle. If this length (usually called the coherence length l_c [1,2]) is large compared to the average interatomic distance, then the induced effect differs to a large extent from the case when the phenomenon could be described as a succession of individual particle-atom interactions.

Landau and Pomeranchuk have pointed out [3] that multiple scattering of high energy electrons in an amorphous medium in the limits of the coherence length l_c can significantly modify the bremsstrahlung radiation spectrum. Compared with Bethe-Heitler [4] results the phenomenon can be described as a reduction of the radiation spectral density in the low frequency domain. Though theoretical predictions have been available for almost four decades, the first detailed experimental studies have been performed only recently [5] at the SLAC accelerator at an electron energy $E = 25$ GeV.

The Landau-Pomeranchuk (LP) effect takes place for a target thickness L large compared to the coherence length ($L \gg l_c$). The opposite case of a thin target $L \ll l_c$. has been considered in ref. [6]. It has been shown that in this particular condition, as it is the case with $L \gg l_c$, multiple scattering can induce a decrease of the radiation cross section compared with the result of Bethe and Heitler. However, the formulas describing the influence of multiple scattering on the radiation in a thin target differ considerably from the corresponding formulas describing the Landau-Pomeranchuk effect (the dependencies on target thickness, radiated photon frequency and particle energy are different).

The influence of multiple scattering on the radiation of high energy particles in a thin layer of matter has not yet been investigated experimentally. The only experimental results

available giving indications in favour of this effect were obtained at SLAC in the framework of LP effect studies. They concern figures of bremsstrahlung radiation spectrum for 25 GeV electrons impinging on a gold target $L = 34$ microns thin [5].)

Hereafter arguments are recollected in order to show that the influence of multiple scattering on the radiation of a relativistic electron in a thin layer of substance can valuably be studied at incident electron energies of the order of 10 GeV. They are based on the results according to which multiple scattering exerts its influence not only on the bremsstrahlung radiation of a relativistic electron in an amorphous medium but also on the coherent radiation in crystals [6]. The conditions for occurrence of the latter effect are met at lower electron energies as it affects a broader frequency range of radiated photons than the former. Therefore an experimental study can be performed at the CERN.

2. Radiation in a thin layer of substance.

In the framework of classical electrodynamics, the spectral density of radiation of a fast charged particle moving in an external field along the trajectory $\vec{r}(t)$ in the frequency range $(\omega, \omega + d\omega)$ can be cast in the form [2,9]

$$\frac{dE}{d\omega} = \frac{e^2}{4\pi^2} \int do \left| \vec{k} \times \int dt \; e^{i(\omega t - \vec{k}\vec{r}(t)} \frac{d}{dt} \frac{\vec{v}(t)}{\omega - \vec{k}\vec{v}(t)} \right|^2 \tag{1}$$

where do is the solid angle element in the direction of the wave vector $\vec{k}$ of the emitted wave.

The above expression is essentially simplified when the coherence length exceeds the target thickness[6,9]. In this case following condition can be written :

$$l_c \gg v \, \Delta t \tag{2}$$

where $\Delta\, t$ represents the presence time of the particle in the external field, and l_c is given by the expression [1,2] :

$$l_c = \frac{2\gamma^2}{\omega} \frac{1}{1 + \gamma^2 \theta^2} \tag{3}$$

here $\gamma = \epsilon/m$ is the Lorentz factor of the particle and θ is the scattering angle. The argument of the exponential term varies slightly during this time interval. Thus the integral can be approximately evaluated by the variation of the term

$$\frac{\vec{v}(t)}{\omega - \vec{k}\vec{v}(t)} \tag{4}$$

induced by the field. The problem is reduced to the simple case of an abrupt deviation of the particle within the coherence length.

For the case examined here, the external field is created by the multitude of atoms that constitute the target material. Within this approximation the spectral density of radiation is determined by a single parameter $\xi = \gamma\theta/2$, i.e. the ratio of the scattering angle and the characteristic radiation angle of a relativistic particle $\theta_k \sim m/\epsilon$ [6,9]

$$\frac{dE}{d\omega} = \frac{2e^2}{\pi}\left\{\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln\left(\xi+\sqrt{\xi^2+1}\right)-1\right\} \tag{5}$$

where $\gamma = \epsilon/m$ is the Lorentz-factor of the particle, $\gamma \gg 1$ (the speed of light being taken unity). The scattering angle θ is assumed small compared to unity.

In order to take into account the multiple scattering of the particle moving through the material, it is necessary to average the formula (5) over the angular distribution $f(\theta)$ of particles leaving the target.

$$\left\langle\frac{dE}{d\omega}\right\rangle = \int d^2\theta f(\theta)\frac{dE\{\theta\}}{d\omega} \tag{6}$$

In amorphous media the angular distribution of particles is gaussian. Its average square value could be expressed as a function of particle energy ϵ and target thickness L by the equation [10] :

$$\langle\theta_{am}^2\rangle = \frac{\epsilon_s^2}{\epsilon^2}\frac{L}{L_{rad}}\left(1+\frac{1}{9}\ln\frac{L}{L_{rad}}\right) \tag{7}$$

where $\epsilon_s^2 = 4\pi 137 m^2, m$ is the electron mass and L_{rad} is the radiation length of the substance the target is made of. $<\theta_{am}^2>$ increases with L the quantity ; two different asymptotic formulas could be derived from (5) according to the value of $\langle\theta_{am}^2\rangle$:

$$\left\langle\frac{dE}{d\omega}\right\rangle \approx \frac{e^2}{3\pi}\frac{\epsilon_s^2 L}{m^2 L_{rad}}, \qquad \gamma^2\theta_{am}^2 \ll 1 \tag{8}$$

$$\left\langle\frac{dE}{d\omega}\right\rangle \approx \frac{2e^2}{\pi}\left(\ln\frac{\epsilon_s^2 L}{m^2 L_{rad}}-1\right), \qquad \gamma^2\theta_{am}^2 >\sim 1 \tag{9}$$

The first expression corresponds to the Bethe and Heitler formula for the spectral density of radiation of an electron in an amorphous medium :

$$\left\langle\frac{dE}{d\omega}\right\rangle_{BH} = \frac{4}{3}\frac{L}{L_{rad}} \tag{10}$$

The second formula in contrast with the first one and with the expression of Landau-Pomeranchuk :

$$\left\langle\frac{dE}{d\omega}\right\rangle_{LP} \approx \frac{2e^2}{\pi}\frac{\epsilon_s}{\epsilon}\frac{L}{L_{rad}}\sqrt{L_{rad}\omega} \tag{11}$$

is practically independent on the target thickness, particle energy and emitted photon frequency. The difference is due to the fact that the validity conditions of these formulas are essentially different. Formula (11) is valid if $L \gg l_c$ whereas formula (8) is valid at $L \ll l_c$.

The change of the radiation pattern of an electron in a thin layer of an amorphous substance occurs if the following conditions are met

$$\gamma^2\theta_{am}^2 \sim 1, \qquad l_c \gg L, \qquad \omega \geq \gamma\omega_p \tag{12}$$

The latter condition is the limitation of the frequency of an emitted photon at which one can neglect the influence of the polarization of a medium on radiation [1]. This effect has not been included in deriving formula (5).

The analysis of conditions (12) shows that the suppression of the bremsstrahlung radiation in a thin layer of an amorphous substance could be experimentally investigated with electron beams of energies of the order of some tens of GeV.

3. Suppression of the coherent radiation in a thin crystal.

In formula (5), the nature of the target is not specified, the results obtained could be applied for the radiation induced by a particle crossing an amorphous medium as well as a crystalline structure. It is only necessary to introduce the exact form of the angular distribution function of particles in order to calculate the effects of the interaction with specific targets.

The radiation of relativistic electrons in thin crystals is strongly dependent on multiple scattering. Compared to amorphous media, the radiative effects in crystals could be observed for lower particle energies and in a broader frequency range of emitted photons. This feature is connected with the fact that the average values of the multiple scattering angle in crystals are exceeding considerably the corresponding values in amorphous media [2,11]. The condition $\gamma^2\theta^2 \sim 1$ under which the radiation pattern in a thin layer of matter changes [8] is therefore met at lower target thicknesses for crystalline materials.

Some further details concerning the influence of multiple scattering on the radiation of an electron in thin crystal should be reminded in order to point out the specificity of the proposed approach.

An electron propagating through a crystalline lattice at a small angle ψ along the axis z collides atomic strings parallel to this axis. The resulting scattering occurs mainly along the azimuth angle ϕ in the plane orthogonal to the string axis : a redistribution of particles over the angle ϕ takes place. If one assumes that the collisions with atomic strings are random, then at $\psi \gg \psi_c$, where

$$\psi_c = \sqrt{\frac{4Ze^2}{\epsilon a}} \tag{13}$$

is the critical angle of axial channeling [12] and a is the distance between the atoms in a string, the distribution of particles over the angles ϕ for a thin crystal is gaussian and the mean square angle of multiple scattering of particles by a crystal could be expressed as follows :

$$\theta_{cr}^2 \sim \frac{R}{8\psi a}\theta_{am}^2 \tag{14}$$

where R is the screening radius of the potential of a single atom of the lattice (we use here the Molière approximation of atomic potential [1,2]). This mean value exceeds by an order of magnitude, due to the factor of $(R/8\psi a)$, the mean value of the angle of multiple scattering in an amorphous medium [2,9]

Equivalent formulas to (8) and (9) giving the spectral density of radiation are obtained :

$$\left\langle \frac{dE}{d\omega} \right\rangle_C \approx \frac{R}{8\psi a}\frac{dE_{BH}}{d\omega}, \qquad \gamma^2\theta_{cr}^2 \ll 1 \tag{15}$$

$$\left\langle \frac{dE}{d\omega} \right\rangle_C \approx \frac{2e^2}{\pi}\left[\ln\left(\frac{R}{8\psi a}\gamma^2\theta_{am}^2\right) - 1\right], \qquad \gamma^2\theta_{cr}^2 \gg 1 \tag{16}$$

Expression (15) is the main result of the theory of coherent radiation of relativistic electrons in a thin crystal [1,2]. Formula (16) describes the influence of multiple scattering on coherent radiation of the electron in a crystal. It shows that by increasing the target thickness or by decreasing the value of the angle ψ so that condition $\gamma^2\theta_{cr}^2 \geq 1$ is met, the spectral density of radiation is increases lower as compared to the result (15) given by the theory of coherent radiation. This effect is similar to the suppression of bremsstrahlung radiation of a relativistic electron in a thin layer of amorphous substance. However, in the case considered here the suppression process affects the coherent radiation rather than the bremsstrahlung. The conditions for observation of this effect are the same as those for which attenuation of radiation occurs in amorphous media :

$$\gamma^2\theta_{cr}^2 \sim 1, \qquad l_{coh} \gg L, \qquad \omega \geq \gamma\omega_p \tag{17}$$

However, at small ψ these conditions can be met for lower particle energies leading to more favorable conditions for experimental investigation of the effect of radiation suppression in matter.

4. Computer simulation of the electron passage through a thin crystal.

Formula (16) describing the influence of multiple scattering on coherent radiation of high energy electrons in a thin crystal is only an estimate, because on deriving it, one has assumed the random character of particle collisions with different atomic strings for all angles ψ. Besides, we have neglected the incoherent effects in scattering as well as the particle channeling [2,12]. However, the atomic strings of a crystal form a regular periodic structure the motion in which may possess chaotic as well as regular patterns [13]. Therefore, in order to obtain quantitative evaluation of the effect, one should include the full dynamics of electrons in a periodic field, as well as the incoherent effects in scattering. The inclusion of the these factors can be made by computer simulation.

In order to describe particles trajectories the binary collisions model has been employed as before (see, e. g., [14, 15]). This model includes automatically all factors listed above on particle motion in the field of the crystal lattice. The main disadvantage is the low speed of calculations, that can be compensated by employing parallel computers [16]. The results presented here have been obtained by the computer simulation performed at KhIPT and at Collège de France with the T-Node parallel computer with 64 T-800 transputers.

5. Discussion of the results.

The method outlined above has been used to study the radiation of relativistic electrons in thin crystals. Calculations have been performed according to the formula :

$$\left\langle \frac{dE}{d\omega} \right\rangle_N = \frac{1}{N} \sum_{i=1}^{N} \frac{dE\{\theta_i\}}{d\omega} \tag{18}$$

where θ_i is the total angle of scattering of the i-th electron by a crystal and $dE\{\theta_i\}/d\omega$ is the spectral density of radiation corresponding to this scattering angle. Summation in (18) is made over all N particles of the beam ($N \gg 1$). It is assumed that the beam is formed of particles impinging the crystal at a small random angle ψ to one of the crystal axes.

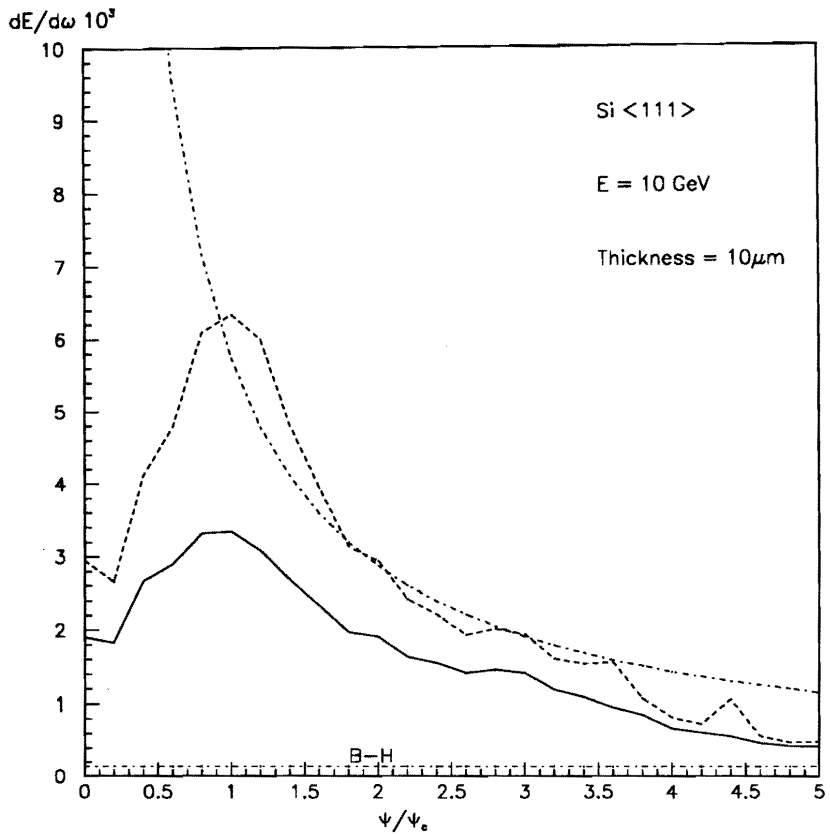

Figure 1: Radiation Spectral Density

The results obtained are gathered on Fig 1. The ordinate is the quantity ψ/ψ_c is represented, in the case considered $\psi_c = 1.3 \cdot 10^{-4}$ rad, while the abscissa is $\langle dE/d\omega \rangle_N$.

Calculations have been made for a silicon crystal of 10μm thick, near the crystal axis $\langle 111 \rangle$. The solid curve represents the spectral density of radiation including the effect of multiple scattering on the coherent radiation of the electron in a crystal according to formula (18). The broken curve depicts the Monte-Carlo results with the asymptotic representation of the quantity $dE\{\theta_i\}/d\omega$ corresponding to small values of the parameter $\gamma\theta_i$:

$$\frac{dE_{coh}\{\theta_i\}}{d\omega} = \frac{2e^2}{3\pi}\gamma^2\theta_i^2. \tag{19}$$

The estimated values for the dependence of the intensity of coherent radiation determined by formula (15) are represented by the dot-dash line. The straight line labeled BH corresponds to

the result of Bethe and Heitler for the radiation spectrum in a thin layer of amorphous substance of the same thickness. The graphs given show that at $\psi \sim 2\psi_c$ the suppression of the coherent radiation approaches $\approx$30%. The discrepancy between the results of numerical simulation and analytic estimates of the spectral density of radiation at $\psi > 3\psi_c$ is due to the fact that analytic estimates are obtained for the case of random collisions of electrons with crystal atomic strings whereas numerical results take into account the dynamics of the particle in the periodic field of atomic strings of a crystal.

For $\psi \leq \psi_c$ the estimate (15) ceases to be valid because the radiation is then considerably influenced by the curvature of the particle trajectory in the field of every atomic string and by the axial channeling of particles. The graphs given show that the influence of multiple scattering on the radiation of electrons in a crystal reaches a maximum at $\psi \sim \psi_c$. In this case, however, there occurs the suppression of the radiation intensity in the crystal of above barrier and channeled particles.

The results obtained are valid when the following inequalities are met

$$\frac{2\gamma^2/\omega}{1+\gamma^2\theta^2} \gg L, \qquad \omega \geq \gamma\omega_p.$$

The analysis of these inequalities shows that the effect of suppression of the coherent radiation of 10 GeV electrons in the silicon crystal $10\mu m$ thick can be studied experimentally in the energy range of radiated photons 0.5 MeV$\leq \omega <<$ 10 MeV at incidence angles of the particles on the crystal with respect to the $\langle 111 \rangle$axis of the order of $\psi \sim 2\psi_c = 2.5 \cdot 10^{-4}$ rad. All these conditions can be met on the SPS accelerator at CERN.

The experimental investigation of the effect could employ a measurement procedure based on formula (18). Namely, by measuring mean square of multiple scattering angle of particles by the crystal, one can easily obtain from formulas (18) and (19) the theoretical value of the spectral density of coherent radiation in the crystal. Comparing then this result with the measured value of the spectral density of radiation obtained under the same conditions in the energy range $\omega \div 1 - 5$ MeV, one should observe the effect under discussion. Its expected value is of the order of 50%.

Finally it is important to remark that by increasing the electron energy, the prospects for observing the effect are widened considerably. At an electron energy of the order of 100 GeV the considered effect of suppression of coherent radiation could be observed in the energy range 5 MeV$\leq \omega \leq$ 1000 MeV and the expected value of the effect is then $\sim$400%.

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