Photon fluxes and the EPA

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ABSTRACT

A derivation of the equivalent-photon approximation is given, for both one and two-photon exchange processes, and its limits of validity are defined.

RESUME

Nous donnons une dérivation de l'approximation du spectre équivalent de photons, à la fois pour les processus à échange d'un et de deux photons, et nous définissons ses limites de validité.

Keywords : Equivalent-photon approximation, photon-hadron interactions, photon-photon interactions.

[1]Talk presented at the Workshop on Two-Photon Physics at LEP and HERA, Lund (Sweden), May 26-28, 1994.

1 History of the EPA

For those who are interested in the history of the equivalent-photon approximation, I am enclosing a list of references. Papers [1-4] refer to the semi-classical derivations of the approximation ; you will perhaps be surprised to find the names of Niels Bohr and Enrico Fermi among the authors of these early references. Papers [5-8] refer to the field-theoretical derivations of the EPA, dating back to the fifties. Finally papers [9,10] are two general review papers written by myself ; they contain in particular a more modern derivation, based on helicity, as well as a number of applications.

2 General principle

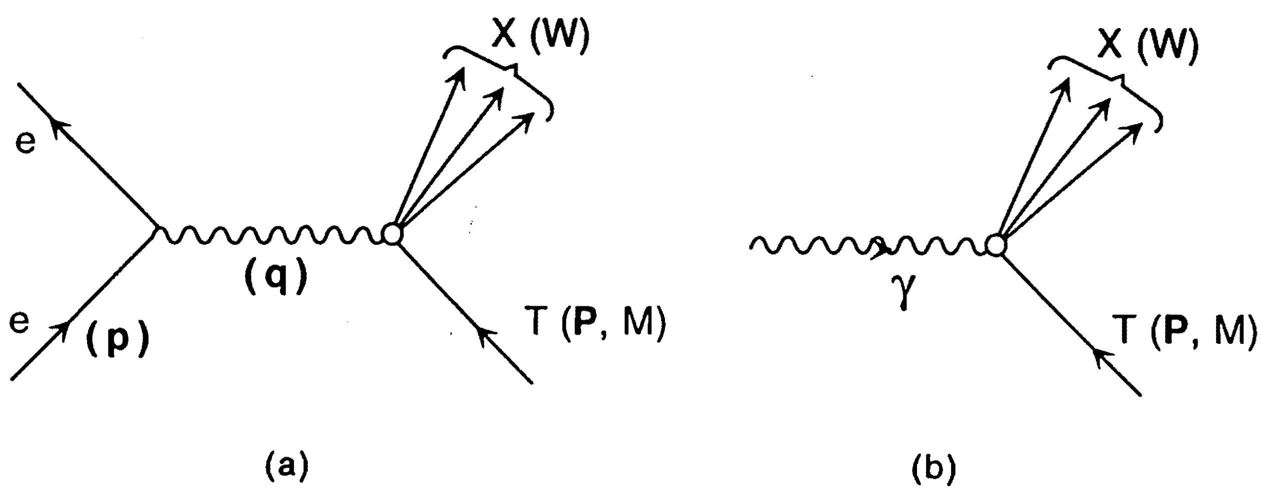

Fig. 1

Diagrams (a) and (b) of Fig. 1 are connected through

$$\sigma^{eT}(s) \simeq \int N(s,x)\hat{\sigma}^{\gamma T}(W^2) \tag{1}$$

where

$$x = \frac{q.P}{p.P} \simeq \frac{W^2 - M^2}{s - M^2} \simeq \frac{q_0}{p_0} \quad \text{(in any reference frame)} \tag{2}$$

$$N(s,x) = \frac{\alpha}{\pi x}\left[(1 - x + \frac{x^2}{2})\ln\frac{Q^2_{\max}}{Q^2_{\min}} - (1-x)\right] \tag{3}$$

with $Q^2_{\min} = m_e^2 x^2/(1-x)$ (notice : $Q^2 = -q^2$). There is a problem with the adequate choice of $Q^2_{\max}$ in case the electron remains untagged (we discuss it farther below). In case the electron is antitagged within a cone of opening angle $\theta_{\max}(\ll 1$ rad), one makes the substitution

$$\frac{Q_{\max}}{Q_{\min}} = \frac{E_e(1-x)\theta_{\max}}{m_e x} \tag{4}$$

where E_e and $\theta_{\max}$ are defined in the lab. frame.

Finally, if the electron is tagged between two angles θ_0, $\theta_1(\ll 1$ rad), one gets :

$$N(s,x) = \frac{\alpha}{\pi x}\left(1 - x + \frac{x^2}{2}\right)\ln\frac{\theta_1^2}{\theta_0^2} \tag{5}$$

3 Derivation of the EPA in one-photon exchange processes

I shall here given a short demonstration ; for more details see [9]. A helicity treatment leads to the well-known formula [11] :

$$\frac{d\sigma}{dx\ dQ^2} = N_T(s,x,Q^2)\ \hat{\sigma}_T(W^2,Q^2)\ +\ N_L(s,x,Q^2)\ \hat{\sigma}_L(W^2,Q^2) \tag{6}$$

where T ("transverse") and L ("longitudinal") refer to the photon polarization in the photon-target c.m. frame. The first problem is to get rid of $\hat{\sigma}_L$.

The longitudinal amplitude is (calling j^μ the electromagnetic current at the target vertex) :

$$j_L = \varepsilon^L_\mu j^\mu = \frac{1}{Q}(q_3 j_0 - q_0 j_3) \tag{7}$$

where the 3-axis is the photon-target collision axis. Using the current conservation relation

$$q_0 j_0 - q_3 j_3 = 0 \tag{8}$$

one is led to

$$j_L = \frac{q_3^2 - q_0^2}{Q q_0} j_3 = \frac{Q}{q_0} j_3 \tag{9}$$

Thus

$$\left|\frac{j_L}{j_T}\right| = \frac{Q}{q_0}\left|\frac{j_3}{(j_1 \pm i j_2)/\sqrt{2}}\right| \tag{10}$$

Nothing general can be said regarding the value of the second factor on the right-hand side. Anyway, in order to ensure $|j_L| \ll |j_T|$, one must assume : $Q \ll q_0$. One may define q_0 either in the target rest frame ($q_0 = \omega$) or in the photon-target c.m. frame ($q_0 = \omega^*$). The latter definition is preferable, as it leads to a more stringent condition, namely : $Q \ll (W^2 - M^2)/2W$.

The photon mass being thus assumed to be negligible with respect to its energy, one is allowed to treat the photon as massless in the transverse term as well, i.e. :

$$\hat{\sigma}_T(W^2, Q^2) \simeq \hat{\sigma}_T(W^2, 0) = \hat{\sigma}_\gamma(W^2) \tag{11}$$

The transverse spectrum is given, integrating over Q^2, by

$$N(s,x) = \frac{\alpha x}{4\pi} \int \frac{dQ^2}{Q^2} \left[(Q^2 + 4m_e^2) \sinh^2\chi + 2Q^2 \right] \tag{12}$$

where χ is the rapidity parameter connected with the Lorentz transformation from the Breit frame of the left-hand vertex to the c.m. frame of the right-hand vertex of Fig.1(a). With the expression of $\sinh^2\chi$ [12] :

$$\sinh^2\chi = \frac{s(Q^2 - Q^2_{\text{min}})(Q^{2\ \text{kin}}_{\text{max}} - Q^2)}{W^2(Q^2 + 4m_e^2)(\omega^{*2} + Q^2)} \tag{13}$$

using our assumption $Q^2 \ll \omega^{*2}$ and in addition $Q^2 \ll Q^{2\ \text{kin}}_{\text{max}}$ (since $Q^{2\ \text{kin}}_{\text{max}} \approx s$), as well as the kinematic relation

$$sQ^2_{\text{min}} Q^{2\ \text{kin}}_{\text{max}} = m_e^2 (W^2 - M^2)^2 \tag{14}$$

one is led to

$$\sinh^2\chi \simeq \frac{4m_e^2(Q^2 - Q^2_{\text{min}})}{Q^2_{\text{min}}(Q^2 + 4m_e^2)} \tag{15}$$

Thus, after integration over Q^2, one gets :

$$N(s,x) = \frac{\alpha x}{4\pi} \left(\frac{2Q^2_{\text{min}} + 4m_e^2}{Q^2_{\text{min}}} \ln \frac{Q^2_{\text{max}}}{Q^2_{\text{min}}} - \frac{1}{Q^2_{\text{min}}} + \frac{1}{Q^2_{\text{max}}} \right) \tag{16}$$

Using $Q^2_{\text{max}} \gg Q^2_{\text{min}}$, and

$$Q^2_{\text{min}} = m_e^2 x^2/(1-x) \tag{17}$$

one is led to formula (3).

When the electron is antitagged or tagged, one should check that $Q_{\text{max}} \ll \omega^*$. In the no-tag case, there is some arbitrariness in the choice of Q_{max}. Several choices can be considered acceptable and are more or less of equal legitimacy :

- $Q_{\text{max}} = Q^{\text{kin}}_{\text{max}}$, which means including the high-Q range where the approximation is not valid (such a procedure can be justified, nevertheless, by arguing that this range does not contribute much, since the Q-spectrum is sharply decreasing).

- $Q_{\max} = \omega^*$, in accordance with the condition of validity we have defined.
- $Q_{\max} = \omega$, where ω is the photon energy in the target rest frame or the lab frame (those two frames are usually the same, except at HERA), which leads to : $\ln Q^2_{\max}/Q^2_{\min} \simeq \ln E_e^2/m_e^2$, E_e being the electron beam energy in the same frame. (For 2-photon processes one commonly uses $\ln E_e^2/m_e^2$, where E_e is the beam energy in the $e\,e'$ c.m. frame, which is usually the lab frame.)
- Finally it may happen that one is led to impose $Q_{\max} = \Lambda$, where Λ is a cut-off accounting for some dynamic structure (for instance, when vector dominance is valid, $\Lambda = m_\rho$).

Various EPA formulas in the literature all agree on the coefficient of the leading-log term, but differ in the argument of the log and in the non-logarithmic term. The arbitrariness of $Q_{\max}$ is the main source of error (and of differences between the predictions of various formulas, that may reach a factor of 2 or so). When $Q_{\max}$ is well determined, the error is roughly of the order of $1/(\ln Q^2_{\max}/Q^2_{\min})$, i.e. only of a few percent for high-energy electrons (this is the big advantage of antitagging over no-tagging).

The EPA formula, as given for electrons, extends of course to high-energy muons. There are also corresponding formulas (incorporating electromagnetic form factors) for charged hadrons and nuclei [13].

4 Extension to two-photon exchange processes (DEPA)

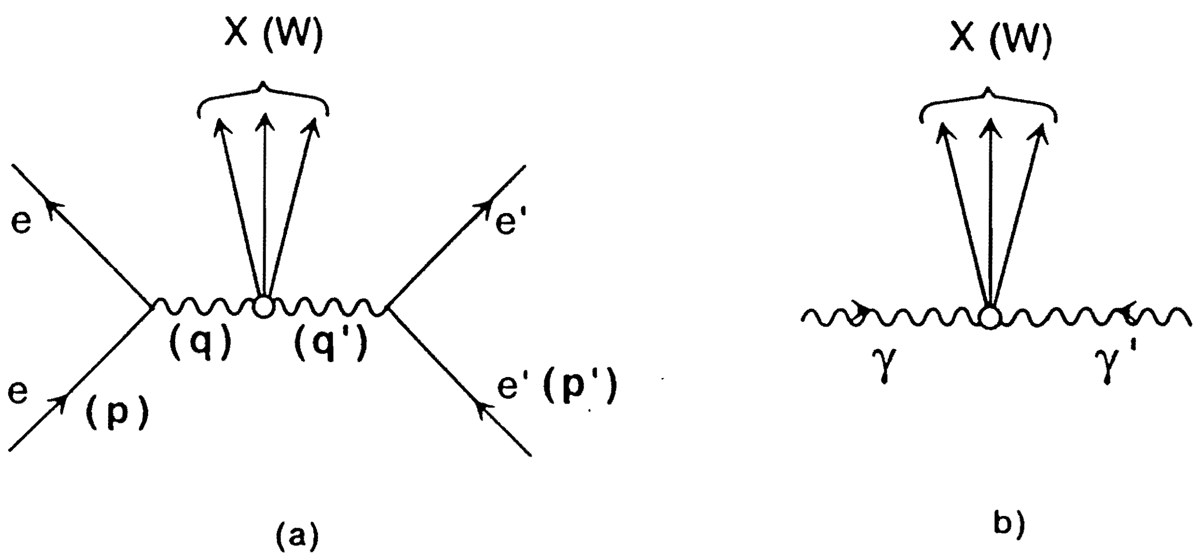

Fig. 2

Here again I refer to [9] for a more detailed demonstration. The problem is how to connect the diagrams (a) and (b) of Fig.2. One starts from (see [14], [15]) :

$$\frac{d\sigma}{dxdQ^2dx'dQ'^2d\varphi} = N_{TT}\hat{\sigma}_{TT} + N_{TL}\hat{\sigma}_{TL} + N_{LT}\hat{\sigma}_{LT} + N_{LL}\hat{\sigma}_{LL}$$
$$+\tilde{N}_{TT}\hat{\tau}_{TT}\cos 2\varphi^* + \tilde{N}_{TL}\hat{\tau}_{TL}\cos\varphi^* \tag{18}$$

where $x = q_0/p_0$, $x' = q_0'/p_0'$; the cofficients N and $\tilde{N}$ are functions of s, x, x', Q^2, Q'^2 ; the cross sections $\hat{\sigma}$ and the interference terms $\hat{\tau}$ depend on W^2, Q^2, Q'^2 ; φ^* is the azimuthal angle between the outgoing e and e' in the $\gamma\gamma'$ c.m. frame, while φ is the same angle in the incident ee' c.m. frame (usually the lab frame).

As before, one shows that, in order to be allowed to neglect longitudinal contributions of both the left-hand and right-hand photon, one should have $Q \ll \omega^*$, $Q' \ll \omega'^*$, where ω^*, ω'^* are the left-hand and right-hand photon energies in the $\gamma\gamma'$ c.m. frame ; i.e. practically : $Q, Q' \ll W/2$.

With that assumption only two terms of the r.h. side of formula (17) are kept :

$$\frac{d\sigma}{dxdQ^2dx'dQ'^2d\varphi} = N_{TT}\hat{\sigma}_{TT} + \tilde{N}_{TT}\hat{\tau}_{TT}\cos 2\varphi^* \tag{19}$$

It can be shown [16] that

$$\cos\varphi^* = \cos\varphi + \mathcal{O}(Q^2/W^2,\ Q'^2/W^2) \tag{20}$$

Therefore, after integrating (19) over φ between 0 and 2π, the term containing $\cos 2\varphi^*$ vanishes.

Moreover the conditions we have set for Q, Q' allow us to treat both photons as massless, thus :

$$\hat{\sigma}_{TT}(W^2, Q^2, Q'^2) \simeq \hat{\sigma}_{TT}(W^2, 0, 0) \equiv \hat{\sigma}_{\gamma\gamma}(W^2) \tag{21}$$

The same type of demonstration as in the previous section (simply replacing M^2 by $-Q^2$ resp. $-Q'^2$) then leads us to

$$\int dQ^2dQ'^2N_{TT} = N(s,x)N'(s,x') \tag{22}$$

where $N(s,x)$ is again given by (3), while $N'(s,x')$ is the same expression with x replaced by x' and $\ln Q^2_{\max}/Q^2_{\min}$ by $\ln Q'^2_{\max}/Q'^2_{\min}$.

It is easy to show, in addition, that

$$xx' = \frac{W^2}{s}\left[1 + \mathcal{O}(Q^2/W^2, Q'^2/W^2, QQ'/W^2)\right] \tag{23}$$

so that one has

$$\sigma_{ee'}(s) \simeq \int N(s,x)dx N'(s,x')dx'\hat{\sigma}_{\gamma\gamma}(xx's) \tag{24}$$

5 Extension to angular distributions

The EPA, resp. DEPA, can be extended as well to differential cross sections, in particular to angular distributions.

Let us consider exclusive reactions, where the system X is made of two particles, i.e. $X \equiv a + b$, or semi-inclusive ones, where $X \equiv a + X'$. Let us call θ_a^*, φ_a^* the orbital and azimuthal angle of a in the γT, resp. $\gamma\gamma'$ c.m. frame.
Then $d\sigma/[dxdQ^2d(\cos\theta_a^*)d\varphi_a^*]$is given by a 4-term formula instead of (6), and $d\sigma/[dxdQ^2dx'dQ'^2d\varphi d(\cos\theta_a^*)d\varphi_a^*]$ is given by a 20-term formula instead of (18) (see [17]). But eliminating all terms with longitudinal polarization (by assuming again $Q \ll \omega^*$ in the one-photon process, and $Q, Q' \ll \omega^*$ in the two-photon process), and integrating over azimuthal angles, one is again left with a one-term formula :

$$\frac{d\sigma^{eT}}{d(\cos\theta_a^*)}(s,\cos\theta_a^*) = \int N(s,x)dx\frac{d\hat{\sigma}^{\gamma T}}{d(\cos\theta_a^*)}(W^2,\cos\theta_a^*) \tag{25}$$

resp.

$$\frac{d\sigma^{ee'}}{d(\cos\theta_a^*)}(s,\cos\theta_a^*) = \int N(s,x)dxN'(s,x')dx'\frac{d\hat{\sigma}^{\gamma\gamma}}{d(\cos\theta_a^*)}(W^2,\cos\theta_a^*) \tag{26}$$

However, if the direction of the photon (resp. photons) cannot be precisely determined through tagging or from the final state, one must assume that its (resp. their) momentum can be treated as being aligned with the beam axis ; one therefore should impose the additional restriction : p_T^γ (resp. p_T^γ, $p_T^{\gamma'}$) $\ll p_T^a$, i.e. practically (neglecting masses) Q (resp. Q, Q') $\ll (W/2)\sin\theta_a^*$.

The EPA can be as well extended to other (e.g. transverse-momentum or lab angle) distributions.

6 The EPA for polarized electrons

O. Philipsen [18] has computed the EPA for polarized photons originating from polarized electron beams :

$$\frac{d\sigma^{eT}}{dx}(s) = N_+(s,x)\hat{\sigma}_+^{\gamma T}(W^2) + N_-(s,x)\hat{\sigma}_-^{\gamma T}(W^2) \tag{27}$$

where the $\pm$ subscripts refer to the photon spin component (± 1) along the γT collision axis, and where one has

$$N_\pm(s,x) = \frac{\alpha}{2\pi x}\left[[1-(x-x^2/2)(1\mp\zeta)]\ln\frac{Q^2_{\max}}{Q^2_{\min}} - (1-x)(1\pm\zeta x/2)\right] \tag{28}$$

ζ being defined as the polarization parameter of the incident electron in the Breit frame of the electron-photon vertex. Its value reaches practically 1 when the electron beam is 100% longitudinally polarized. But even then the polarization effect on the photon stays limited, i.e. of the order of x ; this means that the photon has "lost" most of its polarization in the Lorentz transformation from the Breit frame of the electron-photon vertex to the c.m. frame of the photon-target vertex.

Extending this calculation to two-photon processes, it comes out that polarization effects due to electron beam polarization are at best of order $xx' \simeq W^2/s$.

Such effects are thus relatively (but perhaps not desperately) small.

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