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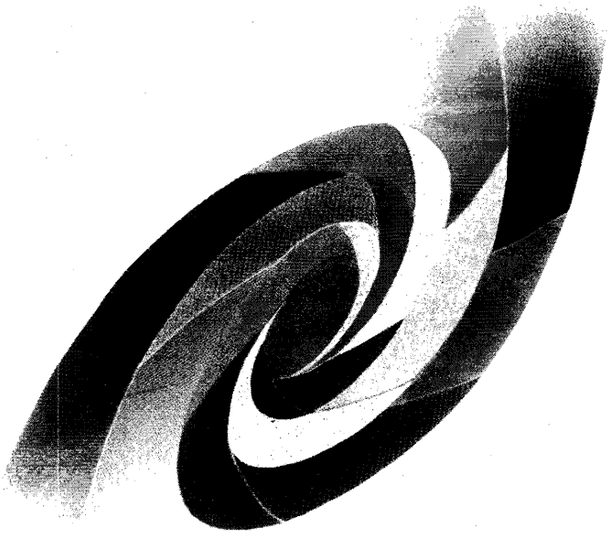

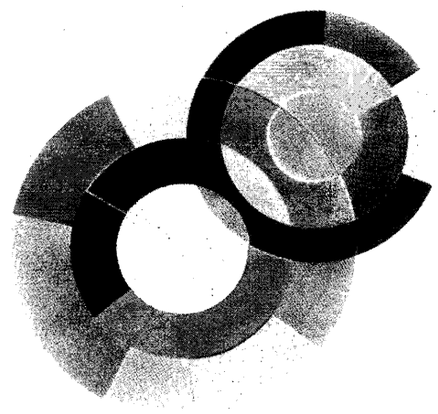

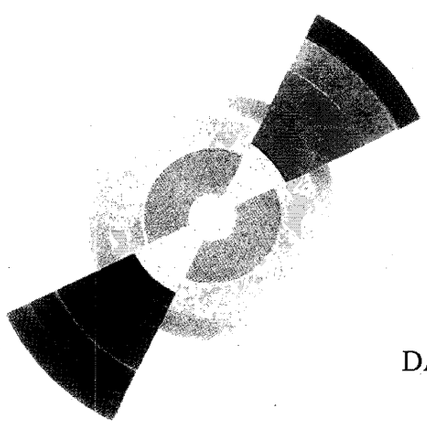

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Measuring the weak phase $2\beta + \gamma$ in color allowed $B^0 \to D^{\pm} K^0 \pi^{\mp}$ decays.

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Rapport

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Measuring the Weak Phase $2\beta + \gamma$ in Color Allowed $B^0 \to D^{\pm} K^0 \pi^{\mp}$ Decays

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Abstract

We present a method to measure the weak phase $2\beta + \gamma$ in the three-body decay of neutral $B^0(\overline{B}^0)$ mesons to the final states $D^{\mp} K_s \pi^{\pm}$. These decays are mediated by interfering amplitudes which are color-allowed and hence relatively large. As a result, large CP violation effects that could be observed with high statistical significance are possible. In addition, the three-body decay helps resolve discrete ambiguities that are usually present in measurements of the weak phase. The experimental implications of conducting these measurements with three-body decays are discussed, and the sensitivity of the method is evaluated using a simulation.

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I. INTRODUCTION

The violation of the CP symmetry is now established in the B meson sector and the parameter $\sin(2\beta)$ is measured with good precision by BaBar [1] and Belle [2]. These measurements are in excellent agreement with the prediction of the Standard Model in which CP violation is accounted for via the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3].

Although improved measurements of $\sin(2\beta)$, where $\beta = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$, combined with results on $B_s - \overline{B}_s$ mixing and more accurate measurements of V_{ub} will probe the theory with greater scrutiny during the next few years, the measurement of the other angles and the sides of the unitarity triangle are necessary for a comprehensive study of CP violation. However, the measurements (or some combination) of the angles suffer from several difficulties related to the uncertainties of the theory, the limited statistics or ambibuities in their extraction. We have recently proposed a new method [4] involving three-body B meson decays, which circumvent these problems and allows to determine the angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$.

In this paper, we extend this method to the measurement of the weak phase $\arg(V_{td}^{*2}V_{tb}^2V_{ub}^*V_{us}/V_{cd}^{*2}V_{cb}V_{cs})$, which can be identified to the combination $2\beta + \gamma'$ of the unitarity triangle angles [5], where $\gamma' = \gamma + \xi$ with $\xi = \arg(-V_{cd}V_{cs}^*/V_{ud}V_{us}^*)$. The angle ξ is a small angle of order λ^4 derived from the unitarity relation $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$. Thus since $2\beta + \gamma' \simeq 2\beta + \gamma$, we will always refer to $2\beta + \gamma$ in the following. Important constraints on the theory will be obtained from the measurement of this phase when combined with the values of 2β and γ extracted elsewhere. To our knowledge, as of today the methods proposed for measuring $2\beta + \gamma$ suffer from several difficulties. The decays $\overline{B}^0 \to D^{(*)+}\pi^-$ allows to extract this phase but it requires the interference between two very different amplitudes in size. Hence the resulting asymmetry is very small requiring a very good control of the experimental systematic errors. Furthermore, in order to extract $2\beta + \gamma$, one is confronted to an 8-fold ambiguity and to the poorer knowledge of the size of the smallest amplitude. In a different method using the final state D^0K_s [6], the interfering amplitudes are of the same order. However since both amplidudes are color-suppressed, the required statistics for

achieving a significant measurement is very large.

The B meson decay to the 3-body final state $D^{\mp}K_s\pi^{\pm}$ has already been considered but the proposed method relies on resonant states such as $K^{*\mp}$ and $D_s^{**\pm}$ [7] leading to a very restricted region of the phase space where the amplitudes can interfere.

In the following, we describe a new method involving the entire area of the Dalitz plot for the 3-body decays $B \to DK\pi$.

II. MEASURING $2\beta + \gamma$ WITH COLOR-ALLOWED $B \to DK\pi$ DECAYS

We investigate a way to circumvent the color suppression penalty by using $B^0(\overline{B}^0)$ decay modes which could potentially offer significantly large branching fractions, as well as large CP asymmetries. The particular decays which are considered here are of the type $B \to D^{(*)}K^{(*)}\pi(\rho)$. These three body decays may be obtained by popping a $q\bar{q}$ pair in color allowed decays. Although modes where one or more of the three final state particles is a vector can also be used, for clarity and simplicity only the mode $B^0(\overline{B}^0) \to D^{\mp}K_s\pi^{\pm}$ is discussed here.

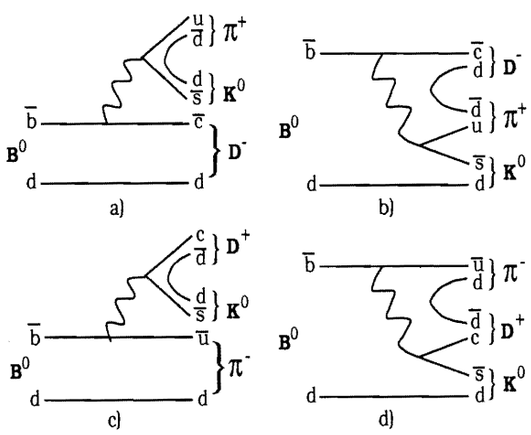

FIG. 1. Feynmann Diagrams for the decay $B^0 \to D^{\mp}K_s\pi^{\pm}$ with the CKM matrix elements $V_{cb}^*V_{us}$ (top) and $V_{ub}^*V_{cs}$ (bottom).

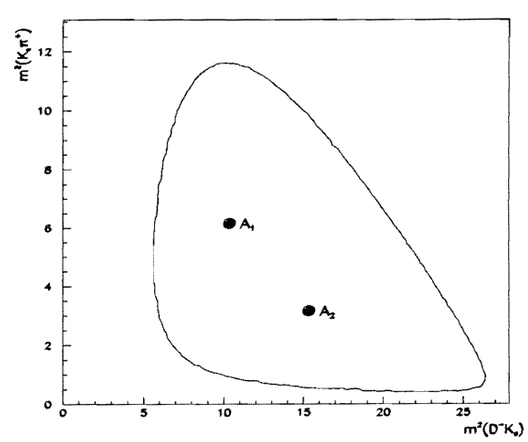

FIG. 2. Two points on the Dalitz plot of the decays $B^0 \to D^{\mp}K_s\pi^{\pm}$.

Figs. 1 shows the diagrams leading to the final states of interest. As can be seen, the leading diagrams (Fig. 1a and 1c) are both color-allowed and of order $\lambda^3 = \sin(\theta_c)$ in the

Wolfenstein parameterization [8], where θ_c is the Cabibbo mixing angle. The B^0 meson can decay to both final states $D^-K^0\pi^+$ and $D^+K^0\pi^-$, while $\overline{B}^0$ decays to $D^-\overline{K^0}\pi^+$ and $D^+\overline{K^0}\pi^-$. Since both K^0 and $\overline{K^0}$ are observed either as a K_S or a K_L it is possible to generate CP violation via the interference between the direct decay and the one proceeding through $B^0-\overline{B}^0$ mixing. Because of the absence of color suppression, both interfering amplitudes are likely to be large, avoiding the complications which arise due to the small magnitude of the $\bar{b} \to \bar{u}c\bar{s}$ amplitude in the two-body decays $D^{(*)-}\pi^+$. As a result, observable CP-violating effects in the three-body decays are expected to be large, and the $\bar{b} \to \bar{u}c\bar{s}$ amplitude is more easily measured from the relatively large branching fraction $\mathcal{B}(B^0 \to D^+K_s\pi^-)$ and $\mathcal{B}(\overline{B}^0 \to D^-K_s\pi^+)$ as it is shown later.

The general formalism describing how the weak angle is extracted from the observation of the time-dependent CP violation has been first presented in ref. [9] and used later for measuring $\sin^2\gamma$ with the decay $B_s \to D_s^-K^+$ [10]. The decays studied in this paper are very similar in many way to the one proposed in this latter reference. However in the present analysis additional information is used to resolve the inherent ambiguities related to the method.

Let us now examine how one could observe CP violation and measure the angle $2\beta+\gamma$ in the decays $B^0 \to D^{\mp}K_s\pi^{\pm}$. We first consider the case of very large statistics, and then discuss how one would proceed when the data sample is limited. Since we are dealing with a three-body decay, we use the Dalitz plot of the system $D^{\mp}K_s\pi^{\pm}$ (see Fig. 2). Selecting a particular point i in this representation, let us write the amplitudes corresponding to the transitions in Fig. 1a and b on the one hand and 1c and d on the other hand as

$$\mathcal{A}_i(B^0 \to D^-K_s\pi^+) = A_{\mathcal{C}i}e^{i\delta_{\mathcal{C}i}} \quad , \quad \mathcal{A}_i(B^0 \to D^+K_s\pi^-) = A_{\mathcal{U}i}e^{i\delta_{\mathcal{U}i}}e^{i\gamma}, \tag{1}$$

and their CP conjugates

$$\mathcal{A}_i(\overline{B}^0 \to D^+K_s\pi^-) = A_{\mathcal{C}i}e^{i\delta_{\mathcal{C}i}} \quad , \quad \mathcal{A}_i(\overline{B}^0 \to D^-K_s\pi^+) = A_{\mathcal{U}i}e^{i\delta_{\mathcal{U}i}}e^{-i\gamma}, \tag{2}$$

where γ is the relative phase of the CKM matrix elements involved in this decay, and $A_{\mathcal{C}}$ ($A_{\mathcal{U}}$) and $\delta_{\mathcal{C}}$ ($\delta_{\mathcal{U}}$) are the real amplitude and CP-conserving strong interaction phase of the

transitions of Fig. 1a and b (Fig. 1c and d). The probability of the transition $\overset{(-)}{B^0} \to D^- K_s \pi^+$ is

$$\mathcal{P}r\left(\overset{(-)}{(B^0} \to D^- K_s \pi^+\right) = \frac{A_{c_i}^2 + A_{u_i}^2}{2} e^{-\frac{t}{\tau}} \left\{1 \overset{(-)}{+} \mathcal{R}_i \cos x\frac{t}{\tau} \overset{(-)}{+} \mathcal{D}_i \sin(2\beta + \gamma + \Delta\delta_i) \sin x\frac{t}{\tau}\right\} \tag{3}$$

where $\mathcal{R}_i = \frac{\rho_i^2 - 1}{\rho_i^2 + 1}$, $\mathcal{D}_i = \frac{2\rho_i}{\rho_i^2+1} = \sqrt{1 - \mathcal{R}_i^2}$ with $\rho_i = \frac{A_{c_i}}{A_{u_i}}$ and $\Delta\delta_i = \delta_{c_i} - \delta_{u_i}$. We have used the approximation $\left|\frac{p}{q}\right| = 1$ where p and q are defined as the coefficient defining the B mass-eigenstates ($|B_{L(H)} >= p|B^0 > +(-)q|\overline{B}^0 >$). In Eq. 3, $x = \Delta m/\Gamma$ and τ is the B^0 meson lifetime.

The probability of the transition $\overset{(-)}{B^0} \to D^+ K_s \pi^-$ is

$$\mathcal{P}r\left(\overset{(-)}{B^0} \to D^+ K_s \pi^-\right) = \frac{A_{c_i}^2 + A_{u_i}^2}{2} e^{-\frac{t}{\tau}} \left\{1 \overset{(+)}{-} \mathcal{R}_i \cos x\frac{t}{\tau} \overset{(-)}{+} \mathcal{D}_i \sin(2\beta + \gamma - \Delta\delta_i) \sin x\frac{t}{\tau}\right\} \tag{4}$$

From the total number of events in each of the four distributions (3) and (4) and a global fit of their time dependence, it is possible to extract the four quantities, $A_{c_i}^2 + A_{u_i}^2$, ρ_i , S , $\bar{S}$ where $S \equiv \sin(2\beta + \gamma + \Delta\delta_i)$ and $\overline{S} \equiv \sin(2\beta + \gamma - \Delta\delta_i)$.

The equations (3) and (4) are valid if the Standard Model holds. Should there be new physics leading to additional diagrams interfering with the ones in Fig. 1, one should write

$$\mathcal{P}r\left(\overset{(-)}{(B^0} \to D^- K_s \pi^+\right) = \frac{A^- c_i^2 + A^- u_i^2}{2} e^{-\frac{t}{\tau}} \left\{1 \overset{(-)}{+} \mathcal{R}_i^- \cos x\frac{t}{\tau} \overset{(-)}{+} \mathcal{D}_i^- S^- \sin x\frac{t}{\tau}\right\} \tag{5}$$

$$\mathcal{P}r\left(\overset{(-)}{B^0} \to D^+ K_s \pi^-\right) = \frac{A^+ c_i^2 + A^+ u_i^2}{2} e^{-\frac{t}{\tau}} \left\{1 \overset{(+)}{-} \mathcal{R}_i^+ \cos x\frac{t}{\tau} \overset{(-)}{+} \mathcal{D}_i^+ S^+ \sin x\frac{t}{\tau}\right\} \tag{6}$$

In the same way, the fit of these four time-dependent distributions would allow to extract $A^- c_i^2 + A^- u_i^2$, $A^+ c_i^2 + A^+ u_i^2$, $\mathcal{R}_i^-$, $\mathcal{R}_i^+$, $\mathcal{D}_i^- S^-$ and $\mathcal{D}_i^+ S^+$. The observation of direct CP violation by measuring $A^- c_i^2 + A^- u_i^2 \neq A^+ c_i^2 + A^+ u_i^2$ and/or $\mathcal{R}_i^- \neq \mathcal{R}_i^+$ would be a clear indication of new physics.

For simplicity we assume no new physics contibutions in the following and therefore one obtains

$$\sin^2(2\beta + \gamma) = \frac{1}{2}\left(1 - S\overline{S} \pm \sqrt{(1 - S^2)(1 - \overline{S}^2)}\right), \tag{7}$$

Hence in the limit of very high statistics, one would extract $\sin^2(2\beta+\gamma)$ for each point i of the Dalitz plot and therefore have many measurements of the same quantity. This would allow one to obtain a large set of redundant measurements from which a precise and consistent value of $\sin^2(2\beta+\gamma)$ could be extracted. However, in every point of the Dalitz plot, $2\beta+\gamma$ is obtained with an eight-fold ambiguity, which is a consequence of the invariance of the $\sin(2\beta+\gamma\pm\Delta\delta_i)$ terms in Eq. (3) and (4) under the three symmetry operations

$$\begin{aligned} S_{\text{ex}} &: 2\beta+\gamma \to \pi/2-\Delta\delta \quad , \ \Delta\delta \to \pi/2-(2\beta+\gamma) \\ S_{\text{sign}} &: 2\beta+\gamma \to -(2\beta+\gamma) \quad , \ \Delta\delta \to \pi-\Delta\delta \\ S_{\pi} &: 2\beta+\gamma \to 2\beta+\gamma+\pi \ , \ \Delta\delta \to \Delta\delta+\pi. \end{aligned} \tag{8}$$

An important benefit is gained from the multiple measurements made in different points of the Dalitz plot. When results from the different points are combined, some of the ambiguity will be resolved, in the likely case that the strong phase $\Delta\delta_i$ varies from one region of the Dalitz to the other. This variation can either be due to the presence of resonances or because of a varying phase in the non-resonant contribution. In this case, the exchange symmetry S_{ex} is numerically different from one point to the other, which in effect breaks this symmetry and resolves the ambiguity.

Similarly, the S_{sign} symmetry is broken if there exists some *a priori* knowledge of the dependence of $\Delta\delta_i$ on the Dalitz plot parameters. This knowledge is provided by the existence of broad resonances, whose Breit-Wigner phase variation is known and may be assumed to dominate the phase variation over the width of the resonance. To illustrate this, let i and j be two points in the Dalitz plot, corresponding to different values of the invariant mass of the decay products of a particular resonance. For simplicity we consider only one resonance. One then measures $\sin(2\beta+\gamma\pm\Delta\delta_i)$ at point i and $\sin(2\beta+\gamma\pm(\Delta\delta_i+\alpha_{ij}))$ at point j, where α_{ij} is known from the parameters of the resonance. It is important to note that the sign of α_{ij} is also known, hence it does not change under S_{sign}. Therefore, should one choose the S_{sign}-related solution $\sin(\pi-(2\beta+\gamma)\mp\Delta\delta_i)$ at point i, one would get $\sin(\pi-(2\beta+\gamma)\mp(\Delta\delta_i-\alpha_{ij}))$ at point j. Since this is different from $\sin(2\beta+\gamma\pm(\Delta\delta_i+\alpha_{ij}))$ the S_{sign} ambiguity is resolved. This is illustrated graphically in Eq. 9:

$$
\begin{array}{ccc}
\sin(2\beta+\gamma\pm\Delta\delta_i) & \overset{S_{\rm sign}}{\longleftrightarrow} & \sin(\pi-(2\beta+\gamma)\mp\Delta\delta_i) \\
BW\downarrow & & \downarrow BW \\
\sin(2\beta+\gamma\pm(\Delta\delta_i+\alpha_{ij})) & \overset{S_{\rm sign}}{\not\longleftrightarrow} & \sin(\pi-(2\beta+\gamma)\mp(\Delta\delta_i-\alpha_{ij}))
\end{array}
\tag{9}
$$

Thus, broad resonances reduce the initial eight-fold ambiguity to the two-fold ambiguity of the S_π symmetry, which is not broken. Fortunately, S_π leads to the well-separated solutions $2\beta+\gamma$ and $2\beta+\gamma+\pi$, the correct one of which is easily identified when this measurement is combined with other measurements of the unitarity triangle.

III. THE FINITE STATISTICS CASE

Since experimental data sets will be finite, extracting $2\beta+\gamma$ will require making use of a limited set of parameters to describe the variation of amplitudes and strong phases over the Dalitz plot. The consistency of this approach can be verified by comparing the results obtained from fits of the data in a few different regions of the Dalitz plot, and the systematic error due to the choice of the parameterization of the data may be obtained by using different parameterizations.

A fairly general parameterization assumes the existence of N_R Breit-Wigner resonances, as well as a non-resonant contribution:

$$
\begin{aligned}
\mathcal{A}_\xi(B^0\to D^-K_s\pi^+) &= \left(A_{C0}\,e^{i\delta_{C0}}+\sum_{j=1}^{N_R}A_{Cj}B_{s_j}(\xi)\,e^{i\delta_{Cj}}\right)e^{i\delta_C(\xi)} \\
\mathcal{A}_\xi(B^0\to D^+K_s\pi^-) &= \left(A_{U0}\,e^{i\delta_{U0}}+\sum_{j=1}^{N_R}A_{Uj}B_{s_j}(\xi)\,e^{i\delta_{Uj}}\right)e^{i\delta_U(\xi)}e^{i\gamma},
\end{aligned}
\tag{10}
$$

where ξ represents the Dalitz plot variables,

$$
B_{s_j}(\xi)\equiv b_{s_j}(\xi)\,e^{i\delta_j(\xi)} \tag{11}
$$

is the Breit-Wigner amplitude for a particle of spin s_j, normalized such that $\int(b_{s_j}(\xi))^2d\xi=1$, A_{U0} and δ_{U0} (A_{C0} and δ_{C0}) are the magnitude and CP-conserving phase of the non-resonant $\bar{b}\to\bar{u}c\bar{s}$ ($\bar{b}\to\bar{c}u\bar{s}$) amplitude, and A_{Uj} and δ_{Uj} (A_{Cj} and δ_{Cj}) are the magnitudes and CP-conserving phase of the $\bar{b}\to\bar{u}c\bar{s}$ ($\bar{b}\to\bar{c}u\bar{s}$) amplitude associated with resonance j [11].

The functions $\delta_C(\xi)$ and $\delta_U(\xi)$ (in fact their difference $\Delta\delta(\xi)$) may be assumed to vary slowly over the Dalitz plot, allowing their description in terms of a small number of parameters. The decay amplitudes of $\overline{B}^0$ mesons are identical to those of Eqs. (10), with γ replaced by $-\gamma$.

The decay amplitudes of Eqs. (10) can be used to conduct the full data analysis. This is done by constructing the probability density function (PDF)

$$P(\xi) = |\mathcal{A}_\xi(f)|^2, \tag{12}$$

where the amplitude $A_\xi(f)$ is given by one of the expressions of Eq. (10), or their CP-conjugates, depending on the final state f. Given a sample of N_e signal events, γ and the other unknown parameters of Eq. (10) are determined by minimizing the negative log likelihood function

$$\chi^2 \equiv -2\sum_{i=1}^{N_e} \log P(\xi_i), \tag{13}$$

where ξ_i are the Dalitz plot variables of event i.

IV. RESONANCES AND AMBIGUITIES

It is worthwhile to consider the resonances which may contribute to the $D^\mp K_s \pi^\pm$ final state. Obvious candidates are broad D^{**} and D_s^{**} states. However, only the ones which can decay as $D^{**0}(\overline{D}^{**0}) \to D^\pm\pi^\mp$ or $D_s^{**\pm} \to D^\pm K_s$ are relevant for the final state of interest. This exclude the 1^+ states, which would decay to $D^*\pi$ or D^*K. Furthermore, since the $D_s^{**\pm}$ is essentially produced through a $W^\pm$, the 2^+ state is forbidden as well. Thus, one does not expect a large contribution from these states. We note, however, that including them in the analysis does not raise particular difficulties, and may in fact increase the sensitivity to γ. Similar arguments can be made for higher excited K states.

One expects narrow resonances, such as $D^{**0}(\overline{D}^{**0})$ and $D_s^{**\pm}$ states, to be produced. However, these resonances do not overlap, and hence do not interfere. In addition, interference between a very narrow resonance and either a broad resonance or a non-resonant term is suppressed in proportion to the square root of the narrow resonance width. Therefore,

narrow resonances contribute significantly to the CP violation measurement only if both the $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ amplitudes proceed through the same resonance. This scenario is favorable, but is not necessary for the success of our method, and will therefore not be focused on in the rest of this study.

In what follows, we discuss important properties of the method by considering the illustrative case, in which the $\bar{b} \to \bar{u}c\bar{s}$ decay proceeds only via a non-resonant amplitude, and the $\bar{b} \to \bar{c}u\bar{s}$ decay has a non-resonant contribution and a single resonant amplitude. For concreteness, the resonance is taken to be the $K^{*\pm}(892)$. We take the ξ-dependent non-resonant phases to be $\delta_{\mathcal{C}}(\xi) = \delta_{\mathcal{U}}(\xi) = 0$. Under these circumstances, the PDF of Eq. (12) depends on four sine terms that are measured in the experiment:

$$\begin{aligned} s_{00}^{\pm} &\equiv \sin(2\beta + \gamma \pm \delta_{\mathcal{U}0}) \\ s_{K^*0}^{\pm} &\equiv \sin(2\beta + \gamma \pm (\delta_{\mathcal{U}0} - \delta_{K^*} - \delta_{K^*}(\xi))), \end{aligned} \tag{14}$$

where $\delta_{K^*}(\xi)$ is the ξ-dependent K^* Breit-Wigner phase of Eq. (11). The sines $s_{00}^{\pm}$ ($s_{K^*0}^{\pm}$) arise from interference between the non-resonant (resonant) $\bar{b} \to \bar{c}u\bar{s}$ amplitude and the non-resonant $\bar{b} \to \bar{u}c\bar{s}$ amplitude.

The phases $\delta_{\mathcal{U}0}$, δ_{K^*}, and $2\beta + \gamma$ are all *a priori* unknown. However, it is important to note that δ_{K^*} is fully determined from the interference between the resonant and non-resonant contributions to the relatively higher statistics decay modes $B^0 \to D^- K_s \pi^+$ and $\overline{B}^0 \to D^+ K_s \pi^-$ as a function of the Dalitz plot variables. Therefore, δ_{K^*} is obtained with no ambiguities, and with an error smaller than those of $\delta_{\mathcal{U}0}$ or $2\beta + \gamma$. Consequently, the only relevant symmetry operations are

$$\begin{aligned}
S_{\text{ex}} &: 2\beta + \gamma \to \pi/2 - \delta_{\mathcal{U}0} &&, \delta_{\mathcal{U}0} \to \pi/2 - (2\beta + \gamma) \\
S_{\text{sign}} &: 2\beta + \gamma \to -(2\beta + \gamma) &&, \delta_{\mathcal{U}0} \to \pi - \delta_{\mathcal{U}0} \\
S_{\pi} &: 2\beta + \gamma \to 2\beta + \gamma + \pi &&, \delta_{\mathcal{U}0} \to \delta_{\mathcal{U}0} + \pi \\
S_{\text{ex}}^{K^*+} &: 2\beta + \gamma \to \pi/2 - \delta_{\mathcal{U}0} + \delta_{K^*} &&, \delta_{\mathcal{U}0} \to \pi/2 - (2\beta + \gamma) + \delta_{K^*} \\
S_{\text{ex}}^{K^*-} &: 2\beta + \gamma \to -\pi/2 + \delta_{\mathcal{U}0} - \delta_{K^*} &&, \delta_{\mathcal{U}0} \to \pi/2 + (2\beta + \gamma) + \delta_{K^*}.
\end{aligned} \tag{15}$$

As discussed above, only S_{π} is a symmetry of all four sines of Eq. (14), and is therefore

fully unresolved. The transformation properties of the sines under any combination of the remaining four operations that can lead to an ambiguity are shown in Table I.

TABLE I. Invariance of each of the sines of Eq. (14) under combinations of the symmetry operations of Eq. (15), excluding S_π. Full (approximate) invariance is indicated by a $\surd(\overline{\surd})$.

Operation	$s^+_{K^*0}$	$s^-_{K^*0}$	s^+_{00}	s^-_{00}
		Non-resonant regime		
$S_{\rm ex}$		$\surd$	$\surd$	$\surd$
$S_{\rm sign}$			$\surd$	$\surd$
$S_{\rm ex}S_{\rm sign}$	$\surd$		$\surd$	$\surd$
		Resonant regime		
$S_{\rm ex}^{K^*+}$	$\overline{\surd}$	$\surd$		$\surd$
$S_{\rm ex}^{K^*-}$	$\surd$	$\overline{\surd}$	$\surd$	
$S_{\rm ex}^{K^*+}S_{\rm ex}^{K^*-}$	$\overline{\surd}$	$\overline{\surd}$		

While none of the operations leaves all four sines invariant, it is important to note cases where $s^\pm_{K^*0}$ are *approximately* invariant under $S_{\rm ex}^{K^*+}$, $S_{\rm ex}^{K^*-}$, or their product. We define approximate invariance under the operation S to be

$$S_{\rm app}\, s^\pm_{K^*0}(\delta_{K^*}(\xi)) = s^\pm_{K^*0}(-\delta_{K^*}(\xi)). \tag{16}$$

Approximate invariance arises due to the fact that far from the peak of the K^* resonance, $\delta_{K^*}(\xi)$ changes slowly as a function of the $K\pi$ invariant mass, and takes values around $\pm\pi$. Therefore, for events in the tails of the Breit-Wigner, $\delta_{K^*}(\xi)$ is almost invariant under any $S_{\rm app}$ satisfying Eq. (16). One can see that approximate invariance of one of the sines $s^\pm_{K^*0}$ implies minimal change in the χ^2 of Eq. (13), which may result in a resolved yet clearly observable ambiguity. Since both $s^\pm_{K^*0}$ terms are only approximately invariant under the product $S_{\rm ex}^{K^*+}S_{\rm ex}^{K^*-}$, this ambiguity is more strongly resolved than either $S_{\rm ex}^{K^*+}$ or $S_{\rm ex}^{K^*-}$.

Observing that no single operation in the Table I is a good symmetry of all sines, one identifies two different regimes: In the non-resonant regime, interference with the non-resonant $\bar{b} \to \bar{c}u\bar{s}$ is dominant, and only $S_{\rm ex}$ and $S_{\rm sign}$ may lead to ambiguities. In the resonant regime, the K^* amplitude strongly dominates the $\bar{b} \to \bar{c}u\bar{s}$ decay, and $S_{\rm ex}^{K^*+}$ and $S_{\rm ex}^{K^*-}$ become the important ambiguities. In the transition between these regimes, the operations of Table I do not lead to clear ambiguities, as we have verified by simulation (See sec. V). Thus, while naively one may expect a 2^5-fold ambiguity, in practice the observable ambiguity is no larger than eight-fold, with only the two-fold S_π being fully unresolved, in the likely case of non-negligible resonant contribution. This is demonstrated in Fig. 5. Furthermore, although one may write down more products of the operations $S_{\rm ex}$, $S_{\rm sign}$, $S_{\rm ex}^{K^*+}$, and $S_{\rm ex}^{K^*-}$, only the products listed in Table I result in full or partial invariance of both sines which dominate the same regime. The additional products do not result in any noticeable ambiguities.

V. MEASUREMENT SENSITIVITY AND SIMULATION STUDIES

To study the feasibility of the analysis using Eq. (13) and verify the predictions of Sec. IV, we conducted a simulation of the decays $B^0 \to D^\mp K_s \pi^\pm$ and $\overline{B}^0 \to D^\pm K_s \pi^\mp$. Events were generated according to the PDF of Eq. (12), with the base parameter values given in Table II. In this table and throughout the rest of the paper, we use a tilde to denote the "true" parameter values used to generate events, while the corresponding plain symbols represent the "trial" parameters used to calculate the experimental χ^2.

The only non-vanishing amplitudes in the simulation were the non-resonant amplitudes in the $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ decays, and the K^* resonant $\bar{b} \to \bar{c}u\bar{s}$ amplitude (except for illustrative examples in Figs. 3 and 4). For simplicity, additional resonances were not included in this demonstration. However, broad resonances that are observed in the data should be included in the actual data analysis.

The simulations were conducted with a benchmark integrated luminosity of 400 fb^{-1}, which each of the asymmetric B-factories plan to collect by about 2005. The final state reconstruction efficiencies were calculated based on the capabilities of current Υ(4S) detec-

TABLE II. Parameters used to generate events in the simulation. The value of $\tilde{A}_{cK^*}$ is chosen so as to roughly agree with the measurement of the corresponding branching fraction [13], taking into account the $K^{*+} \to K_s^0\pi^\pm$ branching fraction.

Parameter	Value	Parameter	Value
$2\tilde{\beta}+\tilde{\gamma}$	2.0	$\tilde{A}_{u0}/\tilde{A}_{c0}$	0.4
$\tilde{\delta}_c(\xi)=\tilde{\delta}_u(\xi)$	0	$\tilde{A}_{cK^*}/\tilde{A}_{c0}$	1.0
$\tilde{\delta}_{K^*}$	1.8	$\tilde{A}_{cK^*}$	$\sim\sqrt{2\times10^{-4}\Gamma_B}$
$\tilde{\delta}_{u0}$	1.0		

TABLE III. The numbers of events obtained by averaging 50 simulations using the parameters of Table II and the reconstruction efficiencies listed in the text.

Mode	Signal events per 400fb^{-1}
$B^0 \to D^- K_s \pi^+$	124
$B^0 \to D^+ K_s \pi^-$	36
$\overline{B}^0 \to D^+ K_s \pi^-$	122
$\overline{B}^0 \to D^- K_s \pi^+$	37

tors. We assumed an efficiency of 50% for reconstructing the K_s, including track quality, and 80% for reconstructing the $\pi^\pm$. The product of reconstruction efficiencies and branching fractions of the $D^\pm$, summed over the final states $K^\mp\pi^\pm\pi^\pm$ and $K_s\pi^\pm$ is taken to yield an effective efficiency of 4%. Finally the analysis efficiency *including tagging* was estimated to be 20%. The numbers of signal events obtained in each of the final states with the above efficiencies and the parameters of Table II are listed in Table III.

In Figs. 3 through 5 we show the dependence of χ^2 on the values of $2\beta+\gamma$ and δ_{u0}. The smallest value of χ^2 is shown as zero. At each point in these figures, χ^2 is calculated with the generated values of the amplitude ratios $A_{u0}/A_{c0} = \tilde{A}_{u0}/\tilde{A}_{c0}$ and $A_{cK^*}/A_{c0} = \tilde{A}_{cK^*}/\tilde{A}_{c0}$. When these amplitude ratios are determined by a fit simultaneously with the phases, the correlations between the amplitudes and the phases are found to be less than 10%. Therefore,

the results obtained with the amplitudes fixed to their true values are sufficiently realistic for the purpose of this demonstration.

For each of these figures, we also show the one-dimensional minimum projection $\chi^2(2\beta+\gamma) = \min\{\chi^2(2\beta+\gamma, \delta_{U0})\}$, showing the smallest value of χ^2 for each value of $2\beta+\gamma$.

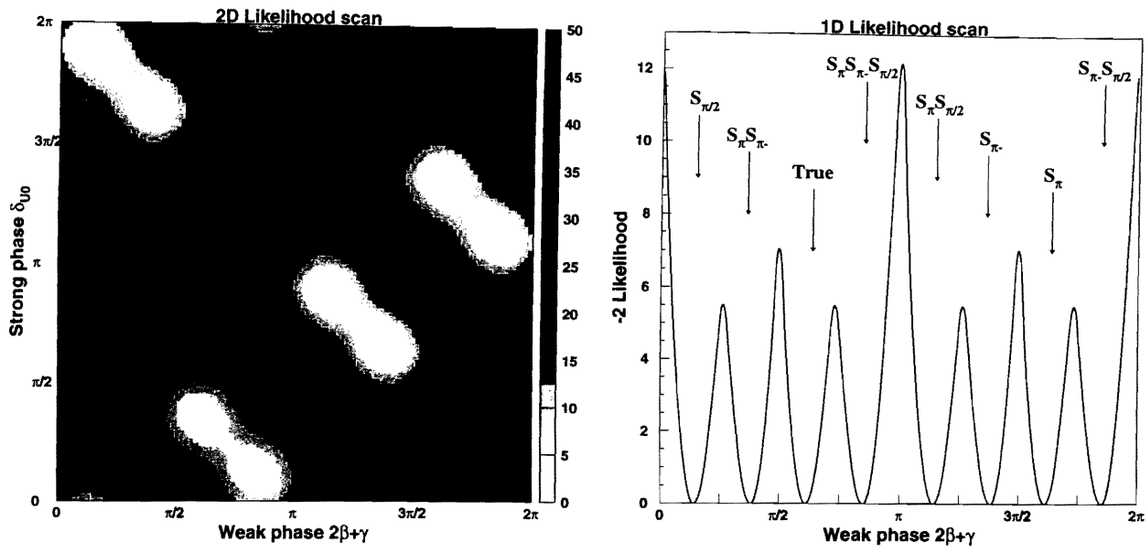

FIG. 3. **Left:** χ^2 as a function of $2\beta+\gamma$ and δ_{U0}, with the parameters of Table II and no resonant contribution ($\tilde{A}_{CK^*} = 0$). **Right:** Minimum projection of χ^2 onto $2\beta+\gamma$.

Fig. 3 is a simulation obtained with the parameters of Table II, but with $A_{CK^*} = 0$. With no resonant contribution, the eight-fold ambiguity of the perfect non-resonant regime is clearly visible.

Fig. 4 is obtained with the parameters of Table II, but with $A_{C0} = 0$. With no non-resonant $\bar{b} \to \bar{c}u\bar{s}$ contribution, the eight-fold ambiguity of the perfect resonant regime is seen. The ambiguities corresponding to approximate invariance are clearly resolved, with the doubly-approximate $S_{\rm ex}^{K^*+} S_{\rm ex}^{K^*-}$ ambiguity resolved more strongly.

Fig. 5 is obtained with the parameters of Table II. With equal resonant and non-resonant $\bar{b} \to \bar{c}u\bar{s}$ amplitudes, only the non-resonant regime ambiguities are observed, due to the relative suppression of the resonant interference terms, discussed in Sec. IV. Nonetheless, the $s^{\pm}_{K^*0}$ terms are significant enough to resolve all but the S_π ambiguity. $S_{\rm sign}$ is more strongly resolved, since it leaves neither of the $s^{\pm}_{K^*0}$ terms invariant.

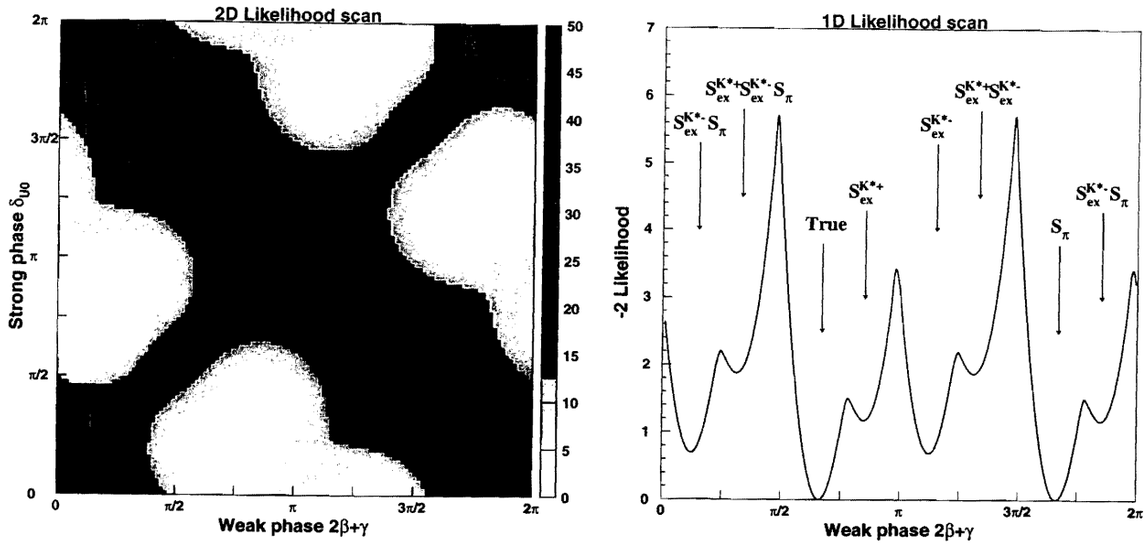

FIG. 4. **Left:** χ^2 as a function of $2\beta+\gamma$ and δ_{U0}, with no non-resonant $\bar{b} \to \bar{c}u\bar{s}$ contribution ($\tilde{A}_{C0} = 0$). The value $\delta_{K^*} = 1.2$ is used to ensure that ambiguities do not overlap. All other parameters are those of Table II. **Right:** Minimum projection of χ^2 onto $2\beta+\gamma$.

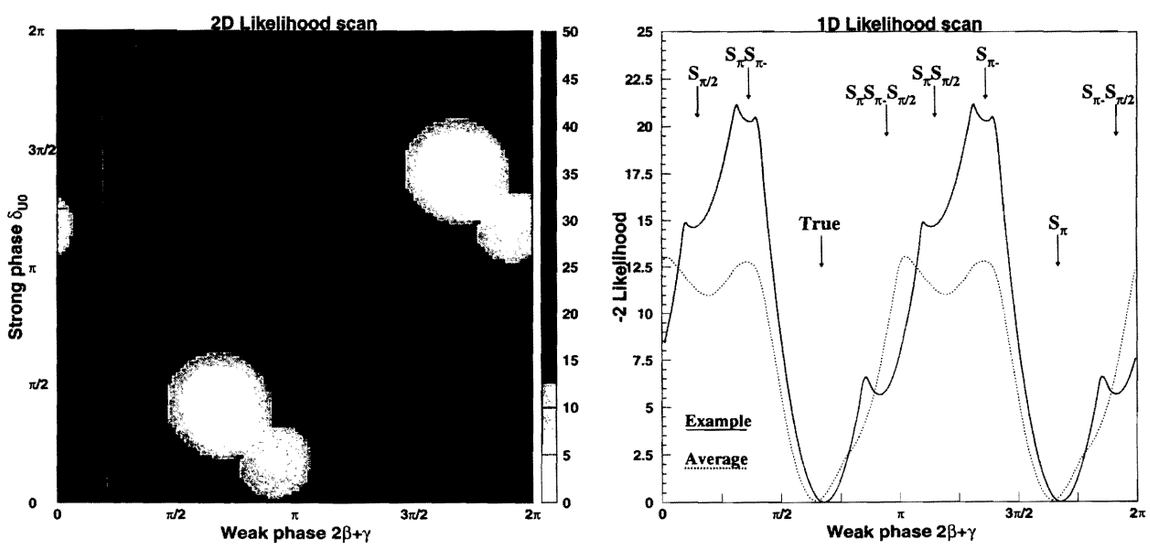

FIG. 5. **Left:** χ^2 as a function of $2\beta+\gamma$ and δ_{U0}, with the parameters of Table II. **Right:** Minimum projection of χ^2 onto $2\beta+\gamma$.

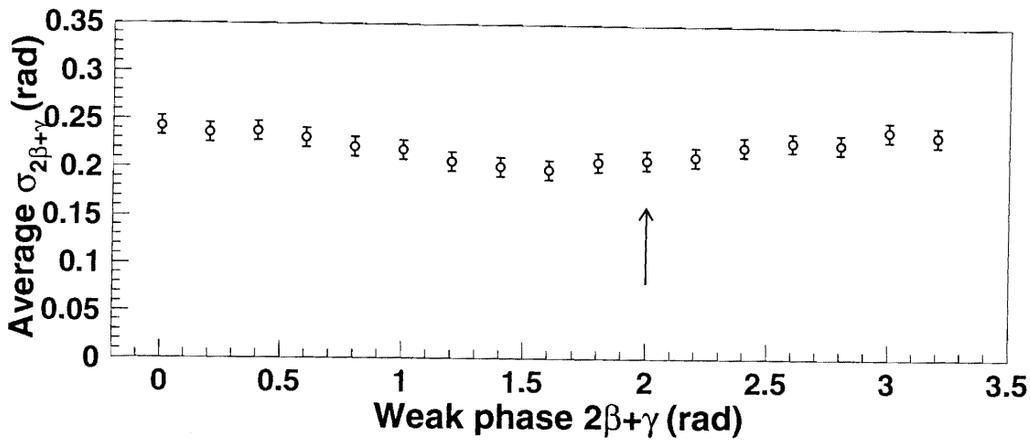

FIG. 6. The error in $2\beta + \gamma$, $\sigma_{2\beta+\gamma}$, as a function of $2\tilde{\beta} + \tilde{\gamma}$.

In Figs. 6 through 9 we present $\sigma_{2\beta+\gamma}$, the statistical error in the measurement of $2\beta + \gamma$, obtained by fitting simulated event samples using the MINUIT package [14], as a function of one of the parameters of Table II. All the other parameters were kept at the values listed in Table II. Each point in these plots is obtained by repeating the simulation 250 times, to minimize sample-to-sample statistical fluctuations. In all cases, all the parameters of Table II were determined by the fit. The arrow in these figures indicates the point corresponding to the parameters of Table II. The total number of signal events in all final states combined is the same for each of the data points.

One observes that $\sigma_{2\beta+\gamma}$ does not depend strongly on the CP-conserving phases. As expected, strong dependence on $\tilde{A}_{u0}/\tilde{A}_{c0}$ is seen in Fig. 9. However, it should be noted that $\sigma_{2\beta+\gamma}$ changes very little for all values of $\tilde{A}_{u0}/\tilde{A}_{c0}$ above about 0.4. This suggests that the likelihood for a significantly sensitive measurement is high over a broad range of parameters.

VI. CONCLUSIONS

We have shown how $2\beta + \gamma$ may be measured in the color-allowed decays $B \to D^{(*)}K^{(*)}\pi(\rho)$, focusing on the simplest mode $B^0(\overline{B}^0) \to D^{\mp}K_s\pi^{\pm}$. The absence of color

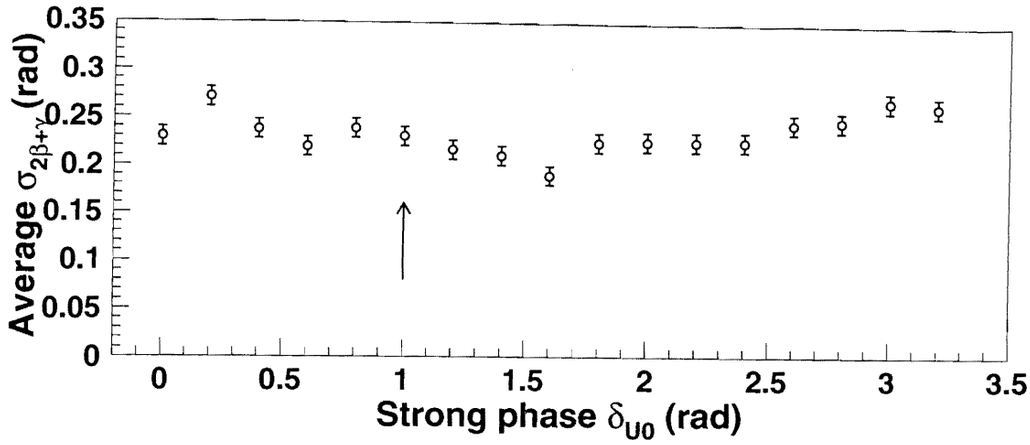

FIG. 7. The error in $2\beta + \gamma$, $\sigma_{2\beta+\gamma}$, as a function of $\tilde{\delta}_{U0}$.

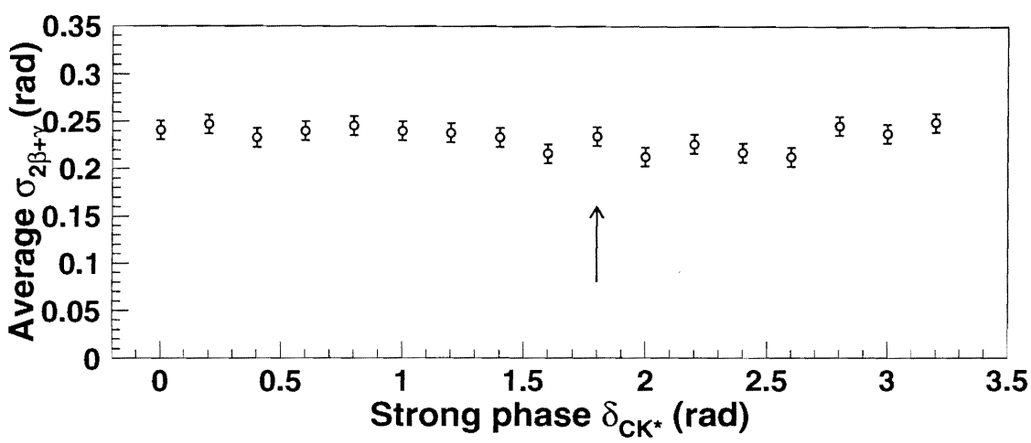

FIG. 8. The error in $2\beta + \gamma$, $\sigma_{2\beta+\gamma}$, as a function of $\tilde{\delta}_{K^*}$.

suppression in the $\bar{b} \to \bar{u}c\bar{s}$ amplitudes is expected to result in relatively large rates and significant CP violation effects, and hence favorable experimental sensitivities. While the Dalitz plot analysis required for this purpose constitutes some experimental complication, it should not pose a major difficulty, while being very effective at reducing the eight-fold

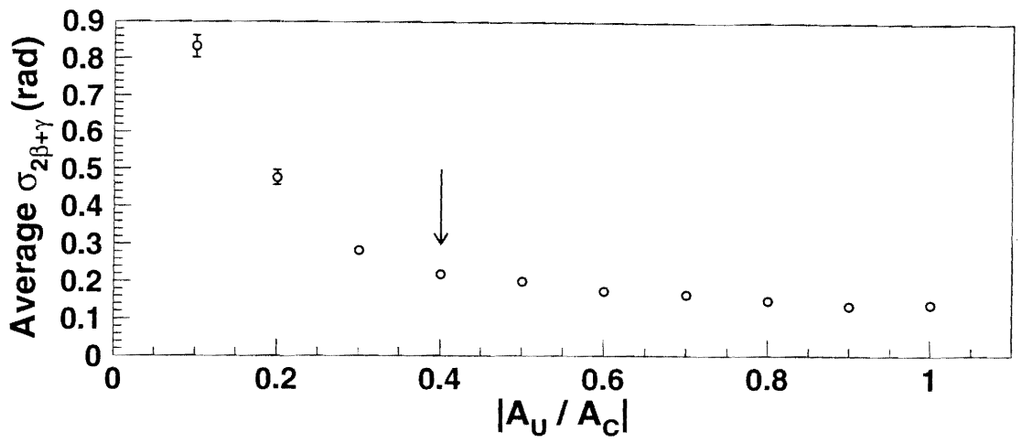

FIG. 9. The error in $2\beta + \gamma$, $\sigma_{2\beta+\gamma}$, as a function of $\tilde{A}_{\mathcal{U}0}/\tilde{A}_{\mathcal{C}0}$.

ambiguities which are a serious problem with other methods for measuring $2\beta+\gamma$. Only the S_π two-fold ambiguity cannot be resolved solely by our method, requiring additional constraints from other measurements of the unitarity triangle. As a result of these advantages, this method is likely to lead to small errors in the measurement of $2\beta + \gamma$ even with the current generation of B factory experiments.

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REFERENCES

[1] BaBar Collaboration (B. Aubert *et al.*), Phys. Rev. Lett. **86** (2001), 2515.

[2] Belle Collaboration (B. Abe *et al.*), Phys. Rev. Lett. **87** (2001), 091802.

[3] N. Cabibbo, Phys. Rev. Lett. **10**, (1963) 531 ; M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973), 652.

[4] R. Aleksan, T. Petersen and A. Soffer, Phys. Rev. D. **67**:096002 (2003).

[5] R. Aleksan, B. Kayser and D. London, Phys. Rev. Lett. **73**, (1994) 18.

[6] M. Gronau and D. London, Phys. Lett. **B253** (1991) 483.

[7] J. Charles, A. Le Yaouanc, L. Oliver, O. Péne and J.-C. Raynal, Preprint LPTHE-Orsay 97/70, hep-ph/9801363 (1997).

[8] L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.

[9] R. Aleksan, I. Dunietz, B. Kayser, F. Le Diberder, Nucl. Phys. B361 (1991) 141.

[10] R. Aleksan, I. Dunietz and B. Kayser, Z. Phys. C **54** (1992), 653.

[11] See, for example, J.C. Anjos *et. al.*, Phys. Rev. **D48**, 56 (1992).

[12] A. Soffer, Phys. Rev. **D60** (1999), 054032; A. Soffer, Proceedings, Snowmass 2001 Workshop on the Future of High Energy Physics, 2001.

[13] Particle Data Group, "Off-year partial opdate of PDG", (http://pdg.lbl.gov/), 2003.

[14] F. James and M. Roos, Comput. Phys. Commun. **10** 343-367, 1975.