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Atom interferometry with arbitrary laser configurations: Exact phase shift for potentials including inertia and gravitation [1]

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Abstract

We study in a general treatment the influence of a class of perturbations acting on the atoms of an atomic interferometer. An exact expression for the resulting shift of the interference pattern is given for an arbitrary number of laser zones with travelling or standing waves. The assumption made is that the two-level model is appropriate for the description of atoms and that the perturbations can be neglected in the laser zones. As an important application we calculate the influence of acceleration, rotation, and space-time curvature on the Ramsey interferometer and the atomic fountain. The measurability of the respective phase shifts is discussed.

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1 Introduction

The idea of using lasers as beam splitters for atomic interferometers was developed by Bordé [1] in 1989. Due to momentum conservation atoms can gain or loose a momentum of $\vec{k}$ if they absorb or emit a photon with wavevector $\vec{k}$. This effect can be used to split and recombine an atomic beam. The interaction with a laser beam which is tuned to be nearly in resonance with a certain atomic transition can be described within the two-level model of atoms. This was used to calculate the complete fringe pattern in the Ramsey spectrometer which consists of two pairs of counterpropagating laser waves [2]. The atomic fountain device [3] may be treated as an effective two level system.

There are extended efforts to study the influence of electromagnetic fields of laser beams on moving atoms with the intention to design mirrors and beam splitters in an atomic interferometer. In the following we study the complementary problem: Using a black box approach for the laser zones we treat exactly the influence of a class of perturbations acting on the atoms when these are moving through the dark zones between the lasers in the interferometer. Our intention thereby is to treat the induced shift of a fringe pattern for a set up with an arbitrary number of laser zones with standing or running waves. We restrict to two level atoms and strive for a result in terms of a recursion formula which allows us to work out the influences of the different parts of the perturbation to any desired order of magnitude. This is of particular importance if these terms differ largely in magnitude. In this case one must be able to control all the mixed terms which may occur in order to obtain the phase shift for a particular experimental specification up to a given accuracy. In this sense we generalize our calculations in Ref. [4].

There are at the moment two potential applications of our general scheme. The perturbations may be unwanted, for example when the atomic interferometer is used as a frequency standard. In this case one has to know the structure of their influence in order to eliminate it. On the other hand, the perturbation may go back to the influence of acceleration, rotation, and space-time curvature. In this connection the intention is to examine whether such an inertial or gravitational influence on quantum matter can be detected in the laboratory using atomic interferometry. Interferometry with neutrons [5, 6], electrons [7], and atoms [8, 3] is at present the only possibility to examine this effect. Its relevance for gravitational theory has been discussed in [9]. For space-time curvature we have studied the measurability by a Ramsey interferometer in Ref. [10]. For the influence of acceleration on the phase shift there is a difference between neutron and atomic interferometry. For neutrons the beam splitters are realized with a single crystal, and a series expansion of the WKB phase is needed [11, 12, 13]. We will show below that for atom interferometers where lasers are used as beam splitters the phase shift due to acceleration alone has only one part: the first order contribution.

The paper is organized as follows. In Sec. 2 we will explain the general scheme for the algebraic calculation of the shift. The idealization of the laser beams is addressed in Sec. 3. The elaboration of the complete phase shift is contained in Sec. 4 and will be applied to the Ramsey interferometer and the atomic fountain in Sec. 5. On this

basis the measurability of space-time curvature is studied in Sec. 6. A quick direct calculation of the phase shift if only acceleration is present is given in Sec. 7. We will use natural units ($\hbar = c = 1$) unless otherwise stated.

2 General scheme

In the two-level model the Hamiltonian of the atoms in the atomic interferometer is given by

$$H_{tot} = H_{cm} + H_{atom} - \vec{d} \cdot \vec{E}(\vec{x}, t) \ , \tag{1}$$

with

$$H_{cm} = \frac{\vec{p}^2}{2M} + H_p \tag{2}$$

$$H_{atom} = H_A - \frac{i}{2}\Gamma \tag{3}$$

where $\vec{x}$, $\vec{p}$, and M are the center of mass position and momentum, and the mass of the atom, respectively. $H_A = E_a|a\rangle\langle a| + E_b|b\rangle\langle b|$ is the internal energy operator restricted to the two atomic energy eigenstates $|a\rangle$ and $|b\rangle$ of interest. We will assume $E_a < E_b$. $\Gamma = \gamma_a|a\rangle\langle a| + \gamma_b|b\rangle\langle b|$ contains the decay factors γ_a, γ_b of the two states. The coupling of the atoms to the laser field in the laser zones is given in the dipole approximation by $-\vec{d} \cdot \vec{E}(\vec{x})$, where $\vec{E}$ is the electric field of the laser and $\vec{d}$ is the dipole moment of the atom. In a two level model one can set $\langle a|\vec{d}|a\rangle = \langle b|\vec{d}|b\rangle = 0$, and $\langle a|\vec{d}|b\rangle = \langle b|\vec{d}|a\rangle = \vec{d}_{ab}$.

H_p in Eq. (1) represents the influence of external potentials on the propagating matter states. Depending upon circumstances these potentials may either be regarded as external disturbances, the influence of which has to be known in order to be able to eliminate them in an experimental setup, or as the external potentials to be measured by matter wave interferometry. We assume H_p to be of the form

$$H_p = M\vec{a} \cdot \vec{x} + \frac{M}{2} R_{0l0m} x_l x_m - \vec{\omega} \cdot (\vec{x} \times \vec{p}) \ . \tag{4}$$

Here and in the remainder we use the convention that any index of R_{0l0m} which appears a second time in the same formula has to be summed from 1 to 3. The first two terms may be regarded as the first terms of a Taylor expansion of some scalar potential. The third term represents an angular momentum coupling. In the applications discussed below H_p will represent the influence of gravitation and of the rotation of the reference system. In this case $\vec{a}$ = const. is the acceleration, $\vec{\omega}$ = const. the angular velocity, and R_{0l0m} = const. are the components of the Riemann tensor with respect to the Fermi coordinates of the observer [10]. It is the aim of this paper to give a general treatment of the resulting shift of the interference pattern if H_p is switched on.

The general arrangement for an atom interferometer using lasers as *beam splitters* consists of an atom beam passing a sequence of N lasers with standing or travelling

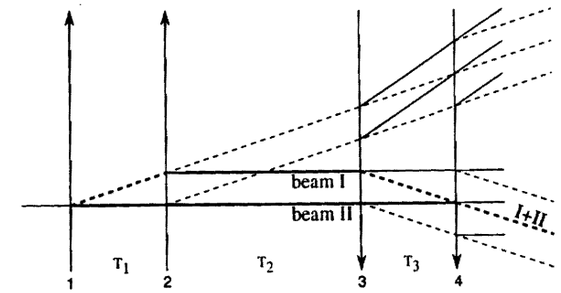

Figure 1: All partial beams of an atomic beam passing the Ramsey device as they are predicted in the rotating wave approximation. The vertical arrows denote the running laser waves and their direction. Dashed lines correspond to excited atoms. The atoms are initially deexcited and travel freely for times T_i between the lasers. Beam I and II are able to interfere.

waves and moving free of laser influences in the dark zones between the lasers for time intervals T_i. The free evolution is governed by the Hamiltonian $\hat{H} = H_{cm} + H_{atom}$ of Eq. (1). Because $\hat{H}$ is time independent the corresponding time evolution operator is simply $\exp[-i\hat{H}T_i]$ in the ith interval of free evolution. Denoting the evolution operator which describes the result of the interaction with the ith laser by $U_L(i)$ the complete evolution of the state of the atoms is given by

$$|\psi(t_f)\rangle = U_L(N)\exp[-i\hat{H}T_{N-1}]U_L(N-1)\cdots U_L(2)\exp[-i\hat{H}T_1]U_L(1)\exp[-i\hat{H}T_0]|\psi_0\rangle \tag{5}$$

where $|\psi_0\rangle$ is the initial state of the atoms and t_f is the final time.

During its evolution the ingoing atomic beam is split into many *partial beams* with different momenta and different internal states which can interfere under certain conditions. As an example Fig. 1 shows the partial beams for the Ramsey interferometer. Note that the word partial beam is introduced for simplicity reasons to denote in a suggestive way parts of the splitted wave function with certain momenta. A partial beam characterizes a particular history of excitations and deexcitations. Eq. (5) shows clearly that there is no concept of a path in configuration space in the quantum evolution. In the following we single out two partial beams I and II resulting at the final time t_f in $|\psi_I\rangle$ and $|\psi_{II}\rangle$ and study the corresponding interference pattern.

A formal description of the partial beams can be made if we decompose the state vector into its deexcited part and its excited part. The Hilbert space of our problem is the direct product of the two dimensional internal Hilbert space and the one of the center of mass motion. We can therefore write

$$|\psi\rangle = \begin{pmatrix} |\psi_b\rangle \\ |\psi_a\rangle \end{pmatrix} \tag{6}$$

with $|\psi_a\rangle = |a\rangle\langle a|\psi\rangle$ and similar for $|b\rangle$. Independent of the details of the atom's interaction with the lasers in the laser zones the transition from the ingoing matter state to the outgoing one can be described phenomenologically by

$$U_L(i) = \begin{pmatrix} \alpha_i & \beta_i \\ \gamma_i & \delta_i \end{pmatrix} \tag{7}$$

where $i = 1, \cdots, N$ denotes the number of the laser zone. The components of this matrix are in general operators containing $\vec{x}$ and $\vec{p}$. A more specific form will be given for idealized laser zones in the next section. The physical meaning of the components is clear: α_i occurs if an excited atom remains excited, δ_i if it continues to be in the state $|a\rangle$. If β_i arises we know that the ith laser has the atom excited, and γ_i describes the process of deexcitation. For a particular partial beam passing a splitter i only one of the components of $U_L(i)$ will become effective. It is therefore obvious that each total partial beam is specified if we write down the corresponding sequence of components. Consider for instance the two interfering beams I and II of Fig. 1. Beam I corresponds to the sequence $\beta_1\gamma_2\beta_3\alpha_4$ and beam II is described by $\delta_1\delta_2\delta_3\beta_4$.

To concentrate in the calculation on the relevant terms the general scheme in this paper will be the following: for some configuration of lasers and two given particular partial beams I and II resulting in $|\psi_I\rangle$ and $|\psi_{II}\rangle$ there is already an interference pattern if the external potentials vanish ($H_p = 0$). This pattern is assumed to be known. We do not address the question which pairs of partial beams lead to a detectable pattern. They may for instance be specified as those with the smallest washing out by Doppler broadening. If we now switch on the disturbance H_p these fringe patterns are shifted. The corresponding terms in the calculation depend on $\vec{a}$, $\vec{\omega}$, or $R_{0/0m}$. We are only interested in this phase shift. In $|\psi_I\rangle$ and $|\psi_{II}\rangle$ the contributions which are already there in the undisturbed case can be isolated as complex factors. If not otherwise stated our *convention* when working out $|\psi_I\rangle$ and $|\psi_{II}\rangle$ will therefore be to omit contributions to these factors whenever possible. Thereby we must keep certain contributions in order not to loose terms of H_p. Finally the resulting phase shift caused by switching on H_p is obtained in restricting in $\langle\psi_I|\psi_{II}\rangle$ to those parts which contain the constituents of H_p.

This convention has consequences. Because H_{atom} commutes with H_{cm} it contributes only to the omitted complex factor. Therefore $|\psi_I\rangle$ and $|\psi_{II}\rangle$ go solely back to the action of H_{cm} in the dark zones and to the respective components of $U_L(i)$ of Eq. (7).

For example, for the two beams in the Ramsey device of Fig. 1 we find

$$\begin{aligned} |\psi_I\rangle &= \alpha_4 e^{-iH_{cm}T_3}\beta_3 e^{-iH_{cm}T_2}\gamma_2 e^{-iH_{cm}T_1}\beta_1 e^{-iH_{cm}T_0}|\psi_0\rangle \\ |\psi_{II}\rangle &= \beta_4 e^{-iH_{cm}T_3}\delta_3 e^{-iH_{cm}T_2}\delta_2 e^{-iH_{cm}T_1}\delta_1 e^{-iH_{cm}T_0}|\psi_0\rangle \,. \end{aligned} \tag{8}$$

In this case both partial beams are excited after the passage of the fourth laser. In many devices such as the Ramsey interferometer the number of excited atoms after the evolution is measured. The observed quantity is therefore $\langle\psi_b|\psi_b\rangle$ and it contains beside other contributions an interference term of the form 2 Re $\langle\psi_I|\psi_{II}\rangle$. In the

Ramsey device almost all the other interference terms, i.e. those between other partial beams, are washed out due to the Doppler effect [2]. This entails that the signal consists of some background and some sinusodial variation coming from the interference term. The phase of this variation can be calculated by evaluating $\langle\psi_I|\psi_{II}\rangle$. The phase shift originating from H_p of Eq. (1) can be read off directly. This is the general scheme of the paper.

Our aim is to give a general expression for $\langle\psi_I|\psi_{II}\rangle$ for any two given partial beams for an arbitrary laser configuration provided the lasers can be idealized as follows.

3 Idealization of the laser beams

We need the knowledge of the evolution operators $U_L(i)$ which refer to laser zones with travelling or standing laser waves serving as beam splitters. If the complete Hamiltonian H_{tot} of Eq. (1) including H_p is taken into account the related microstructure of the time evolution will lead to $U_L(i)$ which are in the exact version complicated functions of the operators of position and momentum, of the ingoing matter state, and of the duration τ of the laser influence. Because we follow two given partial beams I and II we have in fact $U_L^I(i)$ and $U_L^{II}(i)$. Our main idealization is then that the influence of the perturbing Hamiltonian H_p over the interaction time τ in the laser zone is so small that it may be neglected when elaborating the final interference pattern. This is to be expected if τ is very short compared with the times of flight T_i between the lasers. We assume it therefore to be justified to describe the influence of the travelling or standing laser waves in $|\psi_I\rangle$ as a momentum transfer at one instant of time of the form

$$\alpha_i^I = \exp[in_i^I\vec{k}_i\cdot\vec{x}] \tag{9}$$

where $\vec{k}_i$ is the wave vector of the respective laser wave and $n_i^I = 0, \pm1, \pm2, \ldots$. The position operator is the only q-number which appears here. We have corresponding expressions for β_i^I, γ_i^I, δ_i^I, and for the beam II. According to our convention any amplitude is omitted because it does not contribute to the phase shift which goes back to H_p. The same is the case for the laser phase omitted in Eq. (9). The restriction of the laser influence to the momentum transfer which will be justified theoretically below is reasonable. It is this feature which is used for the construction of optical beam splitters as proposed, for instance, by Pfau, Adams, and Mlynek [14] in the context of a three-level atom in a constant magnetic field.

The time evolution of a partial beam is now given by

$$|\psi_I\rangle = e^{in_N^I\vec{k}_N\cdot\vec{x}}e^{-iH_{cm}T_{N-1}}\cdots e^{-iH_{cm}T_1}e^{in_1^I\vec{k}_1\cdot\vec{x}}e^{-iH_{cm}T_0}|\psi_0\rangle \tag{10}$$

and similarly for $|\psi_{II}\rangle$ with H_{cm} of Eq. (2). For $|\psi_I\rangle$ in Fig. 1 we have, for example, $n_1^I = 1$, $n_2^I = -1$, $n_3^I = 1$, $n_4^I = 0$, and $\vec{k}_1 = \vec{k}_2 = -\vec{k}_3 = -\vec{k}_4$.

The approximation made above which leads to the idealization of the laser zones by beam splitters acting instantaneously can formally be obtained by taking the limit to very small τ and very large Rabi's frequency $\Omega_{ab} = \vec{E}_0\cdot\vec{d}_{ab}$ in such a way that $\chi := \tau\Omega_{ab}$ remains fixed characterizing the respective experimental set up. To justify

the structure (9) assumed above we give an exact calculation of the output of a laser zone for the extreme limit $\tau \to 0$ (χ = const.). The electric field $\vec{E}$ may thereby be given by $\vec{E}_0 \cos[\omega t - \vec{k}\cdot\vec{x} + \varphi]$ (running wave) or $\vec{E}_0 \cos[\omega t]\cos[\varphi - \vec{k}\cdot\vec{x}]$ (standing wave). φ is some constant phase. For $\tau < 1/|\vec{k}|$ there is one common result for running and standing waves. The Schrödinger equation with the full Hamiltonian given by Eq. (1) has the formal solution

$$|\psi(\tau)\rangle = \sum_{n=0}^{\infty}(-i)^n \int_0^{\tau} dt_1 \cdots \int_0^{t_{n-1}} dt_n\, H(t_1)\cdots H(t_n)|\psi(0)\rangle \tag{11}$$

whereby the simplifying convention of Sec. 2 has not been used. We now reparametrize the variables of integration by $t_i = \lambda_i \tau$ and perform the described limit. Exploiting

$$\lim_{\tau\to 0} \tau H(t_i = \tau\lambda_i) = -\chi \cos[\varphi - \vec{k}\cdot\vec{x}] \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \tag{12}$$

which is valid for both running and standing waves it is easy to perform the sum in Eq. (11). Remembering that the square of the Pauli matrix σ_1 occurring in Eq. (12) is the identity matrix we arrive at

$$|\psi(0_+)\rangle = \left\{ \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \cos[\chi\cos(\varphi - \vec{k}\cdot\vec{x})] + i \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \sin[\chi\cos(\varphi - \vec{k}\cdot\vec{x})] \right\} |\psi(0)\rangle\,. \tag{13}$$

This form of the evolution operator is not very illuminating (what is the sine of the cosine of the position operator?) but it can be written in a more appropriate form if one represents $\cos[\varphi - \vec{k}\cdot\vec{x})]$ as $(\exp[i\varphi - i\vec{k}\cdot\vec{x}] + \exp[-i\varphi + i\vec{k}\cdot\vec{x}])/2$ and writes down the Taylor series of the remaining sine and cosine. In this way we find, e.g.,

$$\cos[\chi\cos(\varphi - \vec{k}\cdot\vec{x})] = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m)!}\left(\frac{\chi}{2}\right)^{2m} \sum_{l=-m}^{m} \binom{2m}{l+m} \exp[2il(\vec{k}\cdot\vec{x} - \varphi)]\,. \tag{14}$$

A resummation of the r.h.s. leads us to

$$\cos[\chi\cos(\varphi - \vec{k}\cdot\vec{x})] = \sum_{r=-\infty}^{\infty} \exp[2ir(\vec{k}\cdot\vec{x} - \varphi)](-1)^r J_{2|r|}(\chi) \tag{15}$$

where

$$J_{2|r|}(\chi) = \left(\frac{\chi}{2}\right)^{2|r|} \sum_{l=0}^{\infty} \frac{(-1)^l(\chi/2)^{2l}}{l!(l+2|r|)!} \tag{16}$$

are Bessel's functions. A similar calculation can be done for the sine in Eq. (13). The time evolution in its final form is then given by

$$\begin{aligned} |\psi(0_+)\rangle \;=\; & \left\{ \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \sum_{n=-\infty}^{\infty} \exp[2in(\vec{k}\cdot\vec{x} - \varphi)](-1)^n J_{2|n|}(\chi) + \right. \\ & \left. i \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \sum_{n=0}^{\infty} \left[e^{i(2n+1)(\vec{k}\cdot\vec{x}-\varphi)} + e^{-i(2n+1)(\vec{k}\cdot\vec{x}-\varphi)} \right] (-1)^n J_{2n+1}(\chi) \right\} |\psi(0)\rangle\,. \end{aligned} \tag{17}$$

This expression makes the physical effect of the laser waves rather obvious. Since $\exp(in\vec{k}\cdot\vec{x})$ corresponds to a change of $n\vec{k}$ in the momentum of the atoms the atomic beam is split into many partial beams which are weighted with Bessel functions. For even n there is a momentum transfer without excitation and deexcitation. It depends on the apparatus parameter χ which Bessel function and accordingly which momentum transfer n is predominant in Eq. (17). Note that in the extreme limit of Eq. (17) also for running waves excitation with $n < 0$ and deexcitation with $n > 0$ is possible. Such processes are explicitly removed in the rotating wave approximation because they are far from resonance with the atomic transition. In the limit of very short interaction time between lasers and atoms the resonance condition has not to be fulfilled. Seen in this way the result (17) makes it plausible that for very short (running) laser pulses atoms could get the "wrong" momentum transfer.

Having justified the structure of Eq. (9) we stress that the calculation should not be misunderstood as an adequate treatment of the influence of extremely short laser pulses on atoms. This is due to the fact that for short laser pulses the contribution of non-resonant electromagnetic waves cannot be neglected. An important implication is that for too short pulses also atomic states which are not described in the two-level approach can be excited by the laser waves. Since a very short pulse may not extend over one wavelength also the shape of the pulse becomes important. Nevertheless the result (17) is a good approximation if the pulses are not too short and if the flight times T_i in the dark zones are much longer than the interaction times τ_i between the atoms and the lasers. Its range of validity may be constrained to situations where the rotating wave approximation and of course the two-level approximation are applicable.

4 Phase shift for arbitrary interferometer configurations

In this section we will be concerned with the calculation of the phase shift between two partial beams I and II caused by H_p of Eq. (4) in an atomic interferometer with an *arbitrary number of laser beams* serving as beam splitters. In a first step the expression (10) can be brought into a more convenient form if we insert the identity operator in the form $\exp(-iH_{cm}t)\exp(iH_{cm}t)$ for certain times t and make use of the equality

$$\exp(-B)\exp(A)\exp(B) = \exp[e^{-B}Ae^{B}] \tag{18}$$

which is valid for any operators A and B provided the exponentials make sense. We get

$$|\psi_I\rangle = e^{-iH_{cm}t_N} e^{in\vec{k}_N\cdot r_N} \cdots e^{in\vec{k}_1\cdot r_1}|\psi_0\rangle \tag{19}$$

where $t_i := \sum_{j=0}^{i-1} T_j$, and the operators r_i are defined by

$$r_i := e^{iH_{cm}t_i}\vec{k}_i\cdot\vec{x}e^{-iH_{cm}t_i}\,. \tag{20}$$

This approach is similar to the one used by Bordé [15] for the calculation of acceleration and rotation induced shifts in the Ramsey device and for a Mach-Zehnder interferometer.

The essential step in the derivation of the phase shift is now the calculation of these operators by means of the well known formula

$$\exp[itH_{cm}]\vec{k}\cdot\vec{x}\exp[-itH_{cm}] = \sum_{s=0}^{\infty}\frac{(it)^s}{s!}K_s \tag{21}$$

with $K_0 = \vec{k}\cdot\vec{x}$ and $K_{s+1} = [H_{cm}, K_s]$. It is not difficult to prove by induction that each K_s can be written in the form

$$K_s = i^s\{\vec{U}_s\cdot\vec{x} + \frac{1}{M}\vec{V}_s\cdot\vec{p} + \vec{W}_s\cdot\vec{a}\} \tag{22}$$

where $\vec{U}_s, \vec{V}_s$, and $\vec{W}_s$ are c-numbers and *do not depend on the acceleration* $\vec{a}$. This will turn out to be the reason for the independence of the accelerational phase shift of the initial state of the atoms. The c-number vectors obey the recursion law

$$\begin{aligned}(\vec{U}_{s+1})_a &= \varepsilon_{abc}(\vec{U}_s)_b(\vec{\omega})_c + R_{0a0b}(\vec{V}_s)_b\\ (\vec{V}_{s+1})_a &= \varepsilon_{abc}(\vec{V}_s)_b(\vec{\omega})_c - (\vec{U}_s)_a\\ (\vec{W}_{s+1})_a &= (\vec{V}_s)_a\end{aligned} \tag{23}$$

with the initial condition $\vec{U}_0 = \vec{k}$ and $\vec{V}_0 = \vec{W}_0 = 0$. In general the solution of this scheme cannot be given in a closed form, but it is possible to calculate the operators r_i to any wanted accuracy. When this has been done the operators r_i of Eq. (20) are given by

$$r_i = \vec{\mu}_i\cdot\vec{x} + \vec{\nu}_i\cdot\vec{p} + \vec{\rho}_i\cdot\vec{a} \tag{24}$$

with well defined $\vec{\mu}_i$, $\vec{\nu}_i$, and $\vec{\rho}_i$. This is the first step in the evaluation of Eq. (20). The specification of $\vec{\mu}_i$, $\vec{\nu}_i$, and $\vec{\rho}_i$ follows in Eqs. (27), (28), and (30).

The rest of the calculation is only a repeated utilization of the famous Baker-Campbell-Haussdorf formula which gives again exact results because the operators r_i have the simple form (24). In a first step one can prove by induction that

$$e^{in_Nr_N}\cdots e^{in_1r_1} = \exp[\sum_{i=1}^{N} n_ir_i]\exp[\frac{i}{2}\sum_{i=1}^{N-1}\sum_{j>i} n_in_j(\vec{\mu}_i\cdot\vec{\nu}_j - \vec{\nu}_i\cdot\vec{\mu}_j)] \tag{25}$$

holds. This can be used to rewrite the expression (19) in a more convenient form. After the same is done for $|\psi_{II}\rangle$ the interference term $\langle\psi_I|\psi_{II}\rangle$ is quickly calculated to be

$$\begin{aligned}\langle\psi_I|\psi_{II}\rangle &= \exp\left\{i\sum_{i=1}^{N}(n_i^{II} - n_i^I)\vec{\rho}_i\cdot\vec{a}\right\}\times\\ &\exp\left\{\frac{i\hbar}{2}\sum_{i=1}^{N-1}\sum_{j>i}(n_i^{II}n_j^{II} - n_i^In_j^I)(\vec{\mu}_i\cdot\vec{\nu}_j - \vec{\nu}_i\cdot\vec{\mu}_j)\right.\\ &\left.+\frac{i\hbar}{2}\sum_{i,j=1}^{N}\vec{\mu}_i\cdot\vec{\nu}_j(n_i^In_j^I + n_i^{II}n_j^{II} - 2n_i^{II}n_j^I)\right\}\times\\ &\left\langle\psi_0\right|\exp\left\{i\sum_{i=1}^{N}(n_i^{II} - n_i^I)\vec{\mu}_i\cdot\vec{x}\right\}\exp\left\{i\sum_{i=1}^{N}(n_i^{II} - n_i^I)\vec{\nu}_i\cdot\vec{p}\right\}\left|\psi_0\right\rangle\end{aligned} \tag{26}$$

where we have reintroduced $\hbar$ and c for convenience. In realistic situations we will have $|\vec{k}| \approx E_b - E_a$. Doppler shift and recoil are still contained. To read off the phase shift induced by H_p of Eq. (4) one has to work out $\vec{\mu}_i$, $\vec{\nu}_i$, and $\vec{\rho}_i$ explicitly for the interferometer situation in question and to substract from the phase given in Eq. (26) the corresponding expression for $\vec{a} = 0$, $\vec{\omega} = 0$, and $R_{0l0m} = 0$. Eq. (26) is the main result of this paper. It may look somewhat complicated but this is only due to its great generality. In particular, this form is well suited for the application of algebraic computer programs. After specification of the magnitudes of $\vec{a}$, $\vec{\omega}$, and R_{0l0m} in H_p it is now in the expression (26) where one has to make shure that the iteration (23) above has led to the respective contributions of the desired order. An example will be given in the next section.

We remark that in two physically important special cases closed expressions for r_i can be given from which the respective $\vec{\mu}_i$, $\vec{\nu}_i$, and $\vec{\rho}_i$ can be read off. If the rotation vanishes we find

$$r_i = (\vec{k}_i)_l(\cos[t_iN])_{lm}x_m + \frac{1}{M}(\vec{k}_i)_l(N^{-1}\sin[t_iN])_{lm}p_m + (\vec{k}_i)_l(N^{-2}(\cos[t_iN] - 1))_{lm}a_m \tag{27}$$

where the matrix N is defined by $(N^2)_{lm} := R_{0l0m}$. Surely this is a formal expression defined solely via its expansion which contains only even powers of N. In this sense we need not to care for the existence of N.

In absence of curvature the result for the operators is

$$\begin{aligned}r_i &= (\vec{k}_i)_l(\exp[t_iO])_{lm}x_m + \frac{t_i}{M}(\vec{k}_i)_l(\exp[t_iO])_{lm}p_m + (\vec{k}_i)_l(O^{-2}(\exp[t_iO] - 1))_{lm}a_m\\ &\quad -(\vec{k}_i)_l(O^{-1}\exp[t_iO])_{lm}a_mt_i\end{aligned} \tag{28}$$

where the matrix O is defined by $O_{lm} = \varepsilon_{lmk}\omega_k$.

A short discussion of our result (26) will close this section. First, it is obvious that the accelerational induced phase shift is completely independent of the initial state $|\psi_0\rangle$ since the acceleration $\vec{a}$ occurs only in the first row of Eq. (26). The independence of this shift from the initial momentum of the atoms for the atomic fountain was also noted by Kasevich and Chu [3]. Second, there are state dependent shifts as the last row indicates. In order to get an impression of their form we assume that the initial state is $|a\rangle\otimes|G\rangle$ where $|G\rangle$ denotes a Gaussian wavepacket (in momentum space) with mean momentum $\langle\vec{p}\rangle$ and half width σ in all directions. Setting $\vec{A} := \sum_i(n_i^{II} - n_i^I)\vec{\mu}_i$ and $\vec{B} := \sum_i(n_i^{II} - n_i^I)\vec{\nu}_i$ we quickly find that the state dependent part of Eq. (26) contains a phase factor and a factor modifying the amplitude:

$$\exp\{i\vec{B}\cdot\langle\vec{p}\rangle - \frac{i}{2}\vec{A}\cdot\vec{B}\}\exp\{-\frac{\sigma^2}{6}\vec{B}^2 - \frac{3}{8\sigma^2}\vec{A}^2\}\,. \tag{29}$$

We will see in the next section that for the Ramsey interferometer only the term $\vec{B}\cdot\langle\vec{p}\rangle$ contributes to the phase shift to the desired order in our applications since the term $\vec{A}\cdot\vec{B}$ will be entirely of higher order.

5 Application to the Ramsey interferometer and to the atomic fountain

It is now straightforward to apply Eq. (26) to two special devices, the Ramsey interferometer and the atomic fountain. As specification of the experimental set up we may assume flight times T of about one second. The order of the initial velocity v , the laser wave vector $\vec{k}$, and the atomic mass M are about $v \sim 1$ m s^{-1}, $|\vec{k}| \sim 10^7$ m^{-1}, and $M \sim 10^{-26}$ kg, respectively. If one applies such an atomic interferometer for the measurement of the influence of the earth's acceleration $\vec{a}$, the earth's rotation $\vec{\omega}$, and space-time curvature R_{0l0m} of the earth or of some lead block we have the magnitudes $a \sim 10$ m s^{-1}, $\omega \sim 7\cdot 10^{-5}$ s^{-1}, and $R_{0l0m} \sim 5\cdot 10^{-23}$ m^{-2}, respectively. These specifications make it reasonable to work out $\vec{\mu}_i, \vec{\nu}_i$, and $\vec{\rho}_i$ of Eq. (24) by means of the recursion law (23) up to second order in the rotation and first order in the curvature. In the exponents of Eq. (26) we will take into account only phase shifts which are at least of the order of 0.1 rad. In detail we obtain

$$
\begin{aligned}
(\vec{\mu}_i)_l &= (\vec{k}_i)_l - t_i(\vec{k}_i\times\vec{\omega})_l + \frac{1}{2}t_i^2((\vec{k}_i\times\vec{\omega})\times\vec{\omega})_l - \frac{1}{2}t_i^2c^2R_{0l0m}(\vec{k}_i)_m \\
(\vec{\nu}_i)_l &= \frac{1}{M}\{t_i(\vec{k}_i)_l - t_i^2(\vec{k}_i\times\vec{\omega})_l + \frac{1}{2}t_i^3((\vec{k}_i\times\vec{\omega})\times\vec{\omega})_l - \frac{1}{6}t_i^3c^2R_{0l0m}(\vec{k}_i)_m\} \\
(\vec{\rho}_i)_l &= -\frac{1}{2}t_i^2(\vec{k}_i)_l + \frac{1}{3}t_i^3(\vec{k}_i\times\vec{\omega})_l - \frac{1}{8}t_i^4((\vec{k}_i\times\vec{\omega})\times\vec{\omega})_l + \frac{1}{24}t_i^4c^2R_{0l0m}(\vec{k}_i)_m\,. \qquad (30)
\end{aligned}
$$

The first apparatus we will consider is the *Ramsey interferometer* which is described in some detail in Ref. [2]. For our purpose the following facts are sufficient. First, in the Ramsey interferometer essentially only two atomic states are involved and the laser frequency is close to the transition frequency between these states. It is therefore allowed to use the two level model for atoms as we have done in the preceding sections. Four lasers serve as beam splitters, and their wave vectors are given by $\vec{k} := \vec{k}_1 = \vec{k}_2 = -\vec{k}_3 = -\vec{k}_4$. The times of flight as they appear in Eq. (10) are given by $T_0 = 0$, $T_1 = T_3 =: T$, and $T' := T_2$. We introduced T and T' just to be in concordance with the usual notation in the literature. We will examine the phase shift between the partial beams I and II of Fig. 1 as it is induced by H_p. According to Sec. 2 these beams are characterized by the sequences $\beta_1\gamma_2\beta_3\alpha_4$ and $\delta_1\delta_2\delta_3\beta_4$, respectively. From this we can infer to the sequences $\{n_i^I\} = \{1,-1,1,0\}$ and $\{n_i^{II}\} = \{0,0,0,1\}$. Inserting these values in Eq. (26) and use of Eq. (30) leads after some algebra to the expression

$$
\begin{aligned}
\langle\psi_I|\psi_{II}\rangle &= \exp\Big\{i\Big[\vec{k}\cdot\vec{a}T(T+T') - \vec{a}\cdot(\vec{k}\times\vec{\omega})T(T+T')(2T+T') \\
&\quad + \frac{\hbar}{6M}\left(c^2R_{0l0m}k_lk_m + 3(\vec{k}\times\vec{\omega})^2\right)T^2(2T+3T') \\
&\quad + \frac{1}{4}(((\vec{k}\times\vec{\omega})\times\vec{\omega})\cdot\vec{a} - \frac{c^2}{3}R_{0l0m}k_la_m)T(T+T')(7T^2+7TT'+2T'^2)\Big]\Big\} \\
&\times\Big\langle\psi_0\Big|\exp\Big\{i[c^2R_{0l0m}x_lk_m - ((\vec{k}\times\vec{\omega})\times\vec{\omega})\cdot\vec{x}]T(T+T')\Big\} \\
&\quad \exp\Big\{\frac{i}{2M}\Big[\left(c^2R_{0l0m}k_lp_m - 3((\vec{k}\times\vec{\omega})\times\vec{\omega})\cdot\vec{p}\right)T(T+T')(2T+T') \\
&\quad + 4\vec{p}\cdot(\vec{k}\times\vec{\omega})T(T+T')\Big]\Big\}\Big|\psi_0\Big\rangle \qquad (31)
\end{aligned}
$$

which contains only the phase shift due to $H_p \neq 0$. Again we have reintroduced $\hbar$ and c. The first order contribution due to rotation was measured with the Ramsey interferometer by Riehle *et. al.* [8]. For the measurement of space-time curvature the device of Sterr *et. al.* [16] may be better suited because of the longer lifetime of the respective excited state in magnesium. As announced the product of the two vectors $\vec{A}$ and $\vec{B}$ entering Eq. (29) is of higher order in this case. A discussion of the leading terms of the phase shift was given elsewhere [10]. The order of magnitude, however, is the same as for the atomic fountain which will be estimated below.

Next we turn to the *atomic fountain* as it was developed by Kasevich and Chu [3]. In this case 3 atomic levels are involved, two hyperfine levels and one excited level. The atomic beam is split and recombined by three laser pulses. Each pulse includes two counterpropagating laser beams (here denoted by a and b) with frequencies ν_a and ν_b. They are tuned in a way that their frequencies are slightly smaller than the transition frequency between the two hyperfine levels and the excited level so that a two photon process induces transitions between the hyperfine levels. The momentum transfer for such a process is $\vec{k}_a - \vec{k}_b$. The splitting of the atomic beam is given in Fig. 2.

It now seems that our scheme is not applicable to this experiment since it is only valid for two level atoms. However, as pointed out in Ref. [3] the particular tuning of the laser pulses makes it possible to treat the hyperfine states as an effective two level system with an effective momentum transfer of $\vec{k}_a - \vec{k}_b$. This observation allows us to use the results of this paper for the calculation of the phase shift in the atomic fountain apparatus.

We will model the situation by three copropagating running laser beams with wave vector equal to the effective momentum transfer $\vec{k} = \vec{k}_a - \vec{k}_b$. From Fig. 2 we can read off the defining sequences of operators for the interfering beams. For beam I it is $\delta_1\beta_2\alpha_3$ and $\{n_i^I\} = \{0,1,0\}$. Beam II is described by $\beta_1\gamma_2\beta_3$ and $\{n_i^{II}\} = \{1,-1,1\}$. Setting $T_1 = T_2 = T$ and $T_0 = 0$ the application of Eq. (26) leads us to the phase shift

$$
\begin{aligned}
\langle\psi_I|\psi_{II}\rangle &= \exp\Big\{i\Big[-\vec{k}\cdot\vec{a}T^2 + 3\vec{a}\cdot(\vec{k}\times\vec{\omega})T^3 - \frac{\hbar c^2}{2M}R_{0l0m}k_lk_mT^3 \qquad (32) \\
&\quad - \frac{7}{4}\left(((\vec{k}\times\vec{\omega})\times\vec{\omega})\cdot\vec{a} - \frac{c^2}{3}R_{0l0m}k_la_m\right)T^4\Big]\Big\} \\
&\times\Big\langle\psi_0\Big|\exp\Big\{i\left[-2c^2R_{0l0m}x_lk_m + ((\vec{k}\times\vec{\omega})\times\vec{\omega})\cdot\vec{x}\right]T^2\Big\} \\
&\quad \exp\Big\{\frac{i}{M}\Big[\left(3((\vec{k}\times\vec{\omega})\times\vec{\omega})\cdot\vec{p} - c^2R_{0l0m}k_lp_m\right)T^3 - 2\vec{p}\cdot(\vec{k}\times\vec{\omega})T^2\Big]\Big\}\Big|\psi_0\Big\rangle
\end{aligned}
$$

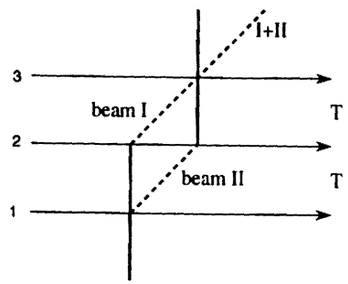

Figure 2: The interfering beams in the atomic fountain device understood as an effective two level system with running laser pulses. The horizontal lines represent the three laser pulses. The atoms are initially in the lower hyperfine level. The higher hyperfine level is denoted by dashed lines. T is the flight time between the laser pulses.

The state-independent part of the phase shift can be directly read off from this expression. Assuming that the center of mass state of the atoms is a Gaussian wavepacket we can apply Eq. (29) to calculate the state-dependent phase factor. For the atomic fountain it is simply given by the exponential in the last line of Eq. (32) if one replaces the operator $\vec{p}$ by the initial mean momentum $\langle\vec{p}\rangle$ of the wavepacket.

In the case of pure acceleration (i.e., $R_{0l0m} = \omega_l = 0$) the complete shift reduces to the first term in Eq. (32) and agrees with the result of Ref. [3]. The last term in Eq. (32) describes the well known Sagnac phase shift. Because the earth's acceleration alters the motion of the atoms substantially it is modified by the second term in Eq. (32). The structure of this modification is easily understood if one considers a classical free particle with initial momentum $\vec{p}_0$. In an accelerated frame of reference its momentum is changed according to $\vec{p}(t) = \vec{p}_0 - M\vec{a}t$. The replacement of the mean initial momentum $\langle\vec{p}\rangle$ in the Sagnac phase shift by $M\vec{a}T$ reflects this fact by leading to the correct structure of the modifying term. Corrections to the Sagnac phase which are of higher order in the earth's angular velocity are the first term in the last line and the first term in the second line. The latter can be understood in the same way as the modification to the first order Sagnac phase as a consequence of the modified atomic motion an the accelerated reference frame. The influence of space-time curvature on atoms, i.e., of the variation of the acceleration over the interferometer, can be splitted into two parts. The last term in the second line and the second term in the last line of Eq. (32) are connected to the motion of the atoms whereas the last term in the first line is only due to the non-vanishing relative acceleration experienced by two partial beams which are separated along the direction of the laser beams. Note again that the third line does not contribute to the phase shift to the order under consideration if the atomic wavepacket is sufficiently localized in momentum space.

6 Measurability of the influence of space-time curvature on quantum systems

A detailed discussion of the measurability of curvature effects with the Ramsey device was given in Ref. [10]. It is necessary to restate here some remarks on the physical value of the measurement of the curvature-dependent phase shift. In general relativity the acceleration $\vec{a}$ is tied to the particular observer who is accelerated with $\vec{a}$. Loosely speaking everything seems to be accelerated with acceleration $-\vec{a}$ in his frame of reference. This implies that in the frame of a freely falling observer there is no acceleration at all. Acceleration is a non-inertial effect, tied to the observer, and is not connected with the structure of space-time. The latter is described by space-time curvature which is present in any frame of reference. A curved space-time implies a relative acceleration between nearby observers, i.e. the distance between two freely falling observers changes and the acceleration of two nearby observers which are held in a fixed distance is different.

It was suggested that one can use the atomic fountain as a gravimeter [3]. This is true in a twofold sense. If one varies the position of the whole device in order to detect the acceleration of the earth at two different points this is indirectly also a detection of space-time curvature since the variation of the acceleration over space is measured. This is, however, by no means the demonstration of how space-time curvature acts on atoms. Rather, the classical apparatus rests in two different frames of reference with two different accelerations. In both frames it is only the acceleration which causes the effect. On the other hand, a demonstration of the influence of space-time curvature on quantum mechanical systems requires the measurement of the curvature-induced phase shift in one particular frame of reference. This is the second and more fundamental sense in which the atomic fountain may be used as a gravimeter. While the influence of acceleration on quantum systems is a well established fact, the demonstration of the influence of space-time curvature is still lacking.

In Ref. [10] it was shown that the Ramsey device seems not to be able to detect the influence of the earth's curvature on atoms. This is due to the great difficulties with the separation of the influence of the earth's curvature from the phase shift caused by the earth's acceleration and rotation which are much larger. As was suggested by Clauser [17] it may be that one can exploit special interferometer geometries to get rid of the disturbing shifts. On the other hand, provided the flight times in the Ramsey device can be enlarged to be of the order of one second, it should be possible to detect the curvature of two lead blocks which is of the same order of magnitude as the curvature of the earth and for which the problem of separation is trivially solved by removing the blocks.

For the atomic fountain these statements remain essentially the same. But since the flight times are already in the desired order of magnitude for the existing experimental set up [3] it should be no problem to measure with it the curvature of the lead blocks. For the existing atomic fountain the relevant datas are the following. The experiment is done with sodium atoms ($M = 3.8 \cdot 10^{-26}$ kg) which have an in-

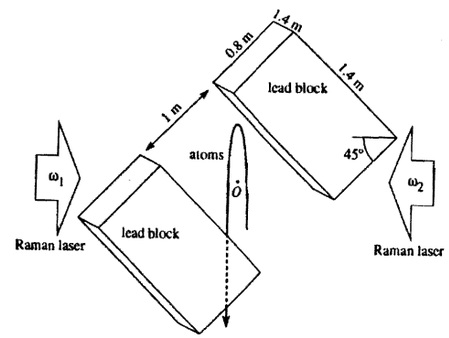

Figure 3: A scheme of the proposed experiment to measure the influence of the space-time curvature caused by two lead blocks on the atomic fountain. The big arrows represent the three Raman laser pulses performed with two counterpropagating laser beams with frequencies ω_1 and ω_2. The point O denotes the origin of the coordinate system.

itial velocity of 2.5 m/s pointing upwards (see Fig. 3). The effective wave vector $\vec{k}$ used in the previous section is about $2 \cdot 10^7$ m^{-1} and is assumed to be parallel to the earth's surface. For our estimations we will insert for the flight times the value of $T_1 = T_2 = T = 0.25$ s. With these data the purely acceleration-induced phase shift is of the order of 10^7 rad and the Sagnac phase is about 450 rad.

In Ref. [10] we have calculated the curvature components of two identical lead blocks with a volume of $1.4 \times 1.4 \times 0.8$ m^3 = 1.57 m^3 which corresponds to a mass of about 17 tons for each block. They are separated by a distance of 1 m, and the symmetry axis is tilted by an angle of 45 degrees with respect to the earth's surface. For this configuration the relevant components of the curvature tensor in the middle between the two blocks take the values $R_{0103} \approx 2.4 \cdot 10^{-23}$ m^{-2} and $R_{0303} \approx 8.5 \cdot 10^{-24}$ m^{-2} if the atoms move initially in the x^1-direction and the lasers propagte in the x^3-direction (for all details we refer to Ref. [10]).

Bearing in mind that the acceleration $\vec{a}$ is that of an observer resting on earth and thus is the negative of the earth's acceleration one obtains for the total curvature-dependend part of the phase shift in Eq. (32) the value of -0.8. It is therefore reasonable that the influence of space-time curvature on atoms should be measurable with the contemporary technology.

7 Acceleration induced phase shift and the equivalence principle

Eq. (26) shows that the phase shift induced by the term $M\vec{a} \cdot \vec{x}$ in H_p of Eq. (4) is linear in $\vec{a}$ and independent of the initial state $|\psi_0\rangle$ of the atoms. To provide a better and deeper understanding of this fact we present a quick direct calculation of this phase shift for the case $\vec{\omega} = 0$ and $R_{0l0m} = 0$. The term $M\vec{a} \cdot \vec{x}$ represents either the influence of the motion of the interferometer with acceleration $\vec{a}$ or the influence of some linear potential for an interferometer at rest with respect to an inertial system. It is the content of the equivalence principle that the latter case, which is typically realized in a homogeneous gravitational field, is physically equivalent to the situation with acceleration. We may therefore restrict to this.

For an accelerated interferometer the atoms move in the dark zones freely (constant velocity with regard to the inertial reference frame) whereas the lasers and their electromagnetic waves are dragged along with the interferometer. The consequence of the acceleration is that the atoms meet the lasers not at $\vec{x}$ but at $\vec{x} - \vec{a}t^2/2$. Equivalently, the Hamiltonian H_{tot} has the form

$$H_{tot} = \frac{\vec{p}^2}{2M} + H_{atom} - \vec{d} \cdot \vec{E}(\vec{x} - \vec{a}t^2/2, t) \; . \tag{33}$$

In consequence of Eq. (33) the idealized pointlike action of the lasers becomes now instead of Eq. (9)

$$\alpha_i^I = \exp[in_i^I \vec{k}_i \cdot \vec{x}] \exp[-in_i^I \vec{k}_i \cdot \vec{a}t_i^2/2] \tag{34}$$

and correspondingly for β_i^I, γ_i^I, and δ_i^I. The modification amounts to a multiplication with a c-number which results in Eq. (10) in an additional $\vec{a}$-dependent phase factor. Because H_{cm} of Eq. (33) does not contain $\vec{a}$ furthermore, and because we restrict according to the convention of Sec. 2 in $\langle\psi_I|\psi_{II}\rangle$ only to those terms which represent the shift of the interference pattern when $\vec{a}$ is switched on, we have

$$\langle\psi_I|\psi_{II}\rangle = \exp[-\frac{i}{2}\sum_{i=1}^{N}(n_i^{II} - n_i^I)\vec{k}_i \cdot \vec{a}t_i^2] \tag{35}$$

with the phase giving the shift. This simple expression is exactly the result which can also be read off from the $\vec{a}$-dependent term in Eq. (26) in putting $\vec{\omega} = 0$ and $R_{0l0m} = 0$ in $\vec{\rho}_i$ of Eq. (30). For an experimental discrimination between alternative general relativistic theories of gravitation it may be interesting to test small corrections in the acceleration [18]. Eq. (35) gives the term which contains $\vec{a}$ for all laser configurations in arbitrary order of the corrections.

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