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# TESTING LORENTZ INVARIANCE WITH ATOMIC BEAM INTERFEROMETRY

J. Audretsch, U. Bleyer and C. Lämmerzahl

*Fakultät für Physik der Universität Konstanz*
*Postfach 5560, D-7750 Konstanz, Germany*

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## Abstract

Atomic and neutron beam interferometry offer a sensible technique for tests of the space-time structure. One important ingredient of the space-time structure is given by the null cones which are related to local Lorentz invariance. A possible deviation from this structure described as splitting of the null cones gives rise to non-Lorentz invariance effects. It is showm that in the non-relativistic limit this typically amounts to a new type of spin-momentum coupling. Starting from a generalisation of the Dirac equation, based on fundamental principles, reasons are given for this special kind of coupling. The outcome of this model theory is taken to propose experiments which use atomic beam interferometry and give improved upper limits on possible non-Lorentz invariance parameters.

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## 1 INTRODUCTION

Matter wave interferometry with atoms, neutrons, or electrons plays an important role for the experimental verification of the basic assumptions of General Relativity [1]. The great accuracy of these devices has already been used to test the principle of equivalence on the quantum level. *Local Lorentz invariance* (LLI) is a fundamental ingredient of Einsteins equivalence principle and the basic assumption of Special Relativity ([2]). Although LLI seems to become manifest essentially in the high energy limit and atomic beam interferometry on the other hand works with cooled atoms with very low velocity, we will demonstrate that nevertheless atomic beam interferometry represents a powerful tool to set experimental limits on a possible *Lorentz non-invariance* (LNI) of particles with spin $\frac{1}{2}$. There are two main topics in this paper: Our intention is to describe a simple direct and quantitative matter wave test for the fact that in each space-time point there is only one future mass shell and accordingly not more than one future null cone. To do so we firstly have to develop a theoretical scheme by generalising the well known Dirac equation.

### 1.1 Lorentz non-invariance (LNI)

Tests of LLI are typically tests of specific model theories. They are based on the proposition and discussion of a LNI parametrised generalisation of a particular theory, whereby the generalisation reduces to the Lorentz invariant theory when the parameters are vanishing. If the parameters are non-vanishing, they give rise to new physical effects depending on the parameters, which are characteristic for the generalisation of the theory in question and can be tested experimentally. Setting upper bounds to these parameters gives a measure to what extent LLI is confirmed by these experiments.

Known LLI tests are, for example, related to the violation of Lorentz invariance if different kinds of matter follow different propagation cones. The mutual orientation of these cones, if existing, depends on the reference frame and therefore breaks the Lorentz invariance. Such effects may occure in model theories with different propagation cones for particles and for electromagnetic field or for different types of particles. This is discussed in the so-called $TH\epsilon\mu$-scheme ([3]). The Mansouri-Sexl-scheme provides a kinematical framework for studying LLI. For further details see the review ([2]).

Alternatively, breaking of Lorentz invariance can also occur in principle for a single matter field if it is multicomponent: The different components may define different propagation cones. This leads to polarisation depending propagation. Such type of propagation is given by double-breaking media, where the speed of light depends on the polarisation direction (Nicols prism). A breaking of Lorentz invariance of this kind may be caused by a generalisation of the Dirac equation. Because of the relation

$$\gamma^{(\mu}\gamma^{\nu)} = g^{\mu\nu} \qquad (1)$$

the Dirac matrices are deeply related with the metric of space-time and accordingly with the local null cones and the causal structure. It is therefore to be expected that modifications of equation (1) will lead to a splitting of the mass shell and correspondingly of the future null cone causing Lorentz non-invariance.

### 1.2 Reasons to consider LNI

There may be many physically very different reasons to consider LNI of massive particles. They appear typically the case in schemes in which the metric is considered as a derived structure. We shortly describe 3 reasons:

(i) One possibility to derive the space time geometry from fundamental physical experiences is a constructive axiomatics of space time. The procedure is to discover and to describe the geometrical structure of space-time by means of the behaviour of appropriately selected physical systems (called primitive objects) in particular physical effects (taken as basic experiences) [4]. Following a scheme of a constructive axiomatics of space-time geometry which is analogous to that of Ehlers, Pirani and Schild who base their axiomatics on light rays and test particles [5], the use of the concept of free matter waves as primitive elements in a space time axiomatics leads to Riemannian geometry, see [6], [7] and [4]. In this approach plane matter waves are considered as a particular limiting case of wave mechanics defined by a general field equation in a differentiable manifold. As field equation for the vector valued complex field the most general linear system of partial differential equations of arbitrary order was derived from first principles. In addition to such fundamental assumptions like a deterministic and local evolution of fields and the validity of a superposition principle the demand of local Lorentz invariance is one of the assumptions in constructive axiomatics. Demanding Lorentz invariance rises the necessity of independent tests of it's validity. This is what we are looking for in the present paper.

(ii) Another heuristical explanation for a local disturbance of Lorentz invariance can be given by an induction scheme of local causal structure based on the famous Mach principle. Due to this principle the inertial properties of a local system are induced by all masses of the surrounding universe. This construction starts from a sufficiently large symmetry group of motion for the whole of cosmical masses (called by Planck the telescopic group). For a small local system we have to sum up the motion of all surrounding cosmical bodies. As a result the local Lorentz invariance arises as the symmetry group of the local system. In this way local Lorentz invariance can be induced for small subsystems by the structure of the cosmos as a whole [8]. This is in full analogy to the induction of Galilei invariance in inertia free mechanics [9] [10]. But in the real cosmical situation the symmetry may be broken by inhomogeneous matter distributions leading to the breaking of Lorentz invariance. This could be the case on the earth due to the inhomogeneous gravitational potential produced by the masses of our galaxy [11]. Despite the fact that such an induction scheme is not realized constructively we can look for possible consequences of such a Machian induction. It will change the kinematical part of the equations of motion [12]. This can be realized with a generalized Dirac equation



described below.

(iii) In high energy physics models of LNI Yang Mills theories have been constructed which have the Lorentz invariant theory as a low energy limit. This makes Lorentz invariance a derived property and possible deviations from the Lorentz invariant theory can be tested by such model theories. The postulate for different metrics for weakly interacting particles is in the same spirit (see Nielsen *et al.* [13], [14]).

### 1.3 LNI induced by a generalised Dirac equation

In the present paper we do not intend to establish or support a particular theory giving reasons for a breaking of Lorentz invariance. Instead *we calculate phenomenologically possible effects of a LNI model theory in order to confront it with experiments thus ruling out a complete class of these generalised theories.* We do this in discussing effects of null-cone splitting in the framework of a generalized Dirac equation.

In our phenomenological approach we start with the *generalised Dirac equation* (GDE)

$$0 = i\tilde{\gamma}^\mu(x)\partial_\mu\varphi(x) - M(x)\varphi(x)\,. \tag{2}$$

At this level $\tilde{\gamma}^\mu$ are arbitrary complex $4\times 4$-matrices which are not necessarily fulfilling (1). The mass matrix $M$ is a complex $4 \times 4$-matrix, too. The $\tilde{\gamma}$-matrices as well as mass matrix $M$ may be responsible for creating LNI effects. (2) is invariant under coordinate transformations. For position dependent transformations $\varphi \mapsto \varphi' = S\varphi$, $S \in Gl(\mathbf{C}, s)$, $\tilde{\gamma}^\mu$ transforms homogeneously, $\tilde{\gamma}^\mu \mapsto \tilde{\gamma}'^\mu = S\tilde{\gamma}^\mu S^{-1}$, while the mass matrix $M$ transforms inhomogeneously, $M \mapsto M' = SMS^{-1} - iS\tilde{\gamma}^\mu\partial_\mu S^{-1}$. For the usual Dirac equation in Riemann space the mass matrix consists of the connection $-\frac{1}{4}(D_\mu h^\nu_a)h^b_\nu\gamma^a\gamma_b$ and the scalar mass $m$.

In order to describe physical propagation phenomena this first order system has to be hyperbolic. A geometrical characterisation of hyperbolicity of a partial differential equation is that there is one solution within a cone for a $\delta$-like source at the vertex of the cone ([15]). This will give consequences for the characteristics to be treated in ch.2.2.

We dont know if this very general equation can be attached to any reasonable physical interpretation within the scheme of first or second quantisation. We therefore specify this field equation in demanding certain properties known from the original locally Lorentz invariant theory which we want to be kept. This reduces the number of unspecified parameters in $\tilde{\gamma}$ and $M$ considerably. We will thereby end up with a particular type of generalisation of the Dirac equation which shows as new feature essentially the splitting of the mass shells and light cones. This seems to be a very prominent and typical breaking of LLI. The final step then consists in the outline of a *clean experimental test* to put upper limits on the remaining LNI parameters. Clean tests are experiments, by which LLI can be veryfied as directly as possible. For an example see the Phillips experiment [16] which will be discussed below. Experiments which are as theoryloaden as high energy experiments, can not be regarded as being fundamental for this purpose, because it is very difficult to distinguish uniquely LNI from unconventional modifications of other parts of physics.

### 1.4 Outline of this paper

In what follows we start with (2) and introduce a parametrisation of the generalised Dirac matrices. Then we look for some minimal nontrivial ansatz for the generalised Dirac matrices in order to describe possible experimental consequences of the null cone splitting. Our ansatz is characterised by the requirements that (i) for oscillatory initial values the helicity should be conserved (as it is the case for the usual Dirac equation in Minkowski space) and (ii) there is a coordinate system so that the characteristics, i.e. the null cone, appear isotropic. The general mass matrix will be reduced by requiring that in the WKB limit the Hamilton operator should commute with the helicity operator and an additional demand concerning helicity states. In the following we exploit step by step these requirements.

In the second part of this paper we discuss the experimental consequences of the special type of null cone splitting specified above. Especially the possibility is analised to test these structures with the help of atomic beam interferometry. On this experimental basis we give estimates for the validity of LLI. Readers interested in experimental consequences only can start with ch.4.

Unless otherwise stated we use $\hbar = c = 1$.

## 2 Restricting the generalised $\gamma$-matrices

As described in the introduction a generalisation of the usual Dirac theory can be realised by a first order system of partial differential equations (2). At first we introduce a parametrisation of the generalised Dirac matrices. We do not use the generalized Dirac equation in its most general form but restrict it in requiring that certain physical phenomena still should hold. Therefore, we use physical arguments in order to restrict the possible perturbations in a special reference frame. In this way we arrive at some 'minimal' generalisation of the usual Dirac equation which still describes some splitting of the usual null cone thus giving rise to Lorentz non-invariance.

### 2.1 Generalised Dirac matrices

The matrices $\tilde{\gamma}^\mu$ of the GDE are not fulfilling any Clifford algebra. Of course, from a physical point of view any deviation of these $\tilde{\gamma}$-matrices from the usual Dirac matrices fulfilling some Clifford algebra are small[1]. Nevertheless, since the calculation below go through for arbitrary deviations we make not this restriction. Only at the end by comparing our results with experiments we take small deviations.

[1] To be more mathematical: For given $\tilde{\gamma}^\mu$ there are other matrices $\gamma^\mu$ fulfilling a Clifford algebra relation and for which $\|\tilde{\gamma}^\mu - \gamma^\mu\| \ll 1$ where $\|\cdot\|$ is some matrix norm, e.g. the maximum norm.



Also for non-Clifford $\tilde{\gamma}$-matrices we can introduce a second rank tensor by $\tilde{g}^{\mu\nu} := \frac{1}{4}\mathrm{tr}(\tilde{\gamma}^\mu\tilde{\gamma}^\nu)$ which however has no physical meaning and is used for mathematical convenience only.

We can now introduce usual Dirac matrices in arbitrary coordinates by $\gamma^{(\mu}\gamma^{\nu)} = g^{\mu\nu}$. Here $g^{\mu\nu}$ is some non-singular second rank tensor with signature -2. With these $\gamma$-matrices we can introduce the Dirac algebra, that is, the complete set of $4 \times 4$-matrices

$$\Gamma^A := \{\mathbf{1}, i\gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\} \tag{3}$$

where we defined $\gamma_5 := i\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ and $\sigma^{\mu\nu} := \frac{1}{2}\gamma^{[\mu}\gamma^{\nu]}$. We emphasize that the second rank tensor $g^{\mu\nu}$ with the mathematical properties of a metric as well as the matrices $\gamma^\mu$ are (on this level) of no physical importance. They are introduced just for being able to have a complete set of $4 \times 4$-matrices available which are used to expand any $4 \times 4$ matrix. No physical propagation property is related to this tensor $g^{\mu\nu}$.

To formalise the notion of 'deviation' of the $\tilde{\gamma}$-matrices from $\gamma$-matrices fulfilling a Clifford algebra, we introduce the difference and expand it with respect to the Dirac algebra:

$$\tilde{\gamma}^\mu = \gamma^\mu + \epsilon^\mu_A\Gamma^A = \gamma^\mu + \epsilon^\mu\mathbf{1} + \epsilon^\mu_\nu\gamma^\nu + \epsilon^\mu_{\rho\sigma}\sigma^{\rho\sigma} + \bar{\epsilon}^\mu_\nu\gamma_5\gamma^\nu + \bar{\epsilon}^\mu\gamma^5 \tag{4}$$

From the physical point of view the parameters $\epsilon^\mu_A$ are assumed to be small but otherwise completely undetermined. The description of the generalised Dirac matrices as a deviation from the usual Dirac matrices is of technical importance.

The deviation of the generalised Dirac matrices for example now appears in the anticommutator

$$\begin{aligned} \tilde{\gamma}^{(\mu}\tilde{\gamma}^{\nu)} &= g^{\mu\nu}\mathbf{1} + \epsilon^{(\nu}_A\{\gamma^{\mu)}, \Gamma^A\} + \frac{1}{2}\epsilon^\mu_A\epsilon^\nu_B\{\Gamma^A, \Gamma^B\} && (5)\\ &= (g^{\mu\nu} + \delta g^{\mu\nu}(\epsilon^\mu_A))\mathbf{1} + \sum_A{}' \pi^{\mu\nu}_A(\epsilon^\mu_A)\Gamma^A && (6)\end{aligned}$$

whereby $\sum'_A$ runs over the Dirac algebra (3) but the $\mathbf{1}$. It is $\tilde{g}^{\mu\nu} = g^{\mu\nu} + \delta g^{\mu\nu}$. The coefficients $\pi^{\mu\nu}_A$ are a representation independent measure of the deviation of the $\tilde{\gamma}$-matrices from a Clifford algebra. The last equation should be the starting point for describing $\tilde{\gamma}$-matrices which do not fulfill a Clifford algebra. Then the $\epsilon^\mu_A$ of (4) are functions of the parameters $\pi^{\mu\nu}_A$. And if the deviation is small, i.e. if $\pi^{\mu\nu}_A \ll 1$, then there should be a representation of the $\gamma$-matrices, so that also for the parameters $\epsilon^\mu_A \ll 1$ holds. However, we will not deal with this problem here but simply use (4) to describe the sigularities and the null cones of our GDE.

The parameters $\epsilon^\mu_A$ are frame dependent and give notice of the breakdown of Lorentz invariance. This breaking is accompanied by a null cone splitting: The mutual orientation of the propagation cones is frame dependent (see ch.4).

### 2.2 Conservation of helicity

The first step in characterising a special version of the GDE (2) consists in the requirement that during the evolution of the field $\varphi$ a prepared helicity should be conserved. In the following considerations we do not assume that the parameters $\epsilon$ are small. Only for the confronatation with experiments in ch.4 we agein refer to small $\epsilon$.

#### 2.2.1 Oscillatory initial values

The GDE can be reformulated as evolution equation ($\hat{\mu} = 1, 2, 3$)

$$i\partial_0\varphi = -i(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}}\partial_{\hat{\mu}}\varphi + (\tilde{\gamma}^0)^{-1}M\varphi \tag{7}$$

whereby $x^0$ acts as 'time' coordinate and the surfaces $x^0 = const$ are hypersurfaces $\Sigma_{x^0}$.

By means of (7) the field on a hypersurface with parameter $x^0 + \delta x^0$ is then given by

$$\varphi = \varphi_0 + \delta x^0\left(-i(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}}\partial_{\hat{\mu}}\varphi_0 + (\tilde{\gamma}^0)^{-1}M\varphi_0\right) \tag{8}$$

where $\varphi_0$ is the initial value on the hypersurface at $x^0$.

If a function $\phi$ is given which depends on the coordinates of $\Sigma_{x^0}$ only, then any initial value given by $\varphi_0 = e^{i\phi}\tilde{\varphi}_0$ with $\|\partial_{\hat{\mu}}\tilde{\varphi}_0\| \ll \|(\partial_{\hat{\mu}}\phi)\tilde{\varphi}_0\|$ and $\|(\tilde{\gamma}^0)^{-1}M\tilde{\varphi}\| \ll \|(\partial_{\hat{\mu}}\phi)\tilde{\varphi}_0\|$ is called a *oscillatory initial value* (see e.g. Courant Hilbert [17]) for the GDE. Physically one may interpret these functions as fields with high momentum. Inserting oscillatory initial values into (8) we get

$$\varphi = \varphi_0 - i\delta x^0(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}}(\partial_{\hat{\mu}}\phi)\varphi_0 \tag{9}$$

#### 2.2.2 Consequences of conservation of helicity

Now we take oscillatory initial values which are prepared as to possess a certain helicity given by the projection operator $P_\pm := \frac{1}{2}(1 \pm \gamma_5)$, that is, $P_\pm\tilde{\varphi}_{0\pm} = \tilde{\varphi}_{0\pm}$. We demand: **Requirement 1:** ***The state of helicity does not change during propagation for oscillatory initial values.*** That means, if the oscillatory initial state is an eigenstate $P_\pm\tilde{\varphi}_{0\pm} = \tilde{\varphi}_{0\pm}$ then also the time-evolved state (9) $\varphi$ should be an eigenstate $P_\pm\varphi_\pm = \varphi_\pm$. To be more accurate, this means that there are $\gamma$-matrices so that one can build with these $\gamma$-matrices a $\gamma_5$ so that the above requirement is fulfilled.

For the operator $(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}}\partial_{\hat{\mu}}\phi$ in (9) governing the time-evolution of oscillatory initial values this means that it should commute with the projection operator $P_\pm$:

$$[P_\pm, (\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}}\partial_{\hat{\mu}}\phi] = 0 \tag{10}$$

Since $\gamma_5$ commutes with itself and $\sigma^{\mu\nu}$ we infer $(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}} = c^{\hat{\mu}}\mathbf{1} + d^{\hat{\mu}}\gamma_5 + e^{\hat{\mu}}_{\rho\sigma}\sigma^{\rho\sigma}$ for some undetermined parameters $c^{\hat{\mu}}$, $d^{\hat{\mu}}$, and $e^{\hat{\mu}}_{\rho\sigma}$. This can be rewritten as $(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}} = c^{\hat{\mu}}\mathbf{1} + d^{\hat{\mu}}\gamma_5 +$



$\widehat{e}^{\hat{\mu}}_{\rho}\gamma^0\gamma^{\hat{\rho}} + \widetilde{e}^{\hat{\mu}}_{\rho}\gamma_5\gamma^0\gamma^{\hat{\rho}}$. For the following it is most convenient to represent this result in the chiral representation of the $\gamma$-matrices:

$$(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}} = \widehat{c}^{\hat{\mu}}\mathbf{1} + \widehat{d}^{\hat{\mu}}\gamma_5 + \widehat{e}^{\hat{\mu}}_{\rho}\gamma^0\gamma^{\hat{\rho}} + \widetilde{e}^{\hat{\mu}}_{\rho}\gamma_5\gamma^0\gamma^{\hat{\rho}} = \begin{pmatrix} a^{\hat{\mu}} + b^{\hat{\mu}}_{\hat{\nu}}\sigma^{\hat{\nu}} & \mathbf{0} \\ \mathbf{0} & c^{\hat{\mu}} + d^{\hat{\mu}}_{\hat{\nu}}\sigma^{\hat{\nu}} \end{pmatrix} \tag{11}$$

where $\sigma^{\hat{\mu}}$ are the usual Pauli-matrices.

Next we use statements about the shape of the light cones to restrict further the deviations of the GDM.

### 2.3 Isotropy of the null cones

Typical features of partial differential equations like the GDE are described by discontinuities (jumps) in its solutions or in one of their derivatives which can occur only on certain hypersurfaces $\Phi(x) = const$ called *characteristics*. In General Relativity they are related to the notion of the light cone because for all physical theories of matter the characteristics are identical with the usual light cones related to the causal behaviour of the fields.

#### 2.3.1 The jumps

We assume that a jump (compare [17]) of a solution of (2) may occur at some surface $\Phi(x) = 0$, called jump-surface. For describing a function $\varphi$ having jumps up to order $N$ (that means up to the $N^{\text{th}}$ derivative) we begin with the ansatz

$$\varphi(x) = \sum_{i=r}^{N}\left(S^{(i)}(\Phi)\right)(x)a^{(i)}(x) + R(x) \tag{12}$$

with $a^{(i)}, R \in \mathrm{C}$ and $S^{(n)}(\Phi) := \frac{1}{n!}\eta(\Phi)\Phi^n$ ($\eta$ is the Heaviside function). $r$ can of course be zero. It is enough that there is a jump at all, that is, that there is a jump of lowest order $r < \infty$. Since $\varphi$ should be a (generalised) solution of the field equation we insert this series into (2) and perform the differentiation taking $\partial_\mu\left(S^{(n)}(\Phi)\right)(x) = (S^{(n-1)}(\Phi))(x)(\partial_\mu\Phi)(x)$ ($S^{(-1)}$ is the $\delta$-function) into account:

$$0 = \tilde{\gamma}^\mu(\partial_\mu\Phi)a^{(r)}S^{(r-1)}(\Phi) + \sum_{i=r}^{N} z_{(i)}(x)S^{(i)}(\Phi) + i\tilde{\gamma}^\mu\partial_\mu R - MR \tag{13}$$

whereby the functions $z_{(i)}(x)$ are regular and consist in the functions $a^{(i)}$ and derivatives of the hypersurface $\Phi$. The most irregular part of (13) is contained in the first term.

All coefficients to the $S^{(i)}(\Phi)$, $i = r-1, \ldots, N$, have to vanish independently so that a series of equations relating the hypersurface $\Phi$ to the functions $a^{(i)}$ arise. This series starts with the equation

$$0 = \tilde{\gamma}^{\mu} k_{\mu} a^{(r)} \tag{14}$$

where we introduced the ***normal*** $k_{\mu} := \partial_{\mu}\Phi$ of the hypersurface $\Phi$. This equation describes the jump $a := a^{(r)}$ of lowest order along a hypersurface $\Phi = 0$. That means, that if a solution of (2) possesses a jump, that is, is not a regular solution, then (14) must be fulfilled. This relation then connects the normal of the jump surface with the jump function $a$.

### 2.3.2 The null-cones

For (14) to possess any solution, the coefficient matrix $\tilde{\gamma}^{\mu} k_{\mu}$ must be singular giving a condition on the normals of the hypersurfaces $k_{\mu}$. Because the $\gamma$ are $4 \times 4$-matrices, the determinant in fact is a polynomial, the *characteristic polynomial*, in $k$ of order 4:

$$\det(\tilde{\gamma}^{\mu} k_{\mu}) = g^{\mu\nu\rho\sigma} k_{\mu} k_{\nu} k_{\rho} k_{\sigma}. \tag{15}$$

where $g^{\mu\nu\rho\sigma}$ is some 4th rank tensor. The solutions $k_{\mu}$ of (15) give the normal cone, also called *null cone*. This null cone becomes a fourth order surface, which can decay into two second order surfaces. In this way the null cone of special relativistic physics is splitted into two different propagation cones.

For the GDE to be hyperbolic a necessary condition is that there is a hypersurface with normal $n_{\mu}$ so that the characteristic polynomial has as many real solutions $k_0 = f(k_{\hat{\mu}}) \neq 0$ as the order of the characteristic equation is. In our case this means that there are 4 solutions $k_0 \neq 0$. The set of topologically different cones is indicated by fig.1 (see [18]).

### 2.3.3 Consequences of isotropic null cones

The condition for the jumps (14) can be reformulated as

$$0 = \left(k_0 \mathbf{1} + (\tilde{\gamma}^0)^{-1} \tilde{\gamma}^{\hat{\mu}} k_{\hat{\mu}}\right) a =: Ba. \tag{16}$$

With (11) the characteristics are now determined to be

$$\begin{aligned} \det B &= \det(a^{\hat{\mu}} k_{\hat{\mu}} - k_0 + b^{\hat{\mu}}_{\hat{\nu}} k_{\hat{\mu}} \sigma^{\hat{\nu}}) \det(c^{\hat{\mu}} k_{\hat{\mu}} - k_0 + d^{\hat{\mu}}_{\hat{\nu}} k_{\hat{\mu}} \sigma^{\hat{\nu}}) \\ &= \left[(a^{\hat{\mu}} k_{\hat{\mu}} - k_0)^2 - \sum_{\hat{\nu}} (b^{\hat{\mu}}_{\hat{\nu}} k_{\hat{\mu}})^2\right] \left[(c^{\hat{\mu}} k_{\hat{\mu}} - k_0)^2 - \sum_{\hat{\nu}} (d^{\hat{\mu}}_{\hat{\nu}} k_{\hat{\mu}})^2\right]. \end{aligned} \tag{17}$$

We restrict the $\tilde{\gamma}^{\mu}$ further by demanding the isotropy of the null cones: **Requirement 2:** *There is a coordinate system, so that this characteristic polynomial has the form* $H(x,k) = \left(\kappa k_0^2 - \lambda \vec{k}^2\right)\left(\mu k_0^2 - \nu \vec{k}^2\right)$ with $\vec{k}^2 := k_1^2 + k_2^2 + k_3^2$ and some undetermined coefficients $\kappa$, $\lambda$,



$\mu$, and $\nu$. Therefore (17) fulfills this isotropy demand only if $a^{\hat{\mu}} = 0$, $c^{\hat{\mu}} = 0$, $b^{\hat{\mu}}_{\hat{\nu}} = b\delta^{\hat{\mu}}_{\hat{\nu}}$ and $d^{\hat{\mu}}_{\hat{\nu}} = d\delta^{\hat{\mu}}_{\hat{\nu}}$. Thus with this restriction (11) reduces to

$$(\tilde{\gamma}^0)^{-1} \tilde{\gamma}^{\hat{\mu}} = \begin{pmatrix} b\sigma^{\hat{\mu}} & \mathbf{0} \\ \mathbf{0} & d\sigma^{\hat{\mu}} \end{pmatrix} \tag{18}$$

Using now the usual Dirac matrices in the chiral representation, then (16) can be written as

$$\left[\mathbf{1} k_0 + ((1 + a')\gamma^0 \gamma^{\hat{\mu}} + b' \gamma^0 \gamma_5 \gamma^{\hat{\mu}}) k_{\hat{\mu}}\right] a = 0 \tag{19}$$

whereby $a' = \frac{1}{2}(d - b) - 1$ and $b' = \frac{1}{2}(d + b)$. Since solutions $a$ remain unchanged we can multiply this equation with the non-singular matrix $\left(1 - \frac{a'}{b'}\gamma_5\right)\gamma^0$ and get with $\epsilon_0 = -\frac{a'}{b'}$ and $\epsilon_1 = b' - \frac{a'}{b'}(1 + a')$

$$\left[(\mathbf{1} + \epsilon_0 \gamma_5)\gamma^0 k_0 + (\mathbf{1} + \epsilon_1 \gamma_5)\gamma^{\hat{\mu}} k_{\hat{\mu}}\right] a = 0 \tag{20}$$

From this equation the generalised Dirac matrices $\tilde{\gamma}^{\mu}$ leading to conserved helicity and isotropic null cone, can be read off.

## 2.4 The resulting $\gamma$-matrices

To sum up, in the special coordinate system specified above, the generalised Dirac matrices finally have the form

$$\tilde{\gamma}^{\mu} = \begin{cases} \gamma^0 + \epsilon_0 \gamma_5 \gamma^0 \\ \gamma^{\hat{\mu}} + \epsilon_1 \gamma_5 \gamma^{\hat{\mu}} \end{cases}. \tag{21}$$

It should be noted that reading off the $\gamma$-matrices from (20) is not unique. All the characteristic features are not changed if one multiplies (20) with a non-singular matrix.

With these $\tilde{\gamma}^{\mu}$ the characteristic polynomial describing the null cones (15) now reads

$$0 = \det(\tilde{\gamma}^0 k_0 + \tilde{\gamma}^{\hat{\mu}} k_{\hat{\mu}}) = \left[(1 + \epsilon_0)^2 k_0^2 - (1 + \epsilon_1)^2 \vec{k}^2\right]\left[(1 - \epsilon_0)^2 k_0^2 - (1 - \epsilon_1)^2 \vec{k}^2\right]. \tag{22}$$

From hyperbolicity the four solutions $k_0$ are real. Therefore $\left(\frac{1 \pm \epsilon_1}{1 \pm \epsilon_0}\right)^2 > 0$, implying $\epsilon_0, \epsilon_1 \in \mathbf{R}$ and the hermiticity of $(\tilde{\gamma}^0)^{-1}\tilde{\gamma}^{\hat{\mu}}$. In the special coordinate system in which the splitting appears isotropic, there are two future pointing solutions $k_0$ for given $\vec{k}$:

$$k_0^{(1\pm)} = \pm \frac{1 + \epsilon_1}{1 + \epsilon_0} |\vec{k}|, \quad k_0^{(2\pm)} = \pm \frac{1 - \epsilon_1}{1 - \epsilon_0} |\vec{k}|. \tag{23}$$

A splitting parameter may then be defined by

$$\Delta := \frac{k_0^{(1+)} - k_0^{(2+)}}{k_0^{(1+)}} = 2 \frac{\epsilon_1 - \epsilon_0}{(1 - \epsilon_0)(1 + \epsilon_1)} = 2(\epsilon_1 - \epsilon_0) + \mathcal{O}(\epsilon^2). \tag{24}$$

For small parameters $\epsilon_0, \epsilon_1$ the splitting is to first order given by their difference. Therefore also in experiments the main contribution is expected to be of this order.

The GDE gives us a model theory to discuss the effects the breaking of Lorentz invariance in a lot of experiments, which we are going to discuss in ch.4. We finally observe that the Hamiltonian of the GDE is

$$i\partial_0\varphi = H\varphi = \left[-i(1+a')\gamma^0\gamma^{\hat\mu}\partial_{\hat\mu} - ib'\gamma_5\gamma^0\gamma^{\hat\mu}\partial_{\hat\mu} + M\right]\varphi \tag{25}$$

and up to first order in the $\epsilon$

$$i\partial_0\varphi = H\varphi = \left[-i\gamma^0\gamma^{\hat\mu}\partial_{\hat\mu} + i(\epsilon_0-\epsilon_1)\gamma_5\gamma^0\gamma^{\hat\mu}\partial_{\hat\mu} + M\right]\varphi\,. \tag{26}$$

In the case of constant $\gamma$-matrices the helicity operator $s := \vec S\cdot\vec p/|\vec p|$ with $S^{\hat\mu} := \gamma_5\gamma^0\gamma^{\hat\mu}$ commutes with the Hamiltonian but the 'mass' term $M$.

If the usual Dirac matrices are modified according to (21), it describes a splitting of the null cone structure which is characterised by one effective perturbation parameter $\epsilon_0-\epsilon_1$. This represents the minimal nontrivial ansatz for the description of a null cone splitting. In this way we completed the discussion of the null cones resp. the causal structure. In order to discuss consequences of the GDE in the low velocity region, as it is required for experiments using matter wave interferometry, we must consider the structure of the mass matrix too, as can be seen from (25). Therefore we will also restrict the general form of the mass matrix in ch.3.

## 3 WKB approximation and non-relativistic limit

In order to work out possible effects appearing in interferometry experiments we consider the GDE (2) with the related $\tilde\gamma^\mu$ of (21). There are two independent ways to arrive at the prediction for an interference experiments: (i) by means of the generalised Pauli equation, and (ii) using the WKB limit of the GDE. We use the second method

### 3.1 Restricting the mass matrix

The WKB approximation will be introduced by

$$\varphi = e^{iS}a \quad \text{with} \quad |\gamma^\mu\partial_\mu a| \ll |a| \tag{27}$$

Inserting this into (2) we get

$$0 = (\tilde\gamma^\mu p_\mu - M^{(0)})a \tag{28}$$
$$0 = i\tilde\gamma^\mu\partial_\mu a - M^{(1)}a \tag{29}$$



with $M = M^{(0)} + M^{(1)}$ whereby the $4\times 4$-matrix $M^{(0)}$ transforms homogeneously under base transformations (further details can be found in [7]).

For the WKB limit the mass matrix $M^{(0)}$ may be arbitrary. However, we specify this matrix by demanding the following **Requirement: 3:** *The matrix $H$ giving the eigenvalue $p_0$:*

$$p_0 a = \left((\tilde\gamma^0)^{-1}\tilde\gamma^{\hat\mu}p_{\hat\mu} + (\tilde\gamma^0)^{-1}M^{(0)}\right)a =: Ha \tag{30}$$

*commutes with the helicity operator $s$.* This is only possible for $\gamma^0$ and $\gamma_5$. Therefore the matrix $(\tilde\gamma^0)^{-1}M^{(0)}$ can be constructed from these two matrices so that $H = (\tilde\gamma^0)^{-1}\tilde\gamma^{\hat\mu}p_{\hat\mu} + \hat m\gamma^0 + \hat n\gamma_5$ and (28) reduces to

$$\left(\tilde\gamma^\mu p_\mu - m\mathbf{1} + n\gamma^0\gamma_5\right)a = 0 \tag{31}$$

for open parameter $m$ and $n$.

The last requirement for characterising the simplest deviation from the usual Dirac equation rests on the following observation: In the chiral representation of the $\gamma$-matrices we demanded conservation of chirality, that is, a splitting of the GDE into two two-component equations for oscillatory initial values. In the WKB approximation this splitting will be conserved for the $n$-term while it is violated for the $m$-term. We define the two-component function describing the respective helicity of the field by $\begin{pmatrix} a_+ \\ 0 \end{pmatrix} := P_+a$ and $\begin{pmatrix} 0 \\ a_- \end{pmatrix} := P_-a$. Especially for a particle at rest $p_{\hat\mu} = 0$ we have from (30)

$$H\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = n\begin{pmatrix} -a_+ \\ a_- \end{pmatrix} - m\begin{pmatrix} a_- \\ a_+ \end{pmatrix} \tag{32}$$

Now we demand **Requirement 4:** $P_+H\begin{pmatrix} a_+ \\ 0 \end{pmatrix} = 0$ *for a state prepared with the initial value* $\begin{pmatrix} a_+(t_0) \\ 0 \end{pmatrix}$. This then requires $n = 0$. In this way we reduced the mass matrix to the very simple structure $M^{(0)} = m\mathbf{1}$.

### 3.2 Resulting mass shells

The solvability condition of (31) with $n = 0$ then gives

$$\begin{aligned} 0 &= \det(\tilde\gamma^\mu p_\mu - m\mathbf{1}) \\ &= \left((1-\epsilon_0^2)p_0^2 + 2p_0(\epsilon_1+\epsilon_0)p - (1-\epsilon_1^2)p^2 - m^2\right) \\ &\quad \left((1-\epsilon_0^2)p_0^2 + 2p_0(\epsilon_0-\epsilon_1)p - (1-\epsilon_1^2)p^2 - m^2\right) \end{aligned} \tag{33}$$

where $p := \sqrt{p_1^2 + p_2^2 + p_3^2}$. The mass shells are then given by

$$p_0^{(1\pm)} = \frac{(\epsilon_0 - \epsilon_1)p}{1 - \epsilon_0^2} \pm \sqrt{\left(\frac{(\epsilon_0 - \epsilon_1)p}{1 - \epsilon_0^2}\right)^2 + \frac{(1 - \epsilon_1^2)p^2 + m^2}{1 - \epsilon_0^2}} \tag{34}$$

$$p_0^{(2\pm)} = \frac{-(\epsilon_0 - \epsilon_1)p}{1 - \epsilon_0^2} \pm \sqrt{\left(\frac{(\epsilon_0 - \epsilon_1)p}{1 - \epsilon_0^2}\right)^2 + \frac{(1 - \epsilon_1^2)p^2 + m^2}{1 - \epsilon_0^2}} \tag{35}$$

There are 4 mass shells (two future and two past directing) which touch at $p_{\hat{\mu}} = 0$, that is $p_0^{(1\pm)}(p_{\hat{\mu}} = 0) = p_0^{(2\pm)}(p_{\hat{\mu}} = 0) =: \pm m_0 = \pm m/\sqrt{1 - \epsilon_0^2}$. Because of our WKB ansatz $p_0$ is real and because we proved that $\epsilon_0$ has to be real too, we can conclude that also $m$ and therefore $m_0$ must be real. $m_0$ may be called the rest mass of our GDE. It is the only mass parameter which may be measurable.

The splitting of the mass shell $\Delta p_0$ for positive energy is then given in linear order of the parameters $\epsilon_0$ and $\epsilon_1$ by

$$\Delta p_0 := p_0^{(1+)} - p_0^{(2+)} = 2\,(\epsilon_0 - \epsilon_1)\,p + \text{ terms quadratic in } \epsilon_0, \epsilon_1 \tag{36}$$

In the non-relativistic limit we get $\Delta p_0 = 2\epsilon p$ where we defined $\epsilon = \epsilon_0 - \epsilon_1$. Again we recover the difference of the parameters $\epsilon_0$ and $\epsilon_1$ as the entity causing the main effect. (36) also shows that relativistic corrections, i.e. terms of the form $p^2/(mc)^2$, are correlated with squares of $\epsilon$. Therefore, deviations from Lorentz invariance would be very difficult to measure in the relativistiv domain.

# 4 Experimental test by matter wave interferometry

The theoretical scheme developed in the previous sections can be used to discuss experimental consequences of the mass shell and related null cone splitting (see fig.2) resp. LNI. Thereby we have derived the simplest non-trivial generalisation of the Dirac equation depending on the effective perturbation $\epsilon$. From this equation we calculate for experimental applications in the low velocity domain the interaction Hamiltonian in the corresponding non-relativistic limit. This gives us the possibility to come to numerical values for the effective perturbation $\epsilon$. For this purpose there are several methods. At first we are using the theoretical results of the last section to give estimates for the effective perturbation. We discuss this for atomic beam interferometry and compare the result with other estimates from hyperfine splitting in the hydrogen atom and the Phillips experiment.

## 4.1 Generalised Pauli equation

From (30) we can derive the respective Hamiltonian for the non-relativistic domain. For doing so we calculate $p_0^2 a = H^2 a$ from (30), take the square root and develop it with respect to $p$.



Then we get to first order in $p$ and the disturbing parameter $\epsilon$

$$H = \frac{p^2}{2m} + H_{\text{int}}, \qquad H_{\text{int}} = \epsilon \vec{S}\vec{p}. \tag{37}$$

Here we have rescaled for convenience $p \mapsto \left(1 - \frac{\epsilon}{2}\right) p$, which is of no significance.

Equation (37) demonstrates that according to the orientation of the spin with respect to the given 3-momentum, the particles momentum is restricted to one or the other mass shell: For $\vec{S}$ parallel to given $\vec{p}$ the energy is given by $\frac{p^2}{2m} + \epsilon S p$ while for $\vec{S}$ anti-parallel to (the same given) $\vec{p}$ the energy is $\frac{p^2}{2m} - \epsilon S p$. Therefore, a spin-flip results in a change in the energy of the particle although the particles have the same 3-momentum. This effect leads to the splitting of the mass shells. (The analogue of this effect in crystal optics is known as double breaking.)

While in the usual Dirac theory relative velocities are observer dependent and the effects induced by them can therefrore be transformed away, in our model theory the splitting of the null cones fixes a preferred frame which enables one to introduce the notion of absolute velocities. This is the reason why a pure velocity effect can play a physical role.

Below we give possible estimates for the difference of the coefficients $\epsilon$. Accordingly, this difference can be set zero to very good accuracy. In this case, the two mass shells coincide according to (34) and (35). The respective dispersion relation reads $p_0 = \pm\sqrt{p^2 + m_0^2}$ with a rescaled mass $m_0$.

## 4.2 Phase shift

We want to describe the outcome of a matter wave interferometer experiment. Thereby we assume that the interferometer is so small that the parameters $\epsilon_0$ and $\epsilon_1$ can be considered as being constant. The incoming particle beam is prepared as to consist of particles which are in a definite helicity state. This incoming beam will be splitted into two beams and, after having travelled along the interferometer paths I and II, will be recombined. In one of these two paths a *spin flip* will be performed along a definite distance $l$ corresponding to a time of flight $\Delta t$. The particles with the flipped spin live on the other mass shell thus accumulating another energy than the beam in the unflipped state. If all external influences are excluded we can calculate for the non-relativistic limit using the WKB approximation the phase shift experienced by matter waves

$$\begin{aligned} \Delta\phi &= \oint p_\mu dx^\mu \\ &= \oint p_0 dx^0 - \oint p_{\hat{\mu}} dx^{\hat{\mu}} \,. \end{aligned} \tag{38}$$

Inserting the interaction Hamiltonian of (37) we get finally

$$\begin{aligned}\Delta\phi &= \oint p_0 dt = \oint H_{\rm int} dt \\ &= \int_{\rm II} p_0^{(2+)} dt - \int_{\rm I} p_0^{(1+)} dt \\ &= 2\epsilon p \Delta t \\ &= 2\epsilon \frac{l}{\lambda_c} \end{aligned} \tag{39}$$

with the Compton wavelength $\lambda_c := \hbar/mc$ of the particles used. Note that the two beams must not travel along different paths in 3-space inside the device. Therefore it may be possible to have one beam of particles and to make a spin flip for half of the particles. Then the interference pattern may vary with $l$.

We emphasize that the final result is independent of any geometrical notion, because in (39) only the ratio of two lengths occurc. One only needs to know the length $l$ in terms of the given Compton wavelength $\lambda_c$. This is important for our approach because in LNI model theories there is no metric available in principle.

### 4.3 Expected result

At first we note that the matter wave experiment described above is not performed till now. Nevertheless, such experiments can be done and we assume in the following that the outcome of such experiments will be negative, i.e., they will not confirm any splitting of the mass shell or the null cone. Then, with the specifications of interferometer apparatus, estimates for the validity of LLI can be given by finding upper limits for the parameter $\epsilon$. Thereby the phase shift is given by (39).

At first we give estimates for the standard neutron interferometer. If this phase shift does not appear, then this effect can be at best of the order of the accuracy of the apparatus used. For the neutron interferometry we find from (39) with $\lambda_c = 10^{-15}$ m and $l = 10^{-1}$ m a phase shift as depending from the perturbation parameters $\delta\Phi \approx \epsilon 10^{14}$. Together with the accuracy $10^{-3}\pi$ for the Bonse-Hart neutron interferometer this gives us the limitation for the perturbations of the order of magnitude $|\epsilon| < 10^{-17}$.

As can be seen by inspection of (39) the velocity of the interfering particles has no influence on the phase shift. The only parameters determining the phase shift are the mass and spin of the particles as well as the length of the device. Therefore it is favourable to have particles with large mass and spin travelling through a large interferometer. Consequently, atomic beam interferometry is most appropriate to perform such tests of LNI.

It follows that for an atomic beam interferometer the estimate obtained above for neutron interferometry can be improved by several orders of magnitude. This may be achieved by means of the following: (i) the mass of an atom is of order 10 larger giving a Compton wavelength of the order $\lambda_c = 10^{-16}$ m, (ii) the length of the paths of the atoms may be of the order 1 m as



is e.g. for the atom beam interferometers with mechanical beam splitters ([19], [20]) and (iii) the accuracy of measuring phase shifts is $\approx 10^{-3}\pi$, which may increase in future. Therefore we estimate for a null experiment of the above type with atoms for the perturbation parameter to be

$$|\epsilon| < 10^{-19}. \tag{40}$$

The results obtained above are calculated for the preferred frame defined by the isotropy of the null cones. Such a preferred frame can be provided typically by some cosmic preferred frame, by some galactical mass distributions, or similar phenomena. On the other hand, the experiments done on the earth are most probably not done in the preferred frame. Therefore it seems to be necessary to describe these experimental results for other reference frames. However, because the splitting of the null cone is expected to be very small, Special Relativity is at least almost the correct theory. Therefore any result obtained in another reference frames would to first order in the perturbation $\epsilon$ be correlated with the preferred frame result by an unperturbed Lorentz transformation. Since all relative velocities are small as compared with the velocity of light, we can neglect these transformations. Therefore, in the non-relativistic limit, we can use predictions obtained in our preferred reference frame to describe outcomes of experiments done on the earth.

### 4.4 Comparison with alternative estimates

Other effects may also be used to give estimates for the perturbation parameters. So we find an additional hyperfine splitting of the energy levels of the hydrogen atom given by

$$E_{n,k} = m\left[1 + \frac{\alpha^2}{(n+k)^2}\right]^{-\frac{1}{2}} - \frac{m}{2}\left[1 + \frac{\alpha^2}{(n+k)^2}\right]^{-\frac{3}{2}} \frac{\alpha^4(1\pm\epsilon)^2}{k(n+k)^3} + \dots \tag{41}$$

In this case we get [21] $|\epsilon| < 10^{-8}$. Myonic atoms do not lead to a stronger limitation. This shows that the hydrogen atom is not as such a sensitive indicator for deviations from the Lorentz invariant Dirac theory as widly believed. Therefore it is meaningful to look for further experimental consequences, which give us stronger upper limits for the perturbations.

A famous experiment giving strong estimates for LNI effects was carried out by Phillips [16]. This experiment determines the daily variation of the torque acting on a ferromagnet hanging on a string. A spin-velocity coupling as predicted by the GDE model could lead to an additional torque depending on the polarisation direction of the magnet. In this way the existence of a preferred reference frame is examined, in which the velocity of the earth $v$ is connected with the spin of the electrons via a coupling term described by (37). The experiment determines the energy splitting of the two different spin states. If we insert into (37) the values for the electron mass and the velocity of the earth on its path arround the sun ($v$ = 30 km/sec) we find $\Delta E = \epsilon 10^{-17}$ J. The experimental result gives $\Delta E = 7 \cdot 10^{-35}$ J leading to $|\,\epsilon\,| < 10^{-18}$.

## 5 CONCLUSIONS

In this article we develloped a model theory based on a generalisation of the Dirac equation leading to a violation of local Lorentz invariance (LLI). This breaking of LLI is related to the fact that the generalised Dirac matrices do not fulfill any Clifford algebra. Using physically meaningful requirements like conservation of helicity and isotropy of the null cones, we reduced the most general violation of LLI to a minimal nontrivial model. This results in the non-relativistic limit in a special spin-momentum coupling leading to a splitting of the mass shells and consequently of the null cones.

This spin-momentum coupling can be most suitably tested with atomic beam interferometry using spin flip devices. Our model would lead to a phase shift proportional to the parameter $\epsilon$ characterising the splitting of the null cone. Assuming a negative outcome of atomic beam interference experiments and taking into considertion the accuracy of the respective apparatus then gives upper limits for the parameter characterising the violation of LLI. The great and increasing accuracy of atomic beam interferometers makes it very desireable to perform such experiments because this would lead to improved limitations of LLI violations: $|\epsilon| < 10^{-19}$.

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**Figure captions**

Figure 1: Possible shapes of the null cones defined by the characteristic polynomial of the GDE.

Figure 2: Splitting of mass shell and null cone for the simplest non-trivial generalisation of the Dirac equation.

Fig. 1

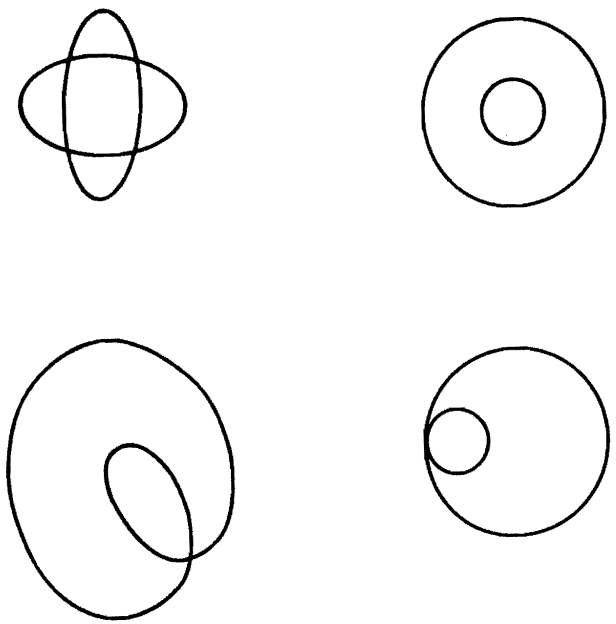

Figure 2: