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Surviving Cosmological Models After the Discovery of Large-Angle Anisotropies of the Cosmic Microwave Background

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ABSTRACT

Using new and first detections of the rms temperature fluctuations at 10° and the quadrupole anisotropy on the cosmic microwave background radiation by *COBE* group together with the upper limits on small scale anisotropies, constraints on various cosmological models are obtained. Complete analysis on the quadrupole anisotropy for flat and open universe models has been done. We calculate not only Sachs-Wolfe effect and the effect of time variation of the gravitational potential, but also the intrinsic fluctuations from the initial perturbations at the decoupling time which play an important role in many cosmological models, in particular, models with initially isocurvature perturbations. With the results of *COBE* and the upper limits on small angle observations of temperature fluctuations, we found that CDMs with $h = 1.0$ and HDMs with $h = 0.5$ are surviving if we consider only models which are consistent with the inflationary scenario. On the other hand, provided that the density parameter Ω is very low, there is no model which is consistent with all scale observations of temperature fluctuations and observations of galaxy correlation functions.

Subject headings:cosmology: cosmic microwave background - dark matter

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I. INTRODUCTION

At last *COBE* has detected the quadrupole amplitude of the Cosmic Microwave Background(CMB) fluctuations and the intrinsic Gaussian temperature fluctuation smoothed on 10°, $\sigma_{sky}(10°)$ (Smoot *et al.* 1992). The CMB anisotropies had been searched for a long time after the discovery of the CMB, but they could not be detected until today. Some scenarios are proposed for formations of galaxies and large-scale structures such as clusters of galaxies. In the scenarios based on the self-gravitational instability, the present structures in the universe originate from primordial fluctuations and the growth of matter density fluctuations eventually result in photon fluctuations, which should be detected today in the form of the CMB anisotropies. Also in other scenarios such as the explosion models and the formation models by cosmic strings and domain walls, the CMB anisotropies should be produced by Sachs-Wolfe effect(Sachs and Wolfe 1967) due to the growth of the matter density fluctuations. Hence we believed and expected that the CMB anisotropies should be detected. In this sense, the results of *COBE* are rather very natural than a great surprise to us. Using the results of *COBE*, however, the galaxy formation models can be constrained more severely than before: the models had been constrained by comparing the expected CMB anisotropies in each model with the observed upper limits of the anisotropies. Now we can use the values of the detected fluctuations instead of the upper limits when constraining on models.

Much work has been done for the estimations of the CMB anisotropies on various cosmological models. The CMB quadrupole moment, of course, has been calculated by many authors (Wilson and Silk 1981; Peebles 1982; Wilson 1983; Tomita and Tanabe 1983; Bond and Efstathiou 1984; Kofman and Starobinskii 1985; Gouda and Sasaki 1986; Efstathiou and Bond 1986; Vittorio *et al.* 1988; Holtzman 1989; Górski and Silk 1989). But all of these were unsatisfactory in the sense that their considerations were restricted either to the spatially flat background or to an incomplete evaluation of the quadrupole moment. In other

words, so far there has appeared no work which considers an open universe model and completely includes both the generalized Sachs-Wolfe effect or Rees-Sciama effect(*i.e.*, that which includes the effect of time variation of the gravitational potential (Wilson 1983)) and intrinsic photon fluctuations at decoupling in evaluating the quadrupole moment. This unsatisfactory situation is mainly due to a technical difficulty in estimating the present quadrupole and/or higher multipoles of the CMB anisotropy in an open universe. However, we have succeeded in deriving a formula by which one may calculate any multipole moment of the CMB anisotropy with practically arbitrary precision (Gouda, Sugiyama and Sasaki 1991a,b). Using this formula, we can estimate the rms CMB quadrupole moment in various cosmological models with both open and flat background(Gouda, Sugiyama and Sasaki 1991a,b).

In this *Letter*, we search the models in which the expected CMB anisotropies are consistent with the observed ones in the following procedure: we calculate the rms temperature fluctuations smoothed on 10°, $\sigma_{sky}(10°)$, in various representative cosmological models including ones with open background. In each model we normalize the amplitudes of the temperature fluctuations by coinciding the expected $\sigma_{sky}(10°)$ with the observed one by *COBE*. According to the above scheme, we estimate the rms quadrupole moment using the formula mentioned before and also the small angle scale anisotropies with the fluctuations normalized. Then we constrain models by comparing the expected rms quadrupole moment and small angle scale anisotropies in each model with the observed ones, respectively. This procedure has a merit as follows: the amplitudes of the fluctuations have been often normalized appropriately, based on a quantitative measure of the present cosmological structures which reflects the amplitude in linear region(Gouda, Sasaki and Suto 1989;Sugiyama 1989). However different normalization schemes in which we use the different quantitative measures give the different amplitude and so the ambiguity of the normalization exists. Moreover, there is a biasing problem that is whether galaxies trace the mass or not, when we normalize fluctuations by familiar methods using the

galaxy correlation function, J_3 and so on. Here we can obtain expected values of CMB fluctuations directly from the observational data $\sigma_{sky}(10°)$ of *COBE*. Using this new normalization scheme, we calculate the expected values of quadrupole and small scale fluctuations and constraint models excluding the ambiguity.

II. METHODS

We consider liner fluctuations around a Robertson-Walker space-time with curvature $K \leq 0$. To evaluate the present-day quadrupole anisotropy, the data at decoupling time have been prepared by solving numerically the evolution equations for density perturbations, for which we have adopted the gauge-invariant formalism (Bardeen 1980; Kodama and Sasaki 1984). The numerical method is the same as the one used in Gouda, Sasaki and Suto (1989) and Sugiyama (1989). The initial spectrum is assumed to be a power-law: $|\Delta(k)|^2 \propto \tilde{k}^n$, where $\tilde{k}^2 \equiv k^2 + K$ and Δ is the gauge-invariant *total* density perturbation. Note that our power-law index differs by 4 from that used in *e.g.*, Peebles (1987) for isocurvature perturbations (*i.e.*, $n = n_{\rm Peebles} + 4$). The merit of this definition is that both adiabatic and isocurvature fluctuations are scale free, *i.e.*, Harrison-Zeldovich spectrum, when it takes $n = 1$. The evolution is numerically solved until the universe becomes sufficiently matter dominated and optically thin.

The observable temperature anisotropy $\Theta_m(\eta_0, \mathbf{x}(\eta_0), \boldsymbol{\gamma})$, defined with respect to the matter restframe(Kodama and Sasaki 1986, 1987), is given analytically by solving the null geodesic equation of photon after the decoupling era, where η_0 is the conformal time at the present-day and $\boldsymbol{\gamma}$ is the direction cosine of photon propagation (Gouda, Sugiyama and Sasaki 1991a). Θ_m consists of three terms, *i.e.*,Θ_{int}, Θ_{sac} and Θ_{dif}. Here Θ_{int} describes intrinsic photon fluctuations at decoupling, Θ_{sac} the Sachs-Wolfe effect, and Θ_{dif} the effect of the time variation of the gravitational potential(Wilson 1983). We call the term $\Theta_{GS} \equiv \Theta_{sac} + \Theta_{dif}$ the generalized Sachs-Wolfe effect. After some algebra, one can get a formula for the present quadrupole component of Θ_m (Gouda, Sugiyama and Sasaki 1991a),

which has not derived by someone else for an open universe model. With that formula the rms quadrupole anisotropy is evaluated.

The rms temperature fluctuation smoothed to a total 10° FWHM, $\sigma_{sky}(10°)$, (Smoot *et al.* 1992) is given by $\sigma_{sky}(\theta_{FWHM} = 10°) = [C(\theta = 0, \theta_s)]^{1/2}$, where $C(\theta, \theta_s)$ is the intrinsic temperature angular correlation convoluted with a Gaussian smoothing angle θ_s and is given by the ordinary method(see *e.g.*, Gouda, Sasaki and Suto 1989). In the observation of *COBE*, the sky maps of temperature are made by smoothing with a total 10° FWHM Gaussian and then θ_s is 4.°25.

At first, in each cosmological model we normalize the fluctuation according to the scheme that the expected rms temperature anisotropy ($\tilde{\sigma}(10°) \equiv \sigma_{sky}(10°)/T_0$) is equal to the observed one($\sigma_{sky}(10°)/T_0 = 1.1 \times 10^{-5}$). And then we estimate the rms quadrupole moment of the anisotropy $\tilde{Q}_{rms}(\equiv Q/T_0)$ according to this normalization in each model. Finally, we compare the expected quadrupole moment $\tilde{Q}_{rms}$in each model with the observed one ($Q/T_0 = 5 \times 10^{-6}$) and search the models whose quadrupole moment is consistent with the observed one. In this comparison, we must note that the ***theoretically*** expected rms quadrupole moment is not directly comparable to the observed one and the quadrupole moment in our universe will be distributed like χ^2 with 5 degrees of freedom under the assumption of random phase Gaussian distribution(Abbott and Wise 1984). From this, it follows that there is a probability of 90% of measuring a quadrupole $\tilde{Q}$,

$$0.48 < \frac{\tilde{Q}}{\tilde{Q}_{rms}} < 1.5.$$

Furthermore, we estimate the temperature anisotropy $\delta T/T$ at 7.′15 and at 1° in each model using the temperature fluctuations normalized according to the above scheme. Comparing the results with the observed upper limits of $\delta T/T$ at 7.′15 by Readhead *et al.* (1989) and at 1° by Meinhold and Lubin (1991), we constrain models more severely than from only the large-angular anisotropy alone.

III. RESULTS

The models that we have examined in this *Letter* are baryon-dominated universe models(BDMs); cold dark matter models(CDMs) with initially adiabatic and isocurvature perturbations; hot dark matter models(HDMs) with initially adiabatic perturbation; and Peebles' reionized universe models(Peebles 1987). As for dark matter dominated models, we set the baryon density parameter $\Omega_b = 0.03$. We have taken the non-dimensional Hubble parameter h normalized by 100km/s/Mpc, 0.5 and 1.0. Models with initial power spectrum $n = 0, 1$ and 2 have been calculated except Peebles' models. As for Peebles' models, higher initial power are needed to reionize the interstellar matter and we have chosen $n = 1, 2$ and 3. Here, we have neglected the secondary effect on CMB fluctuations induced by ionized matter, so-called Vishniac effect (Ostriker and Vishniac 1986; Vishniac 1987) for Peebles' models. This effect plays an important role on small scale fluctuations, but do not produce large scale anisotropies. In our calculations, neglecting this effect may make the expected fluctuations on 7.′15 scale underestimated. As a result, the constrains on Peebles' models by the small scale observation may be slightly more strict than those obtained in this *Letter*.

In Fig.1, the expected quadrupole anisotropies for various models with $h = 1.0$ are shown. Constraints are obtained at 90% confidence level as previously mentioned. Including the constraints by the small scale anisotropies, results for all models are shown in Table 1.

IV. CONCLUSIONS AND DISCUSSIONS

In a variety of open and flat universe models, we have calculated the rms quadrupole moment normalized according to the scheme that the ***theoretically*** expected $\tilde{\sigma}(10°)$ in each model is equal to the observed one by *COBE*. Comparing the results with the observed quadrupole moment by *COBE* and the upper limit of

$\delta T/T$ at 7.'15 by Readhead *et al.*(1989) and at 1° by Meinhold and Lubin (1991), we have found the surviving universe models as shown in Table 1. It shows that models with $n > 1$ are constrained mainly from the result by Readhead *et al.* (1989) and models with $n \sim 1$ and $n < 1$ are restricted mainly from the results by Meinhold and Lubin (1991)and by *COBE*, respectively. If we consider models which are consistent with the inflationary scenario such that $\Omega = 1$ and $n = 1$, BDMs with initially isocurvature perturbations and $h = 1.0$, HDMs with $h = 0.5$ are survived and CDMs with adiabatic perturbations and $h = 1.0$ is marginally alive. Most of the adiabatic CDMs may be ruled out if we would like to seek the models which are consistent with the observations on both large and small angular scales. On the other hand, if we believe the dynamical estimates on the density parameter, *i.e.*, $0.1 < \Omega < 0.3$ (see *e.g.*, Peebles 1987), only Peebles' reionized universe models with $n = 1$ or isocurvature CDMs with $n < 2$ can explain such low value of Ω. Isocurvature BDMs with $h = 0.5$ and $n = 1$ may also have a chance to realize the low Ω universe. It should be noted that we neglect the observational error when constraints on cosmological models are set. Including the observational error, however, constraints obtained here by the quadrupole anisotropy are still 80% confidence level.

In deriving the results stated above, we do not consider whether or not the matter fluctuations in the allowed models in Table 1 are consistent with the observed quantities of matter structures such as galaxy two-point correlation functions and so on. Here, as an example, we try to constrain the allowed models in Table 1 by examining whether the expected $J_3(25h^{-1}\mathrm{Mpc})$ in each model is consistent with the observed value, $780\mathrm{Mpc}^3h^{-3}$. We estimate the biasing factor b in each model, which is defined by $b \equiv \sqrt{C(0,10°,J_3)/C(0,10°,COBE)}$, where $C(0,10°,J_3)$ is the intrinsic temperature angular correlation function estimated according to the normalization scheme by $J_3(25h^{-1}\mathrm{Mpc})$ and $C(0,10°,COBE)$ is the observed one by *COBE*. We find from the definition of b that the allowed models in Table 1 are still survived if $b \sim 1$, which means that the expected J_3 in the models is nearly equal to the observed one. The result is as follows: all allowed

models in Table 1 with $n = 0$ have $b \gtrsim 5$ and then are not desirable. Adiabatic BDMs with $n = 1$ might be marginally allowed since $b \sim 4$. However we can not expect such large baryon densities from the primordial nucleosynthesis. As for isocurvature BDMs with $n = 1$, $b \sim 5 - 10$ and then it may be difficult to be still alive. Although the reionized universe scenario is considered, it is difficult to save isocurvature baryon dominated universe since the Peebles' models with $n = 1$ must be large biasing factor $b \gtrsim 10$. Moreover it is difficult for models with scale invariant initial spectrum $n = 1$ to produce ionized matter since ionization may be produced by higher power spectrum. The biasing parameter of adiabatic CDMs and HDMs models with $n = 1$ is nearly equal or less than 1 and these models with $\Omega \simeq 1$ are desirable. As for isocurvature CDMs, small Ω demands large b and it is difficult to realize low density universe. Even if in these isocurvature CDMs, we choose $\Omega = 1$ for $h = 1$ and $h = 0.5$, $b = 4$ and $b = 6.5$, respectively. So it may be difficult to realize the isocurvature CDMs.

Basically, the normalization scheme based on other observed quantities, e.g., J_3 at other scales, correlation function itself and peculiar velocities yields the similar amplitudes of the fluctuations within the factor 2 or so over a wide range of parameters(Gouda, Sasaki and Suto 1989; Sugiyama 1989). Then we will get the similar results shown above when estimating the constraint on the model from other quantities of matter structures and/or peculiar velocities.

As a result, only dark matter dominated models with $n \simeq 1$ and $\Omega \simeq 1$ are desirable. It is remarkable that these models are just consistent with inflationary scenario. The angular resolution of COBE's data is 7° which is larger than the horizon scale at the recombination. Then the fluctuations observed by COBE are completely uncausal at the recombination epoch. Since only the inflation can make uncausal regions which is once causaly connected (Sato 1981), it may be the proof of the inflationary scenario that such fluctuations are existed.

COBE has also showed the profile of the angular correlation function $C(\alpha)$. Smoot *et al.*(1992) show the best fitted model for $C(\alpha)$ if the power spectrum

of the fluctuation obeys the power law. However the theoretical formula of the rms multipole moment used in their paper when deriving this result is valid for the case that the familiar Sachs-Wolfe effect dominantly contribute to the total large-angle fluctuations of photon, e.g. for the models with both flat background and initially adiabatic perturbations. So one has to analyze the $C(\alpha)$ also in other various models in which the total large-angle anisotropies are dominantly contributed not only from the Sachs-Wolfe effect but also from the effect of time variation of the gravitational potential and/or intrinsic photon fluctuations.

Wright *et al.*(1992) also analyze some cosmological models using the results of *COBE*. However they do not estimate the quadrupole moment for open universe models and/or isocurvature models and thire works are limited in the case of scale invariant initial spectrum. We gave the complete analysis on the open and/or isocurvature models and also surveyed the models with $n \neq 1$. Their estimation of quadrupole moment for flat and adiabatic fluctuations gives the similar results of ours if their normalization scheme is changed to ours. However the conclusion for the restriction of the models give the slightly different ones from ours because the normalization scheme of ours by $\sigma_{sky}(10°)$ is different of theirs by $\delta M/M$ at $8h^{-1}$Mpc and moreover they give the more severe constraint on the models by restricting the allowed range of the biasing factors within the narrow limits. For example, they conclude that adiabatic CDMs with $n = 1$, $\Omega = 1$ and $h = 0.5$ is desirable. It is shown in Table 1 that this model can not be survived by the constraint on 1°. The biasing factor b of this model is 0.6. Then if we use the normalization scheme by galaxy correlations, the small scale anisotropies become smaller and this model is not constrained from observations on 1°. And the produced quadrupole anisotropy is also fitted with observations. In this case, however, the expected value of $\sigma_{sky}(10°)$ becomes slightly smaller than observations.

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TABLE 1
Constraints on Cosmological Models

model			7.15min.	1deg.	Quadrupole	All
BDM	h=1.0	n=0	No Constraint	No Constraint	$0.75<\Omega<0.82$	$0.75<\Omega<0.82$
adiabatic		n=1	$0.16<\Omega<0.86$	$0.10<\Omega<0.74$	$0.62<\Omega<0.90$	$0.62<\Omega<0.74$
		n=2	No Region	No Region	$0.40<\Omega$	No Region
	h=0.5	n=0	$0.17<\Omega$	$0.22<\Omega$	No Region	No Region
		n=1	No Region	No Region	$0.65<\Omega<0.81$	No Region
		n=2	No Region	No Region	$0.40<\Omega$	No Region
BDM	h=1.0	n=0	No Constraint	No Constraint	$0.75<\Omega<0.92$	$0.75<\Omega<0.92$
isocurvature		n=1	No Constraint	No Constraint	$0.55<\Omega$	$0.55<\Omega$
		n=2	No Region	No Region	$0.30<\Omega$	No Region
	h=0.5	n=0	No Constraint	$0.13<\Omega$	No Region	No Region
		n=1	$0.10<\Omega$	$0.27<\Omega$	$0.26<\Omega<0.89$	$0.27<\Omega<0.89$
		n=2	No Region	No Region	No Constraint	No Region
Peebles	h=1.0	n=1	No Constraint	No Constraint	$\Omega<0.84$	$\Omega<0.84$
		n=2	No Constraint	No Region	No Constraint	No Region
		n=3	No Constraint	No Region	$\Omega<0.18$	No Region
	h=0.5	n=1	No Constraint	No Constraint	$\Omega<0.74$	$\Omega<0.74$
		n=2	No Constraint	No Region	No Constraint	No Region
		n=3	No Constraint	No Region	$\Omega<0.49$	No Region
CDM	h=1.0	n=0	$0.13<\Omega$	$0.13<\Omega$	No Region	No Region
adiabatic		n=1	$0.63<\Omega$	$\Omega\sim1.0$	$0.62<\Omega$	$\Omega\sim1.0$
		n=2	No Region	No Region	$0.53<\Omega$	No Region
	h=0.5	n=0	No Constraint	No Constraint	No Region	No Region
		n=1	$0.80<\Omega$	No Region	$0.64<\Omega$	No Region
		n=2	No Region	No Region	$0.53<\Omega$	No Region
CDM	h=1.0	n=0	No Constraint	No Constraint	$\Omega<0.54$	$\Omega<0.54$
isocurvature		n=1	No Constraint	No Constraint	No Constraint	No Constraint
		n=2	$0.55<\Omega$	No Region	No Constraint	No Region
	h=0.5	n=0	No Constraint	No Constraint	$\Omega<0.55$	$\Omega<0.55$
		n=1	No Constraint	$0.10<\Omega$	No Constraint	$0.10<\Omega$
		n=2	$0.40<\Omega$	No Region	No Constraint	No Region
HDM	h=1.0	n=0	No Constraint	No Constraint	$0.40<\Omega<0.60$	$0.40<\Omega<0.60$
adiabatic		n=1	$0.65<\Omega$	No Region	No Constraint	No Region
		n=2	No Region	No Region	$\Omega<0.59$	No Region
	h=0.5	n=0	No Constraint	No Constraint	No Region	No Region
		n=1	$0.78<\Omega$	$0.72<\Omega$	No Constraint	$0.78<\Omega$
		n=2	No Region	No Region	$\Omega<0.68$	No Region

note: If there remains no allowed region on the density parameter, we mark as No Region.
If the model is not constrained, No Constraint is marked.

Figure Caption
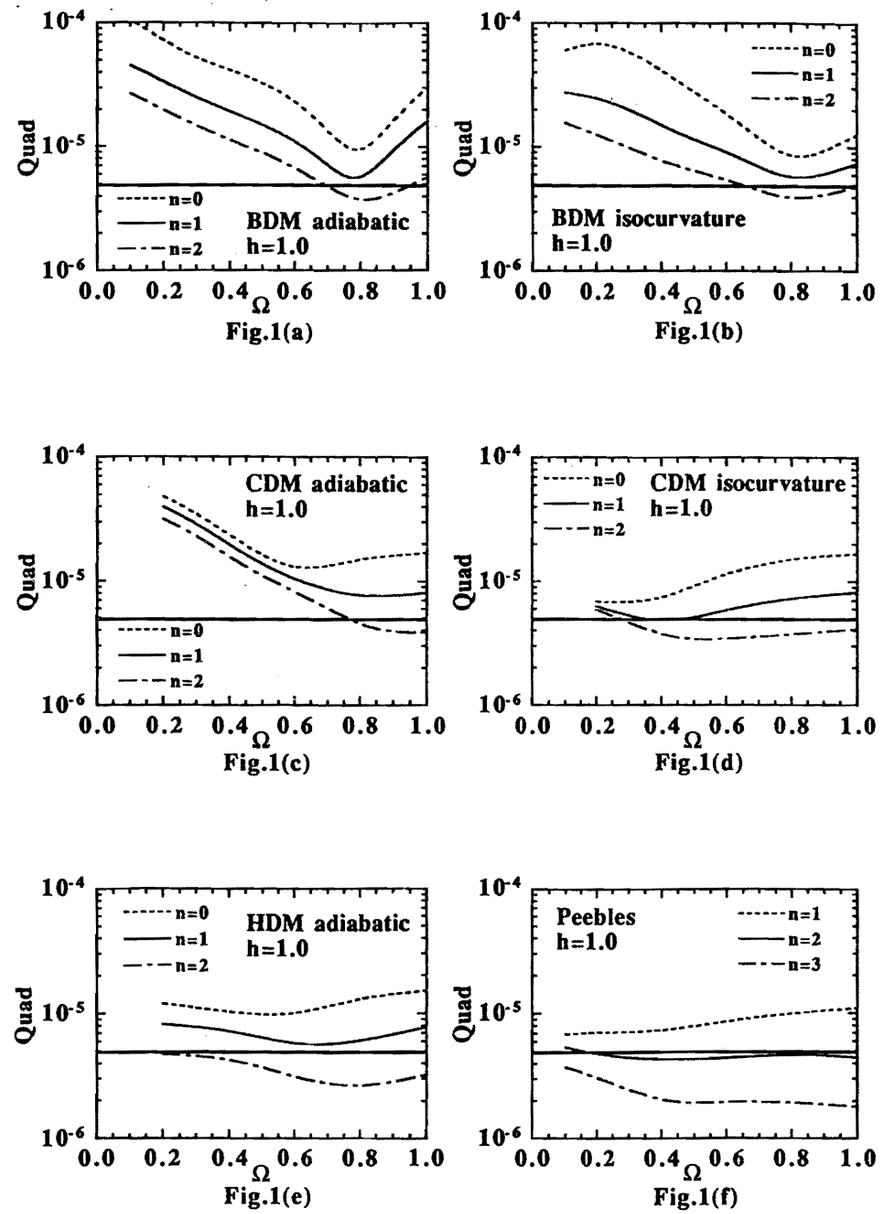
Fig.1. The rms quadrupole moment in a) adiabatic BDMs, b) isocurvature BDMs, c) adiabatic CDMs, d) isocurvature CDMs, e) HDMs and f) Peebles' reionized models with some initial power spectrum n. The observed value is also shown.