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J. Gao

Laboratoire de l'Accélérateur Linéaire
IN2P3-CNRS et Université de Paris-Sud, BP 34, F-91898 Orsay Cedex

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B.P. 34 ~ Bâtiment 200 ~ 91898 ORSAY CEDEX

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Analytical Investigation on the Dynamic Apertures of Circular Accelerators

J. Gao
Laboratoire de L'Accélérateur Linéaire
IN2P3-CNRS et Université de Paris-Sud
B.P. 34, 91898 Orsay cedex, France

Abstract

In this paper by considering delta function sextupole and octupole perturbations and using difference action-angle variable equations, analytical formulae for the dynamic aperture of circular accelerators are derived based on the Chirikov criterion of the onset of stochastics motions. Comparisons with numerical simulations are made.

1 Introduction

One of the preoccupations of the circular accelerator designer is to estimate the influence of nonlinear forces on the single particle's motion. These nonlinear forces manifest themselves as the systematic and random errors from optical elements, the voluntarily introduced functional ones, such as the sextupoles used for the chromaticity corrections and the octupoles used for stabilizing the particles' collective motion, or from nonlinear beam-beam interaction forces. Even though the nonlinear forces mentioned above compared with the linear forces are usually very small, what is observed in reality, however, is that when the amplitudes of the transverse oscillation of a particle are large enough, the transverse motion might become unstable and the particle itself will finally be lost on the vacuum chamber. Apparently, the above implied maximum oscillation amplitudes, $A_{x,y}$, corresponding to the stable motions are functions of the specific longitudinal position, s, along the

machine, and these functions $A_{x,y}(s)$ are the so-called dynamic apertures of the machine. A reasonably designed machine should satisfy the condition $A_{x,y}(s) \geq M_{x,y}(s)$, where $M_{x,y}(s)$ are the mechanical cross section dimensions of the vacuum chamber. In the following sections we will show how the nonlinear forces limit the dynamic apertures and what is the relation between them. From the newly established analytical formula for the dynamic aperture one gets the scaling law between the nonlinear perturbation strength and the dynamic aperture, and this scaling law is compared with some numerical simulations.

2 Hamiltonian Formalism

The Hamiltonian formulation of dynamics is best known not only for its deep physical and philosophical inspiration to the physical world, but also for its technical convenience of solving various nonlinear dynamical problems. The general Hamiltonian for a particle of rest mass, m_0, and charge, e, in a magnetic vector potential, $\mathbf{A}$, and electric potential, Φ, is expressed as:

$$\mathcal{H}(q,p,t) = e\Phi + c\left((\mathbf{p} - e\mathbf{A})^2 + m_0c^2\right)^{1/2} \tag{1}$$

where c is the velocity of light and $\mathbf{p}$ is the momentum with its components, p_i, conjugate to the space coordinates, q_i. The equations of motion can be readily written in terms of Hamilton's equations:

$$\frac{dp_i}{dt} = -\frac{\partial \mathcal{H}}{\partial q_i} \tag{2}$$

$$\frac{dq_i}{dt} = \frac{\partial \mathcal{H}}{\partial p_i} \tag{3}$$

For our specific dynamic problems in a circular accelerator it is convenient to chose curvilinear coordinates instead of Cartesian ones for us to describe the trajectory of a particle near an *a priori* known closed orbit. The Hamiltonian in the new system, (x, s, y) (where x, s, and y denote the coordinates in the Frenet-Serret normal, tangent, and binormal triothogonal and right-handed

coordinate system), is given by [1]-[7]:

$$H_t(q,p,t) = e\Phi + c\left((p_x - eA_x)^2 + (p_y - eA_y)^2 + \left(\frac{p_s - eA_s}{1 + x/\rho}\right)^2 + m_0c^2\right)^{1/2} \tag{4}$$

where ρ is the radius of curvature and the torsion of the closed orbit is everywhere zero. Since it is useful to use the variable, s, as the independent variable rather than time, t, one gets the new Hamiltonian by using a simple canonical transformation:

$$H_s = -eA_s - (1+x/\rho)\left(\frac{1}{c^2}(E^2 - m_0^2c^4) - (p_x - eA_x)^2 - (p_y - eA_y)^2\right)^{1/2} - e\Phi \tag{5}$$

Noting that the term $\frac{1}{c^2}(E^2 - m_0^2c^4)$ in eq. 5 is equal to P^2 with P being the total mechanical momentum of the particle, by using another trivial canonical transformation:

$$\bar{q} = q, \bar{s} = s, \bar{p}_{x,y} = \frac{p_{x,y}}{P_0}, H = \frac{H_s}{P_0} \tag{6}$$

one gets another Hamiltonian:

$$H = -\frac{eA_s}{P_0} - (1 + x/\rho)\left(\frac{P}{P_0} - (\bar{p}_x - \frac{eA_x}{P_0})^2 - (\bar{p}_y - \frac{eA_y}{P_0})^2\right)^{1/2} - \frac{e\Phi}{P_0} \tag{7}$$

where P_0 is the mechanical momentum of the reference particle, and $P = P_0 + \Delta P$. Inserting eA_s/P_0 in eq. 7 by:

$$\frac{eA_s}{P_0} = -\frac{B_y x^2}{2\rho^2 B_0} - \frac{1}{B_0\rho}\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\partial^{n-1}B_y}{\partial x^{n-1}}|_{x=0,y=0}(x+iy)^n \tag{8}$$

one gets finally the Hamiltonian which serves as the starting point of most of the dynamical problems in a circular accelerator:

$$H = \frac{x^2 B_y|_{x=0,y=0}}{2\rho^2 B_0} + \frac{1}{B_0\rho}\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\partial^{n-1}B_y}{\partial x^{n-1}}|_{x=0,y=0}(x+iy)^n$$

$$-(1 + x/\rho)\left(1 + \frac{\Delta P}{P_0} - \left(\bar{p}_x - \frac{eA_x}{P_0}\right)^2 - \left(\bar{p}_y - \frac{eA_y}{P_0}\right)^2\right)^{1/2} - \frac{e\Phi}{P_0} \tag{9}$$

where B_0 is the bending magnetic field on the orbit of the reference particle, and B_y in general is a complex variable.

3 Dynamic apertures due to nonlinear resonances and stochastic motions

In this section a necessary distinction will be made between two essentially different cases: a proton machine and an electron one. The reason is simple. In the first case there is no dissipative forces (which is gernerally true) and the particle's motion can be described in the frame of Hamiltonian system. In the second case, however, one has to take into account the synchrotron radiation damping effect (even neglecting the quantum radiation excitation) which results in putting the particle's motion into a non-Hamiltonian system.

3.1 Proton storage ring

To start with we consider the linear horizontal motion of the reference particle assuming that the magnetic field is only transverse ($A_x = A_y = 0$) and has no screw fields, and Φ is a constant. The Hamiltonian can be simplified as

$$H = \frac{p^2}{2} + \frac{K(s)}{2}x^2 \tag{10}$$

where x denotes normal plane coordinate, $p = dx/ds$, and $K(s)$ is a periodic function satisfying the relation

$$K(s) = K(s + L) \tag{11}$$

where L is the circumference of the ring. The solution of the deviation, x, is found to be

$$x = \sqrt{\epsilon_x \beta_x(s)} \cos(\phi(s) + \phi_0) \tag{12}$$

where

$$\phi(s) = \int_0^s \frac{ds}{\beta_x(s)} \tag{13}$$

As an essential step towards further discussion on the motions under nonlinear perturbation forces, we introduce action-angle variables and the Hamiltonian expressed in these new variables:

$$\Psi = \int_0^s \frac{ds'}{\beta_x(s')} + \phi_0 \tag{14}$$

$$J = \frac{\epsilon_x}{2} = \frac{1}{2\beta_x(s)}\left(x^2 + \left(\beta_x(s).x' - \frac{\beta_x' x}{2}\right)^2\right) \tag{15}$$

$$H(J,\Psi) = \frac{J}{\beta_x(s)} \tag{16}$$

Since the $H(J,\Psi) = J/\beta_x(s)$ is still a function of the independent variable, s, we will make another canonical transformation to freeze the new Hamiltonian:

$$\Psi_1 = \Psi + \frac{2\pi\nu}{L} - \int_0^s \frac{ds'}{\beta_x(s')} \tag{17}$$

$$J_1 = J \tag{18}$$

$$H_1 = \frac{2\pi\nu}{L} J_1 \tag{19}$$

Before going on further, let's remember the relation between the last action-angle variables and the particle deviation x:

$$x = \sqrt{2J_1\beta_x(s)}\cos\left(\Psi_1 - \frac{2\pi\nu}{L}s + \int_0^s \frac{ds'}{\beta_x(s')}\right) \tag{20}$$

Being well prepared, we start our journey to find out the limitations of the nonlinear forces on the stability of the particle's motion. To facilitate the analytical treatment of this complicated problem we consider at this stage only sextupoles and octupoles (no screw terms) and assume that the contributions from the sextupoles and octupoles in a ring can be made equivalent to a point sextupole and a point octupole. The perturbed Hamiltonian can thus be expressed:

$$H = \frac{p^2}{2} + \frac{K(s)}{2}x^2 + \frac{1}{3!B\rho}\frac{\partial^2 B_z}{\partial x^2}x^3 L \sum_{k=-\infty}^{\infty}\delta(s-kL) + \frac{1}{4!B\rho}\frac{\partial^3 B_z}{\partial x^3}x^4 L \sum_{k=-\infty}^{\infty}\delta(s-kL) \tag{21}$$

Representing eq. 21 by action-angle variables (J_1 and Ψ_1), and using

$$B_z = B_0(1 + xb_1 + x^2 b_2 + x^3 b_3) \tag{22}$$

one has

$$H = \frac{2\pi\nu}{L}J_1 + \frac{(2J_1\beta_x(s_1))^{3/2}}{3\rho}b_2 L\cos^3\Psi_1 \sum_{k=-\infty}^{\infty}\delta(s-kL)$$

$$+\frac{(J_1\beta_x(s_2))^2}{\rho}b_3 L\cos^4\Psi_1\sum_{k=-\infty}^{\infty}\delta(s-kL) \tag{23}$$

where s_1 and s_2 are just used to differentiate the locations of the sextupole and the octupole perturbations. By virtue of Hamiltonian one gets the differential equations for Ψ_1 and J_1

$$\frac{dJ_1}{ds} = -\frac{\partial H_1}{\partial \Psi_1} \tag{24}$$

$$\frac{d\Psi_1}{ds} = \frac{\partial H_1}{\partial J_1} \tag{25}$$

$$\frac{dJ_1}{ds} = -\frac{(2J_1\beta_x(s_1))^{3/2}}{3\rho}b_2 L\frac{d\cos^3\Psi_1}{d\Psi_1}\sum_{k=-\infty}^{\infty}\delta(s-kL)$$
$$-\frac{(J_1\beta_x(s_2))^2}{\rho}b_3 L\frac{d\cos^4\Psi_1}{d\Psi_1}\sum_{k=-\infty}^{\infty}\delta(s-kL) \tag{26}$$

$$\frac{d\Psi_1}{ds} = \frac{2\pi\nu}{L} + \frac{\sqrt{2}J_1^{1/2}\beta_x(s_1)^{3/2}}{\rho}b_2 L\cos^3\Psi_1\sum_{k=-\infty}^{\infty}\delta(s-kL)$$
$$+\frac{2\beta_x^2(s_2)}{\rho}J_1 b_3 L\cos^4\Psi_1\sum_{k=-\infty}^{\infty}\delta(s-kL) \tag{27}$$

Now it is the moment to change this differential equations to the difference equations which is *suitable* to analyse the possibilities of the onset of stochasticity [8][9]. Since the perturbations have a natural periodicity of L we will sample the dynamic quantities at a sequence of s_i with constant interval L assuming that the characteristic time between two consecutive adiabatic invariance breakdown intervals is shorter than L/c. The differential equations in eqs. 26 and 27 are reduced to

$$\overline{J_1} = \overline{J_1}(\Psi_1, J_1) \tag{28}$$

$$\overline{\Psi_1} = \overline{\Psi_1}(\Psi_1, J_1) \tag{29}$$

where the bar stands for the next sampled value after the corresponding unbared previous value.

$$\overline{J_1} = J_1 - \frac{(2J_1\beta_x(s_1))^{3/2}}{3\rho}b_2 L\frac{d\cos^3\Psi_1}{d\Psi_1} - \frac{(J_1\beta_x(s_2))^2}{\rho}b_3 L\frac{d\cos^4\Psi_1}{d\Psi_1} \tag{30}$$

$$\overline{\Psi_1} = \Psi_1 + 2\pi\nu + \frac{\sqrt{2}\beta_x(s_1)^{3/2}\overline{J_1}^{1/2}}{\rho} b_2 L \cos^3 \Psi_1 + \frac{2\beta_x(s_2)^2}{\rho}\overline{J_1} b_3 L \cos^4 \Psi_1 \quad (31)$$

Eqs. 30 and 31 are the basic difference equations to study the nonlinear resonance and the onset of stochasticities considering sextupole and octupole perturbations. By using trigonometric relation

$$\cos^m \theta \cos n\theta = 2^{-m} \sum_{r=0}^{m} \frac{m!}{(m-r)!r!} \cos(n - m + 2r)\theta \quad (32)$$

one has

$$\cos^3 \theta = \frac{2}{2^3}(\cos 3\theta + 3\cos\theta) \quad (33)$$

$$\cos^4 \theta = \frac{1}{2^4}(\cos 4\theta + 4\cos 2\theta + \frac{4!}{((4/2)!)^2}) \quad (34)$$

Apparently, the right hand sides of eqs. 30 and 31 contain sinusoidal functions of phases, Ψ_1, $2\Psi_1$, $3\Psi_1$, and $4\Psi_1$. If the tune ν is far from the resonance lines $\nu = m/n$, where m and n are integers (n=1, 2, 3, and 4 for this specific problem), the invariant tori of the unperturbed motion are preserved under the presence of the small perturbations by virtue of the Kolmogorov-Arnold-Moser (KAM) theorem. If, however, ν is close to the above mentioned resonance line, the situation is getting complicated and under some conditions the KAM invariant tori can be broken. Taking the third order resonance, $m/3$, for example, we keep only the sinusoidal function with phase $3\Psi_1$ in eq. 30 and the dominant phase independent nonlinear term in eq. 31, and as the result, we have eqs. 30 and 31 reduced to

$$\overline{J_1} = J_1 + A \sin 3\Psi_1 \quad (35)$$

$$\overline{\Psi_1} = \Psi_1 + B\overline{J_1} \quad (36)$$

with

$$A = \frac{(J_1\beta_x(s_1))^{3/2}}{\sqrt{2}\rho} b_2 L \quad (37)$$

$$B = \frac{3\beta_x(s_2)^2}{4\rho} b_3 L \quad (38)$$

where we have dropped the constant phase in eq. 31. It is helpful to transform eqs. 37 and 38 into the form so-called *standard mapping* [9] expressed as

$$\overline{I} = I + K_0 \sin\theta \quad (39)$$

$$\overline{\theta} = \theta + \overline{I} \tag{40}$$

with $\theta = 3\Psi$, $I = 3BJ_1$ and $K_0 = 3AB$. By virtue of the Chirikov criterion [9] it is known that when $|K_0| \geq 0.97164$ [10] resonance overlaping occurs which results in particles' stochastic motions and diffusion processes. Therefore,

$$|K_0| \leq 1 \tag{41}$$

can be taken as a natural criterion for the determination of the dynamic aperture of the machine. Putting eqs. 37 and 38 into eq. 41, one gets

$$|K_0| = \frac{9}{4\sqrt{2}}|b_2 b_3|\beta_x^2(s_2)\beta_x^{3/2}(s_1)J_1^{3/2}\frac{L^2}{\rho^2} \leq 1 \tag{42}$$

and consequently, one finds maximum J_1 corresponding to $m/3$ resonance

$$J_1 \leq J_{max,m/3} = \left(\frac{4\sqrt{2}\rho^2}{9|b_2 b_3|\beta_x^2(s_2)\beta_x(s_1)^{3/2}L^2}\right)^{2/3} \tag{43}$$

The dynamic aperture of the machine is therefore

$$A_{dyn,m/3} = \sqrt{2J_{max,m/3}\beta_x(s)} = \left(\frac{16\rho^2\beta_x(s)^{3/2}}{9|b_2 b_3|\beta_x^2(s_2)\beta_x(s_1)^{3/2}L^2}\right)^{1/3} \tag{44}$$

Eq. 44 gives the dynamic aperture of a sextuple and octupole strength determined case which is believed to be true for the most small emittance electron storage rings. Obviously, the dynamic aperture scales with the one third power of the sextupole and the octupole strength, respectively. If in a storage ring the perturbation from the sextupoles can be omitted, in a similar way one finds the maximum J_1 corresponding to $m/4$ resonance mode

$$J_1 \leq J_{max,m/4} = \frac{2\rho}{\sqrt{3}\beta_x(s_2)^2 L|b_3|} \tag{45}$$

and the corresponding dynamic aperture

$$A_{dyn,m/4} = \sqrt{2J_{max,m/4}\beta_x(s)} = \left(\frac{4\rho\beta_x(s)}{\sqrt{3}\beta_x(s_2)^2 L|b_3|}\right)^{1/2} \tag{46}$$

From eq. 46 one reads that the dynamic aperture, $A_{dyn,m/4}$, is proportional to the square root of the octupole strength. Usually, one has $A_{dyn,m/3} < A_{dyn,m/4}$.

Now I would like to spend some ink on the scenario of those particles whose motions do not satisfy the condition given by eq. 41. Once a particle begins to execute stochastic motion the phase mixing occurs, and the mapping given by eqs. 39 and 40 can be regarded as a Markov process [11], and in consequence, the possibility distribution function $\mathcal{F}(s, I)$ satisfies the Fokker-Planck equation:

$$\frac{\partial \mathcal{F}}{\partial s} = -\frac{\partial(\mathcal{A}\mathcal{F})}{\partial I} + \frac{1}{2}\frac{\partial^2(\mathcal{D}\mathcal{F})}{\partial I^2} \tag{47}$$

where

$$\mathcal{A} = \frac{<< \Delta I >>}{L} \tag{48}$$

and

$$\mathcal{D} = \frac{<< (\Delta I)^2 >>}{L} \tag{49}$$

where the notation $<<>>$ denotes the average over phase θ. From eq. 39 one knows

$$\Delta I = K_0 sin\theta \tag{50}$$

and obviously, one has

$$\mathcal{A} = 0 \tag{51}$$

and

$$\mathcal{D} = \frac{K_0^2}{2L} \tag{52}$$

Inserting eqs. 51 and 52 into eq. 47 one gets

$$\frac{\partial \mathcal{F}}{\partial s} = \frac{1}{2}\frac{\partial^2(\mathcal{D}\mathcal{F})}{\partial I^2} \tag{53}$$

Due to the diffusion, needless to say, the amplitude of the particle's motion is increasing with the distance and finally the particle is lost on the vacuum chamber wall.

3.2 Electron storage ring

In an electron storage ring the physical picture is more complicated due to the synchrotron radiation damping. To treat this problem let's resort to the so-called *standard dissipative mapping* [8] which is different but similar to the standard mapping shown in eqs. 39 and 40, and expressed as follows:

$$\overline{I} = \exp(-\Gamma)I + K_0\mu\sin\theta \tag{54}$$

$$\overline{\theta} = \theta + \overline{I} \tag{55}$$

where $\Gamma = \frac{2L}{\tau_x c}$, τ_x is the damping time of the betatron oscillation in the horizontal direction, and $\mu = (1 - \exp(-\Gamma))/\Gamma$. Apparently, when $\Gamma \to 0$, eqs. 54 and 55 return to the standard mapping given by eqs. 39 and 40. The criterion for avoiding the onset of the stochastic motion in the dissipative system is given by

$$|\mu K_0| \leq 1 \tag{56}$$

The expressions for the dynamic apertures of the electron storage rings corresponding to the two expressions for the proton ones derived in the previous subsection are

$$A_{dyn,m/3} = \sqrt{2J_{max,m/3}\beta_x(s)} = \left(\frac{16\rho^2\beta_x(s)^{3/2}}{9\mu|b_2b_3|\beta_x^2(s_2)\beta_x(s_1)^{3/2}L^2}\right)^{1/3} \tag{57}$$

and

$$A_{dyn,m/4} = \sqrt{2J_{max,m/4}\beta_x(s)} = \left(\frac{4\rho\beta_x(s)}{\sqrt{3}\beta_x(s_2)^2L\mu|b_3|}\right)^{1/2} \tag{58}$$

The whole discussion in this section is focused on the motion in the horizontal plane, and the passage to the vertical plane is considered to be trivial. Following the same idea one can investigate more complicated cases, such as the coupled transverse motion and the coupled betatron-synchrotron motion, however, they will be excluded from this paper.

4 Comparison with some simulation results

In this section we make some comparisons between the numerical simulation results and the analytical formula shown in eq. 57. The first example is

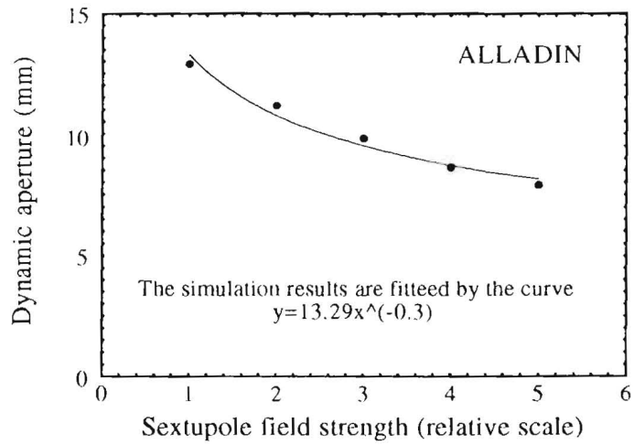

Figure 1: ALLADIN dynamic aperture dependence on the sextupole strength.

the ALLADIN [12] dynamic aperture dependence on the sextupole strength. The numerical simulation results and the fitting curve are shown in Fig. 1 where we can find $A_{dyn} \propto 1/I_{sext}^{\Xi}$ with $\Xi \approx 1/3$ (I_{sext} is the sextupole strength). Secondly, let's see the KEK Photon Factory [13] dynamic aperture dependence on the octupole strength. Fig. 2 shows the numerical simulation results and the fitting curve. Again we find the $A_{dyn} \propto 1/I_{oct}^{\Xi}$ with $\Xi \approx 1/3$ (I_{oct} is the sextupole strength). Both examples agree quite well with what predicted by eq. 57.

To validate quantitatively the analytical dynamic aperture expression shown in eqs. 44 and 57 more comparison works need to be done in the future.

5 Conclusion

Considering delta function sextupole and octupole perturbations, analytical expression for the dynamic aperture of a circular machine is obtained by using the Chirikov criterion. It is shown that when the dynamics aperture is sextupole and octupole strength determined, which is true for most cases, the dynamic aperture scales with the one third power of the sextupole and octupole strength, respectively, and this conclusion is confirmed by the simu-

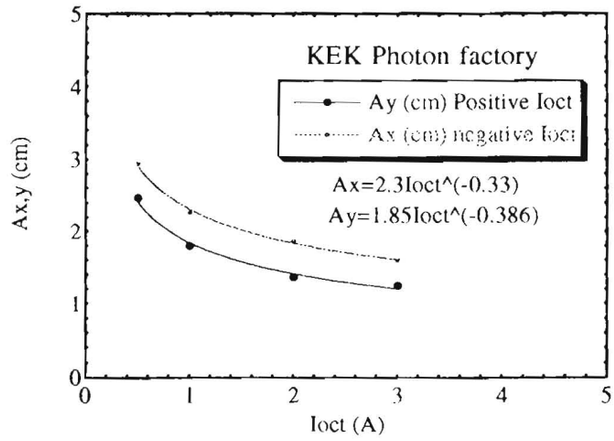

Figure 2: KEK Photon Factory dynamic aperture dependence on the octupole strength.

lation results from the ALLADIN and KEK Photon Factory machines. This work is by no means aimed to replace the powerful numerical simulations in the study of dynamic aperture, but gives useful theoretical insight.

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References

[1] R. Ruth, "Single particle dynamics in circular accelerators", AIP conference proceedings, No. 153, p. 150.

[2] B.W. Montague, "Basic Hamiltonian mechanics", CERN 95-06, Vol. I.

[3] E.J.N. Wilson, "Nonlinear resonances", CERN 95-06, Vol. I.

[4] J.S. Bell, "Hamiltonian mechanics", CERN 87-03.

[5] A.A. Kolomesky and A.N. Lebedev, "Theory of cyclic accelerators", 1966.

[6] T. Suzuki, "Hamiltonian formulation for synchrotron oscillations and Sacherer's integral equation", *Particle Accelerators*, Vol. 12 (1982), p. 237.

[7] H. Wiedemann, "Particle Accelerator Physics II", Springer-Verlag, 1993.

[8] R.Z. Sagdeev, D.A. Usikov, and G.M. Zaslavsky, "Nonlinear Physics, from the pendulum to turbulence and chaos", Harwood Academic Publishers, 1988.

[9] B.V. Chirikov, "A universal instability of many-dimensional oscillator systems", Physics Reports, Vol. 52, No. 5 (1979), p. 263.

[10] J.M. Greene, J. Math. Phys. 20 (1979), p. 1183.

[11] R.L. Stratonovich, "Topics in the theory of random noise", Vol. 1, Gordon and Breach Science Publishers Inc., 1963.

[12] J. Bridges, et al., "Dynamic aperture measurement on ALADDIN", *Particle Accelerators*, Vol. 28 (1990), p. 1.

[13] Y. Kobayash, et al., "Experimental measurement of dynamic aperture at the photon factory storage ring", Proceedings of PAC93 (1993), p. 215, see also: M. Katoh, "Dynamics aperture of the photon factory storage ring", *Particle Accelerators*, Vol. 28 (1990), p. 17.