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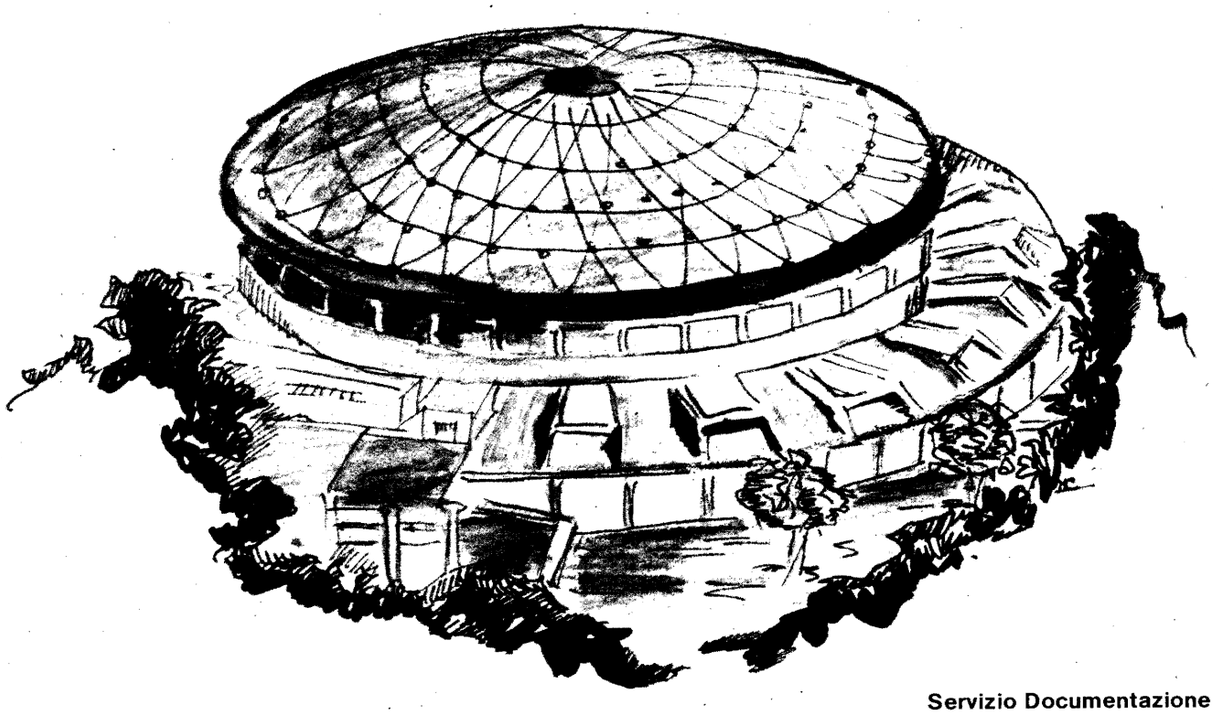

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COMPOSITE GAUGE FIELDS IN RENORMALIZABLE MODELS

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ABSTRACT

We construct models of gauge theories renormalizable by power counting in four dimensions, with gauge fields composite of bosonic or fermionic constituents.They are obtained by a regularization of CP^{n-1} models where the constraint on constituent fields is replaced by a constraint on expectation values.

In this paper we present a generalization of CP^{n-1} models [1] in their non abelian versions [2],constructing models of gauge theories renormalizable by power counting in four dimensions,with gauge fields composite of bosonic or fermionic constituents.The latter will be referred to as fermionic CP^{n-1} models.

One obvious motivation is the conceptual economy of reducing the number of fundamental fields.But a complementary motivation is that gauge fields composite of fermionic fields are expected to give rise to softer divergences than fundamental gauge fields,a perspective especially relevant to quantum gravity if in such models a phase of unbroken symmetry exists.We will not investigate this point, but we will keep in mind that in this connection the constituent fields can be just auxiliary fields which need not be identified with physical particles or "physical constituents" like quarks.They can be for instance scalar fermions and/or have *scaling dimensions different from the canonical ones.*We will actually assume scaling dimension zero to have renormalizability by power counting.

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To define composite fields we use a new lattice regularization [3] where the gauge fields are noncompact, a feature which is essential to deal with fermionic constituents.Since we will need the basic formulae of this regularization anyway, we will briefl composite gauge fields and we will discuss renormalizability by power counting.Finally we will mention how a proof of reflection positivity can be given.Such a proof,ensuring a quantum mechanical interpretation, is especially important dealing with composites.

We will confine ourselves to the case of non abelian CP^{n-1} models,both bosonic and fermionic, with SU(2) local invariance, but most of our results can be extended to $SU(N), N > 2$ and to the abelian case.The lagrangian density of the bosonic models in the continuum can be written

$$\mathcal{L} = \sum_{h=1}^{N_f} \sum_{\mu} (\mathcal{D}_\mu \lambda_h)^+ \mathcal{D}_\mu \lambda_h, \tag{1}$$

where the covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + iA_\mu, \qquad A_\mu = \frac{1}{2}\sigma_a A_{\mu a}, \tag{2}$$

is defined in terms of the composite gauge field

$$A_{\mu a} = \sum_{h=1}^{N_f} \lambda_h^+ \sigma_a \partial_\mu \lambda_h - \partial_\mu \lambda_h^+ \sigma_a \lambda_h. \tag{3}$$

λ_h are constituent fields in the fundamental representation of SU(2), subject to the constraint

$$\sum_{h=1}^{N_f} \lambda_h^+ \lambda_h = \sum_{h=1}^{N_f} \sum_{\alpha=1}^{2} \lambda_{h\alpha}^* \lambda_{h\alpha} = 1, \tag{4}$$

N_f being the number of flavours.

Due to this constraint

i) it is not possible to write a mass term for the constituent fields

ii) the expression of the lagrangian in terms of independent fields involves an infinite power series arising from the solution of the constraint

iii) it is not possible to have fermionic constituents.

We will be able to avoid such limitations by regularizing the model in such a way that the constraint on the fields is replaced by a constraint on expectation values.

Fermionic $CP^{(n-1)}$ models have actually been considered by Amati et al. [4],but their analysis meets with a difficulty due the fact that δ-functions of Grassmann variables do not have all the properties of ordinary δ-functions, as discussed below.

In the non compact lattice regularization we are going to use the covariant derivative is

$$\mathcal{D}_\mu(x)\lambda(x) = D_\mu \lambda(x+\mu) - \frac{1}{a}\lambda(x), \tag{5}$$

where μ is the unit vector with components $\mu_\nu = \delta_{\mu\nu}$ and

$$D_\mu = V_\mu + iA_\mu \tag{6}$$

is the parallel transporter expressed in terms of the gauge field A_μ and the auxiliary field V_μ.Under gauge transformations

$$\lambda_h(x) \to g(x)\lambda_h(x), \tag{7}$$

$$D_\mu(x) \to g(x)D_\mu(x)g^+(x+\mu) \tag{8}$$

for g$\in$ SU(2).

The transformations of A_μ and V_μ are

$$\begin{aligned} \delta A_\mu &= i[A_\mu,\theta] + a(V_\mu \triangle_\mu \theta + \frac{i}{2}[A_\mu, \triangle_\mu\theta]) \\ \delta V_\mu &= -\frac{1}{2}aTrA_\mu \triangle_\mu \theta, \end{aligned} \tag{9}$$

where θ_a are the gauge parameters. We see that for $a \to 0$, they do not reproduce gauge transformations. These can be recovered if the auxiliary field V_μ acquires a nonvanishing expectation value

$$< V_\mu >= \frac{1}{a}, \tag{10}$$

so that defining the shifted field

$$W_\mu = \frac{1}{a} - V_\mu, \tag{11}$$

we have

$$\begin{aligned} \delta A_\mu &= \triangle_\mu\theta + i[A_\mu,\theta] - a(W_\mu \triangle_\mu \theta - \frac{i}{2}[A_\mu, \triangle_\mu\theta]) \\ \delta W_\mu &= \frac{1}{8}aTrA_\mu \triangle_\mu \theta. \end{aligned} \tag{12}$$

In conclusion a non compact regularization can be constructed if a) spontaneous breaking of GL to iGL can be ensured by a suitable potential and b) the auxiliary field decouples in the continuum limit.

The strength and the Yang-Mills lagrangian density can be written in analogy to the continuum

$$F_{\mu\nu}(x) = \frac{1}{i}[D_\mu(x)D_\nu(x+\mu) - D_\nu(x)D_\mu(x+\nu)] \tag{13}$$

$$\mathcal{L}_{YM} = \frac{1}{8}\beta\sum_{\mu,\nu} F^+_{\mu\nu}(x)F_{\mu\nu}(x). \tag{14}$$

The possibility of enforcing conditions a) and b) is related to the existence of the other invariant

$$t_\mu = \frac{1}{2}Tr[D^+_\mu D_\mu - \frac{1}{a^2}] = A^2_\mu + W^2_\mu - \frac{2}{a}W_\mu. \tag{15}$$

The total action will contain an arbitrary function of this invariant.This function is determined [3] by some requirements including a divergent mass $\sim 1/a$ for W_μ to ensure its decoupling .The resulting total lagrangian is

$$\mathcal{L}_G = \mathcal{L}_{YM} - \frac{1}{16}\beta a^2 \sum_{\mu\nu}(\Delta_\mu t_\nu - \Delta_\nu t_\mu)^2 + \frac{1}{2}\gamma^2 \sum_\mu t_\mu^2. \tag{16}$$

The minimum of the potential occurs at $V_\mu = \pm 1/a$,namely W_μ =0,-2/a.Since the second root has been shown to add only unessential complications the conditions a) and b) are satisfied.

It is worth while mentioning that in the limit $\gamma \to \infty$ we recover Wilson's definition.

The scaling properties of this regularization have been studied perturbatively.It has been found that in the continuum limit $\gamma \to \beta$, so that the fixed point is the same as in Wilson scheme.A Monte Carlo calculation has shown that also the Wilson loop has essentially the same behaviour.

This regularization is suitable to put CP^{n-1} models on the lattice .This can be accomplished by the following definition of the parallel transporter

$$\begin{aligned} D_{\mu\alpha\beta} = & \sum_{h,k=1}^{N_f} \frac{1}{2a}[\bar{\lambda}^*_{h\alpha}(x) M_{hk} \bar{\lambda}_{k\beta}(x+\mu) \\ & \pm\lambda_{h\alpha}(x) Q_{hk} \lambda^*_{k\beta}(x+\mu) \\ & +\bar{\lambda}^*_{h\alpha}(x) N_{hk} \lambda^*_{k\beta}(x+\mu) \\ & \pm\lambda_{h\alpha}(x) P_{hk} \bar{\lambda}_{k\beta}(x+\mu)] \end{aligned} \tag{17}$$

where

$$\bar{\lambda} = \sigma_2 \lambda \tag{18}$$

and the $\pm$ sign refers to bosonic/fermionic constituents resp.In the latter case if there are spinor indices they are included in the flavour index h.

The above is the most general expression compatible with the gauge transformations (8).We must now constrain the matrices M,Q,N and P in such a way that A_μ and W_μ be hermitian and reflection positivity be satisfied.

The gauge and auxiliary fields resulting from eq.(17) are

$$\begin{aligned} A_{\mu a}(x) = & \frac{i}{2a}[\lambda^+(x) M \sigma_a \lambda(x+\mu) \\ & -\lambda^+(x+\mu) \tilde{Q} \sigma_a \lambda(x) \\ & +\lambda^+(x) N \sigma_a \sigma_2 \lambda^*(x+\mu) \\ & +\tilde{\lambda}(x+\mu) \tilde{P} \sigma_2 \sigma_a \lambda(x)] \\ V_\mu(x) = & \frac{1}{4a}[\lambda^+(x) M \lambda(x+\mu) \\ & +\lambda^+(x+\mu) \tilde{Q} \lambda(x) \\ & +\lambda^+(x) N \sigma_2 \lambda^*(x+\mu) \\ & -\tilde{\lambda}(x+\mu) \tilde{P} \sigma_2 \lambda(x)] \end{aligned} \tag{19}$$

where the constituent fields have been ordered in such a way to get rid of the $\pm$ sign.

Hermiticity of A_μ and W_μ requires

$$Q = M^*, \qquad P = -N^*, \tag{20}$$

while to ensure the condition

$$D_{-\mu}(x) = D^+_\mu(x-\mu) \tag{21}$$

related to reflection positivity it sufficient that

$$M = M^+, \quad N = \pm\tilde{N} \;\; for\; fermions/bosons. \tag{22}$$

Taking the above conditions into account

$$\begin{aligned} A_{\mu a} &= \frac{i}{2}[\lambda^+ M\sigma_a \triangle_\mu \lambda - \triangle_\mu \lambda^+ M\sigma_a\lambda \\ &\quad +\lambda^+ N\sigma_a\sigma_2 \triangle_\mu \lambda^* - \triangle_\mu\tilde{\lambda} N^+\sigma_2\sigma_a\lambda] \\ V_\mu &= \frac{r}{4a} + \frac{1}{4}[\lambda^+ M \triangle_\mu \lambda + \triangle_\mu\lambda^+ M\lambda \\ &\quad +\lambda^+ N\sigma_2 \triangle_\mu \lambda^* + \triangle_\mu\tilde{\lambda} N^+\sigma_2\lambda], \end{aligned} \tag{23}$$

where

$$r = 2\lambda^+ M\lambda + \lambda^+ N\sigma_2\lambda^* + \tilde{\lambda} N^+\sigma_2\lambda. \tag{24}$$

So far we have constructed a parallel transporter [5] in terms of constituent fields,but to have a gauge theory we must impose the condition (10).This can obviously be done only in the context of a definite lagrangian that we now assume to be $\mathcal{L}_G$(we could as well have chosen the lattice transcription of the CP^{n-1} lagrangian). At this point we must distinguish between the bosonic and fermionic case.In the bosonic case condition (10) can be imposed at the semiclassical level.For constant constituent fields it reads

$$r = 4. \tag{25}$$

Such a condition follows from the potential of $\mathcal{L}_G$

$$\frac{2}{a^4}\gamma^2[(\frac{r}{4})^2 - 1]^2 \tag{26}$$

which has in fact the degenerate minima $r = \pm 4$ .We recover a constraint on the fields for $\gamma \to \infty$,where the exponential of the last term of $\mathcal{L}_G$ becomes a δ-function.

Let us now come to the issue of renormalizability by power counting.This requires that the lagrangian have a quadratic term in the constituent fields and that these fields have scaling dimension zero.For the lagrangian to contain a quadratic term it is necessary that r contain a constant.This can be easily obtained.Taking advantage of the hermiticity of M we can perform on the constituent fields a transformation such that

$$r = \sum_{h=1}^{N_f} \epsilon_h \phi_h^+ \phi_h + \phi^+ N'\sigma_2\phi^* + \tilde{\phi} N'^+\sigma_2\phi \tag{27}$$

where $\epsilon_h = \pm 1$ and N' is the transformed of N. Next we introduce the real and imaginary parts of the new constituent fields

$$\phi_{h\alpha} = \phi_{h\alpha 1} + i\phi_{h\alpha 2} \tag{28}$$

and fix the gauge according to

$$\phi_{112} = \phi_{121} = \phi_{122} = 0. \tag{29}$$

This gauge fixing shows that the case of one single flavour is trivial because $A_\mu = 0$. Assuming ϵ_1=1 we shift the field ϕ_{111}

$$\phi_{111} = \sqrt{2} + \rho \tag{30}$$

getting

$$\begin{aligned} r &= 4 + 4\sqrt{2}\rho + 4\rho^2 + 2\sum_{h=2}^{N_f} \epsilon_h \phi_h^+ \phi_h \\ &\quad + \phi^+ N' \sigma_2 \phi^* + \tilde{\phi} N'^+ \sigma_2 \phi. \end{aligned} \tag{31}$$

It remains to give the constituent fields scaling dimension zero.This can be accomplished by including in the lagrangian a term with a forth covariant derivative

$$\mathcal{L}_{B4} = \sum_{h=1}^{N_f} (\mathcal{D}_2 \lambda_h)^+ (\mathcal{D}_2 \lambda_h) \tag{32}$$

, where

$$\begin{aligned} \mathcal{D}_2 \lambda(x) &= \frac{1}{a} \sum_\mu D_\mu(x) \lambda(x+\mu) \\ &\quad + D_\mu^+(x-\mu)\lambda(x-\mu) - \frac{2}{a}\lambda(x) \\ &\quad - \frac{a}{2}[t_\mu(x) + t_\mu(x-\mu)]\lambda(x). \end{aligned} \tag{33}$$

The situation is different for fermions.First of all a condition on the fields cannot be imposed at the semiclassical level,neither by use of a δ-function, because a δ-function of Grassmann variables with all the properties and representations of ordinary δ-functions does not exist.In the case of one single flavor,for instance,the function

$$\delta(\lambda^+\lambda - v) = \frac{1}{2} v^2 exp[\frac{1}{v}\lambda^+\lambda + \frac{1}{2v^2}(\lambda^+\lambda)^2] \tag{34}$$

acts as a δ-function on powers of $\lambda^+\lambda$ but not on arbitrary functions of this argument nor with respect to integration over v.This is at the origin of the difficulties in the mentioned attempt [4] to relate fermionic bilinears to bosonic fields.In the anticommuting case it is only possible to impose a condition on expectation values,and to impose the condition (10) we can include in the lagrangian a term of the form (34).

As far as renormalizability is concerned, an expression corresponding to Eq.(31) does not exist, so that the lagrangian has no quadratic terms in the constituent fields. A possible way out which,however,remains to be investigated , is to add a term

$$\mathcal{L}_{F4} = \Box\Lambda_h^* \Box\Lambda_h \tag{35}$$

where $\Box$ is the euclidean laplacian on the lattice and

$$\Lambda_h = \lambda_h \sigma_2 \lambda_h \tag{36}$$

are new gauge invariant variables. $\mathcal{L}_{F4}$ is quadratic in the Λ-fields that we now take as fundamental fields according to the integration rule

$$\int d\lambda_{h1}^* d\lambda_{h1} d\lambda_{h2}^* d\lambda_{h2} = \frac{1}{4}\int d\Lambda_h^* d\Lambda_h. \tag{37}$$

We can adopt such a rule since the only non vanishing contributions to the integration come from terms $\Lambda_h^*\Lambda_h$.For this reason we can perform the change of variables (36) at any nonvanishing order of perturbation theory even though we cannot express $\mathcal{L}_G$ in terms of the new variables.

Let us finally come to the issue of physical positivity.This requires that for any polynomial $\mathcal{P}$ of the D_μ at positive euclidean times

$$< \Theta\mathcal{P}\mathcal{P} > \geq 0, \tag{38}$$

where Θ is an operator which performs complex conjugation and euclidean time inversion

$$\begin{aligned} \Theta D_4(x) &= \tilde{D}_4(\theta x - e_4) \\ \Theta D_k(x) &= D_k^*(\theta x). \end{aligned} \tag{39}$$

In the above equations e_4 is the unit vector in the euclidean time direction and

$$\theta x_4 = -x_4, \qquad \theta x_k = x_k. \tag{40}$$

The proof of Eq.(38) can be splitted into two parts.First one gives the proof for the case where the gauge fields are elementary.This is most conveniently done using (with minor changes)the formalism that Mack[6] has developed in the framework of the colordielectic theory where also non unitary link variables appear.Then one shows that the action of Θ on the constituent fields can be defined in such a way that equations (39) are satisfied.It is easy to check that this happens for the natural definition

$$\Theta[\lambda_\alpha(x)\lambda_\beta^*(y)] = \lambda_\beta(\theta y)\lambda_\alpha^*(\theta x) \tag{41}$$

by which also Eq.(21) is fulfilled.

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