Fermilab Library
0 1160 0034824 7

KOBE-TH-93-09

KOBE-TH-93-09

QUANTUM POTENTIAL INTERPRETATION AND A THIRD-QUANTIZED UNIVERSE

Tsutomu Horiguchi

The Graduate School of Science and Technology

Kobe University

Nada, Kobe 657, Japan.

(October 1993)

Abstract

We apply Bohm's quantum potential interpretation to a third-quantized universe theory. Using this interpretation, we obtain trajectories of the universe field in a third-quantized de Sitter minisuperspace model.

19

MAR 09 1994

Typeset using REVTEX

I. INTRODUCTION

The wave function of the universe is defined on the superspace (the space of all three-metrics and matter-field configurations). The wave function ψ is a solution to the Wheeler-DeWitt (WDW) equation (second quantization of gravity), $H\psi = 0$ [1]. The WDW equation has a serious difficulty in its probabilistic interpretation. Because the WDW equation is, in general, a hyperbolic second-order differential equation, there is no conserved positive definite probability density as in the case of the Klein-Gordon equation. It is usually thought that this difficulty makes it more appropriate to regard ψ as a quantum field rather than a state amplitude. This procedure is so-called third quantization [2]. In this formalism, negative probability does not arise formally. However, if we try to apply the "Copenhagen" interpretation to third-quantized universe theory, we encounter the following difficulties. Firstly, the conventional "Copenhagen" formalism postulates a radical dualism between an observer and a system, and assumes a possible division of the world into "observer" and "observed". However, in quantum theory of the universe, there can be no fundamental division into observer and observed. Also, measurement and observer can not be fundamental at early universes, since both can not exist at the early stage of the evolution of the universe. Secondly, in the "Copenhagen" scenarios, measurements are made at a fixed time, and the "collapse" of a wave function (state vector) requires a description involving time evolution. It is usually not easy to introduce "time" to the WDW equation, since the WDW equation is a "timeless" equation. It is difficult to see how the collapse of a state vector can be consistent with the WDW equation. Thus we think that the conventional "Copenhagen" interpretation is inapplicable to third quantization (or quantum cosmology) [1].

In 1952 Bohm proposed an alternative interpretation of quantum theory [4], which is based on the notion of a particle acted on not only by classical potential, but also by "quan-

[1] For discussion about an interpretational problem about the wave function of the universe, see, for example, Ref. [3]

tum potential". The quantum potential is determined by a solution to Schrödinger equation (see eq.(4)). In this quantum potential interpretation of quantum theory, for example, an electron is constituted of a particle following continuous and well-defined trajectories and a wave satisfying Schrödinger equation. Both particle and wave are assumed to be objectively real wether they are observed or not. In quantum potential interpretation, it was explained that all sorts of quantum processes, such as transition between states, quantum interference in the two-slit experiment are able to take place, in principle, without the need for a observer and also for collapse of a wave function [5]. Therefore, we think that Bohm's quantum potential interpretation is very suitable for quantum theory of the universe. This interpretation was applied to the WDW equation (second quantization of gravity) [7].

In this paper, we apply quantum potential interpretation to third-quantized universe theory. We study quantum potential interpretation of the third-quantized de Sitter minisuperspace model by using a Schrödinger picture in the field representation. Comparison with other interpretations will not be discussed in this paper.

The paper is organized as follows. In Sec.II we give a brief review of Bohm's quantum potential interpretation. In Sec.III we apply quantum potential interpretation to third-quantized de Sitter minisuperspace model. In Sec.IV we show that there is an interesting relation between the universe field and the quantum potential. We summarize and conclude in Sec.V.

II. BOHM'S QUANTUM POTENTIAL INTERPRETATION

First we review some main features of Bohm's quantum potential interpretation. The Schrödinger equation for a single particle is

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2m}\nabla^2\Psi + V(\mathbf{x})\Psi, \quad (\hbar = 1), \tag{1}$$

where m is the inertial mass and $V(\mathbf{x})$ is the potential energy due to an external classical potential. We write the wave function in the form $\Psi = R\exp(iS)$, where R and S are real. Then Schrödinger equation reduces to the following two equations for the fields R and S:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0, \tag{2}$$

$$\frac{\partial P}{\partial t} + \nabla\cdot(P\frac{\nabla S}{m}) = 0, \quad \text{with } P = R^2, \tag{3}$$

$$Q = \frac{-1}{2m}\frac{\nabla^2 R}{R}. \tag{4}$$

These two equations come from the real and the imaginary part of eq.(1), respectively. We note that the quantum potential Q is not changed when we multiply the field Ψ by an arbitrary constant. Quantum potential interpretation of the quantum theory is based on the following assumptions [4]. (i) The Ψ-field satisfies Schrödinger equation. (ii) A quantum-mechanical system, such as an electron, consists of a particle with a definite coordinate, which is a well defined continuous function of the time. Its velocity is assumed to be given by $\mathbf{v}(\mathbf{x},t) = \frac{\nabla S}{m}$. (iii) $P = R^2$ is assumed to represent the probability distribution of particles in a statistical ensemble of well-defined trajectories. This probability density P satisfies a continuity equation $\frac{\partial P}{\partial t} + \nabla\cdot(P\mathbf{v}) = 0$, which is obtained from eq.(3). The trajectories of the particle are orthogonal to the surfaces $S =$ constant and may be calculated by integrating the differential equation $\dot{\mathbf{x}} = \frac{\nabla S}{m}|_{\mathbf{x}=\mathbf{x}(t)}$. Eq.(2) resembles the Hamilton-Jacobi equation except for an additional term Q. This suggests that we may regard the electron as a particle acted on, not only by the classical potential V, but also by the quantum potential Q. The equation of motion of the particle acted on by classical potential and quantum potential is given by [4]

$$\frac{d}{dt}(m\dot{\mathbf{x}}) = -\nabla(V+Q)|_{\mathbf{x}=\mathbf{x}(t)}. \tag{5}$$

This equation can be obtained by applying the operator ∇ to eq.(2) and by using

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \dot{\mathbf{x}}\cdot\nabla. \tag{6}$$

Usually, in quantum potential interpretation, trajectories of the particle are obtained by solving the differential equation $\dot{\mathbf{x}} = \frac{\nabla S}{m}|_{\mathbf{x}=\mathbf{x}(t)}$, rather than by solving eq.(5) [6]. We will follow this idea.

III. APPLICATION TO THIRD-QUANTIZED DE SITTER MINISUPERSPACE MODEL

We apply the quantum potential interpretation to third-quantized universe theory. The Einstein action with a cosmological constant is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R^{(4)} - 2\Lambda) + \text{boundary terms}, \tag{7}$$

where $R^{(4)}$ is a scalar curvature, G is the gravitational constant, and Λ is a cosmological constant. In inflationary scenario [8], it is usually believed that our universe passed through a de Sitter phase of exponential expansion. So, the investigation of the de Sitter minisuperspace model would be of importance physically [9] [10] [11] [12]. The de Sitter minisuperspace model is described by the closed Robertson-Walker metric

$$ds^2 = \frac{2G}{3\pi}[-N^2dt^2 + a(t)^2 d\Omega_3^2], \tag{8}$$

where $a(t)$ is the scale factor and $d\Omega_3^2$ is the metric on the unit three-sphere. The Einstein action with (rescaled) cosmological constant for this metric is

$$S = \frac{1}{2}\int dtN[-\frac{a\dot{a}^2}{N^2} + a - \Lambda a^3]. \tag{9}$$

By varying the action S with respect to the lapse function N, we obtain the Hamiltonian constraint

$$\mathrm{H} = \frac{1}{2}(-\frac{p_a^2}{a} - a + \Lambda a^3) \approx 0, \tag{10}$$

where p_a is a conjugate momentum to a. In the conventional quantization prescription, this constraint is replaced by an equation for the quantum state:

$$\{\frac{\partial^2}{\partial a^2} + \frac{p}{a}\frac{\partial}{\partial a} - a^2[1 - a^2\Lambda]\}\psi(a) = 0, \tag{11}$$

where the parameter p represents the factor-ordering ambiguity. Eq.(11) is the WDW equation. This equation is also expressed, for the new variable $\sigma = a^2$, as

$$[\frac{d^2}{d\sigma^2} + U(\sigma)]\psi(\sigma) = 0, \tag{12}$$

$$U(\sigma) = -\frac{1}{4}(1 - \Lambda\sigma), \tag{13}$$

where we have chosen $p = -1$. In third quantization, $\psi(\sigma)$ is regarded as a quantum field in the minisuperspace. The third-quantized action to yield the WDW eq.(12) is

$$\begin{aligned} S &= \frac{1}{2}\int d\sigma[(\frac{\partial\psi}{\partial\sigma})^2 - U(\sigma)\psi^2] \\ &\equiv \int d\sigma L(\sigma). \end{aligned} \tag{14}$$

The canonical momentum is given by

$$\pi(\sigma) = \frac{\partial L}{\partial(\frac{\partial\psi}{\partial\sigma})} = \frac{\partial\psi}{\partial\sigma}. \tag{15}$$

The Hamiltonian is constructed as

$$H = \frac{1}{2}[\pi^2(\sigma) + U(\sigma)\psi^2]. \tag{16}$$

To quantize this system (16), we impose the equal "time" commutation relations

$$[\hat{\psi}(\sigma), \hat{\pi}(\sigma)] = i. \tag{17}$$

We work in the Schrödinger picture with the substitution $\hat{\psi}(\sigma) \to \psi_c$, $\hat{\pi}(\sigma) \to -i\frac{d}{d\psi_c}$, where ψ_c is a real c-number. Thus we obtain the Schrödinger equation

$$i\frac{\partial}{\partial\sigma}\Psi(\sigma, \psi_c) = \hat{H}\Psi(\sigma, \psi_c), \tag{18}$$

$$\hat{H} = \frac{1}{2}[-\frac{\partial^2}{\partial\psi_c^2} + U(\sigma)\psi_c^2]. \tag{19}$$

$\Psi(\sigma, \psi_c)$ is a quantum mechanical state in third quantization. The inner product of two functionals $\Psi_1(\sigma, \beta)$ and $\Psi_2(\sigma, \beta)$ is defined by

$$< \Psi_1|\Psi_2 >_\sigma = \int d\psi_c \Psi_1(\sigma, \psi_c)\Psi_2^*(\sigma, \psi_c). \tag{20}$$

To arrive at quantum potential interpretation of the third-quantized-universe field system, we write Ψ in a polar decomposition as $\Psi(\sigma,\psi_c) = R(\sigma,\psi_c)\exp\{iS(\sigma,\psi_c)\}$, in terms of which eq.(18) can be rewritten as

$$\frac{\partial S}{\partial \sigma} + \frac{1}{2}(\frac{\partial S}{\partial \psi_c})^2 + \frac{1}{2}U(\sigma)\psi_c^2 + Q_{3G} = 0, \tag{21}$$

$$\frac{\partial P}{\partial \sigma} + \frac{\partial}{\partial \psi_c}(P\frac{\partial S}{\partial \psi_c}) = 0, \quad \text{with } P = R^2. \tag{22}$$

These two equations come from the real and the imaginary part of eq.(18), respectively, and the gravitational quantum potential in third quantization is

$$Q_{3G}(\sigma,\psi_c) = -\frac{1}{2R}\frac{\partial^2 R}{\partial \psi_c^2}. \tag{23}$$

Following quantum potential interpretation, we identify $\frac{\partial S}{\partial \psi_c}$ with $\frac{d\psi_c}{d\sigma}$. Then eq.(21) and eq.(22) become

$$\frac{\partial S}{\partial \sigma} + \frac{1}{2}(\frac{d\psi_c}{d\sigma})^2 + \frac{1}{2}U(\sigma)\psi_c^2 + Q_{3G} = 0, \tag{24}$$

$$\frac{\partial P}{\partial \sigma} + \frac{\partial}{\partial \psi_c}(P\frac{d\psi_c}{d\sigma}) = 0, \quad \text{with } P = R^2. \tag{25}$$

Eq.(24) is the Hamilton-Jacobi equation modified by the quantum potential(23). Clearly, ψ_c plays the role of the space $\mathbf{x}$ in the particle Schrödinger equation. Eq.(25) is a continuity equation, which implies conservation of the probability density $P = R^2$. $P = R^2$ is assumed to be the probability distribution of the trajectories of the universe field $\psi_c(\sigma)$ on the configuration space $\psi_c(\sigma)$ for all σ. When we differentiate eq.(21) by ψ_c, we get

$$[\frac{d^2}{d\sigma^2} + U(\sigma)]\psi_c = -\frac{\partial Q_{3G}}{\partial \psi_c}|_{\psi_c=\psi_c(\sigma)}, \tag{26}$$

where we have identified $\frac{\partial S}{\partial \psi_c}$ with $\psi_{c,\sigma}$ and

$$\frac{d}{d\sigma} = \frac{\partial}{\partial \sigma} + \psi_{c,\sigma}\frac{\partial}{\partial \psi_c}. \tag{27}$$

Eq.(26) is analogous to the modified Newton's second law (5). The modified WDW equation (26) should be compared with the usual WDW equation (12). The " gravitational quantum

force" term on the right-hand side of eq.(26) is responsible for all the characteristic effects of third quantized gravity.

Our universe (our space-time) is now essentially classical. Therefore we try to find coherent states as solutions to the Schrödinger equation(18) by using the following Gaussian ansatz.

$$\Psi(\sigma,\psi_c) = C(\sigma)\exp(-\frac{1}{2}A(\sigma)[\psi_c - \eta(\sigma)]^2 + iP(\sigma)[\psi_c - \eta(\sigma)]), \tag{28}$$

$$A(\sigma) = D(\sigma) + iI(\sigma), \tag{29}$$

where the real functions $D(\sigma)$, $I(\sigma)$, $P(\sigma)$ and $\eta(\sigma)$ have to be determined from eq.(18). $C(\sigma)$ is a normalization of the wave function. When we compare eq.(28) with $\Psi = Re^{iS}$, we get

$$R = |C(\sigma)|\exp(-\frac{1}{2}D(\sigma)[\psi_c - \eta(\sigma)]^2), \tag{30}$$

$$S = -\frac{1}{2}I(\sigma)[\psi_c - \eta(\sigma)]^2 + P(\sigma)[\psi_c - \eta(\sigma)] + \Omega(\sigma). \tag{31}$$

where $C(\sigma) = |C(\sigma)|\exp(i\Omega(\sigma))$. $\Omega(\sigma)$ is a real function. Thus, from eq.(23), we have

$$Q_{3G} = \frac{1}{2}D(\sigma) - \frac{1}{2}D^2(\sigma)[\psi_c - \eta(\sigma)]^2, \tag{32}$$

$$\frac{\partial Q_{3G}}{\partial \psi_c} = -D^2(\sigma)[\psi_c - \eta(\sigma)], \tag{33}$$

and

$$\frac{d\psi_c}{d\sigma} = \frac{\partial S}{\partial \psi_c} = -I(\sigma)[\psi_c - \eta(\sigma)] + P(\sigma), \tag{34}$$

where we have used the assumption $\frac{d\psi_c}{d\sigma} = \frac{\partial S}{\partial \psi_c}|_{\psi_c=\psi_c(\sigma)}$ in quantum potential interpretation. Trajectories of the universe field $\psi_c(\sigma)$ on the configuration space $\psi_c(\sigma)$ for all σ can be calculated by integrating the guidance formula (34). This formula is analogous to $m\dot{\mathbf{x}} = \nabla S$. The general solution to eq.(34) is given by

$$\psi_c(\sigma) = \exp(-\int I(\sigma)d\sigma)\{\int (I(\sigma)\eta(\sigma) + P(\sigma))\exp(\int I(\sigma)d\sigma)d\sigma + \tilde{c}\}, \tag{35}$$

where $\tilde{c}$ is an integration constant.

Substituting the ansatz (28) into eq.(18), we get the following equations

$$-iA_{,\sigma} = -A^2 + U, \tag{36}$$

$$i(A_{,\sigma}\eta + A\eta_{,\sigma}) - P_{,\sigma} = A(A\eta + iP), \tag{37}$$

$$-\frac{i}{2}A_{,\sigma}\eta^2 - iA\eta\eta_{,\sigma} + P_{,\sigma}\eta + P\eta_{,\sigma} + i\frac{|C|_{,\sigma}}{|C|} - \Omega_{,\sigma} = \frac{1}{2}A - \frac{1}{2}(A\eta + iP)^2. \tag{38}$$

From the real and the imaginary part of eq.(37), we get

$$\eta_{,\sigma} = -\frac{D_{,\sigma}}{D}\eta + 2I\eta + P, \tag{39}$$

$$P_{,\sigma} = (-I_{,\sigma} + \frac{ID_{,\sigma}}{D} - I^2 - D^2)\eta. \tag{40}$$

From the real and the imaginary part of eq.(36), we have

$$I_{,\sigma} = -D^2 + I^2 + U, \tag{41}$$

$$D_{,\sigma} = 2DI. \tag{42}$$

By using eq.(41) and eq.(42), eq.(39) and eq.(40) become

$$\eta_{,\sigma} = P, \tag{43}$$

$$P_{,\sigma} = -U\eta. \tag{44}$$

Thus, from eq.(43) and eq.(44), we have the WDW equation

$$[\frac{d^2}{d\sigma^2} + U]\eta(\sigma) = 0. \tag{45}$$

The general solution to eq.(45) can be written as

$$\eta(\sigma) = c_1\mathrm{Ai}(z) + c_2\mathrm{Bi}(z), \tag{46}$$

where $z = (4\Lambda^2)^{-1/3}(1 - \Lambda\sigma)$, c_1 and c_2 are real constants (we have assumed that $\eta(\sigma)$ is real), and $\mathrm{Ai}(z)$, $\mathrm{Bi}(z)$ are Airy functions [14]. If we choose Hartle-Hawking boundary condition [9], the solution is given by [12]

$$\eta(\sigma) = c_1\mathrm{Ai}(z). \tag{47}$$

From the real part of eq.(38), $\Omega(\sigma)$ is obtained as:

$$\Omega(\sigma) = \int^{\sigma} d\sigma\{-\frac{1}{2}U\eta^2 + \frac{1}{2}(\eta_{,\sigma})^2 - \frac{D}{2}\} + \mathrm{const}. \tag{48}$$

From eq.(42), the imaginary part of eq.(38) is automatically satisfied. To get the concrete expression of the solution to eq.(34), we first calculate the concrete expression for $I(\sigma)$. In eq.(36), A is expressed as

$$A = -i\frac{d}{d\sigma}\ln u, \tag{49}$$

where $u(\sigma)$ is any solution to the WDW equation (12). The general solution to eq.(12) can be written as (up to an irrelevant constant)

$$u(\sigma) = \mathrm{Ai}(z) + r\mathrm{Bi}(z), \tag{50}$$

where $z = (4\Lambda^2)^{-1/3}(1 - \Lambda\sigma)$, and r is a complex constant. Thus the general solution to eq.(36) is

$$A(\sigma) = i\Lambda(4\Lambda^2)^{-1/3}\frac{\mathrm{Ai}'(z) + r\mathrm{Bi}'(z)}{\mathrm{Ai}(z) + r\mathrm{Bi}(z)}, \tag{51}$$

where dash stands for the differentiation with respect to z. The real part and the imaginary part of A are

$$D = \mathrm{Re}A = -\frac{1}{\pi}\Lambda(4\Lambda^2)^{-1/3}\frac{\mathrm{Im}\ r}{|\mathrm{Ai} + r\mathrm{Bi}|^2}, \tag{52}$$

$$I = \mathrm{Im}A = \Lambda(4\Lambda^2)^{-1/3}\frac{\mathrm{Ai}'\mathrm{Ai} + |r|^2\mathrm{Bi}'\mathrm{Bi} + (\mathrm{Re}\ r)(\mathrm{Bi}'\mathrm{Ai} + \mathrm{Ai}'\mathrm{Bi})}{|\mathrm{Ai} + r\mathrm{Bi}|^2}, \tag{53}$$

where we have used the Wronskian $\mathrm{Ai}(z)\mathrm{Bi}'(z)-\mathrm{Ai}'(z)\mathrm{Bi}(z)=1/\pi$. We choose $r=-i$, then fluctuations of the third-quantized universe field measured by the Heisenberg uncertainty converges on its minimum rapidly in the course of cosmic expansion [13]. This coincides with our experience that space-time in which we live is now essentially classical. The asymptotic forms of the Airy functions at large value of $z(z\to+\infty)$ are [14]:

$$Ai(z)\sim\frac{1}{2}\pi^{-\frac{1}{2}}z^{-\frac{1}{4}}\exp(-\zeta), \tag{54}$$

$$Bi(z)\sim\pi^{-\frac{1}{2}}z^{-\frac{1}{4}}\exp(\zeta), \tag{55}$$

$$Ai(-z)\sim\pi^{-\frac{1}{2}}z^{-\frac{1}{4}}\sin(\zeta+\pi/4), \tag{56}$$

$$Bi(-z)\sim\pi^{-\frac{1}{2}}z^{-\frac{1}{4}}\cos(\zeta+\pi/4), \tag{57}$$

where $\zeta=\frac{2}{3}z^{\frac{3}{2}}$. We assume that $0<\Lambda\ll 1$ in Planck unit. Using these asymptotic forms, (52) and (53) for small scales ($\sigma\to 0$) and large scales ($\sigma\to\infty$) are given by

$$D=\frac{1}{2}\exp(-\frac{2}{3\Lambda})\ll 1 \quad \text{for } \sigma\to 0, \tag{58}$$

$$D=\frac{1}{2}(\Lambda\sigma-1)^{1/2}\gg 1 \quad \text{for } \sigma\to\infty, \tag{59}$$

$$I(\sigma)\to\frac{1}{2} \quad \text{for } \sigma\to 0, \tag{60}$$

$$I(\sigma)\to 0 \quad \text{for } \sigma\to\infty. \tag{61}$$

From (61) and the guidance formula (34), it turns out that third quantization effect becomes small at large scales. In fact, from (61) and eq.(34), we have

$$\frac{d\psi_c}{d\sigma}=\frac{d\eta}{d\sigma} \quad \text{for } \sigma\to\infty. \tag{62}$$

Thus we get

$$\psi_c(\sigma)=\eta(\sigma)+c_0 \quad \text{for } \sigma\to\infty, \tag{63}$$

where c_0 is an integration constant. From (30) and (63), the probability distribution of trajectories of the universe field $\psi_c(\sigma)$ at large scales is given by

$$P=R^2=|C|^2\exp(-D(\sigma)c_0^2) \quad \text{for } \sigma\to\infty. \tag{64}$$

Since $D(\sigma)\to\infty$ for $\sigma\to\infty$ (see (59)), $c_0\simeq 0$ is most probable. Thus the probability distribution of the universe field $\psi_c(\sigma)$ at large scales is strongly peaked at Hartle-Hawking wave function $\eta(\sigma)$. On the other hand, from (35) and (60), the behavior of the universe field $\psi_c(\sigma)$ at small scales is given by

$$\psi_c(\sigma)=\eta(\sigma)+\tilde{c}\exp(-\frac{1}{2}\sigma) \quad \text{for } \sigma\to 0, \tag{65}$$

where we have assumed that $0<\Lambda\ll 1$ in Planck unit. We note that $\tilde{c}\neq c_0$. Substituting eq.(47) and eq.(53) to eq.(34), we have solved eq.(34) numerically (see Fig.1). Fig.1 and (65) show that, in quantum potential interpretation, the universe field $\psi_c(\sigma)$ at small scales($\sigma\to 0$) deviates from Hartle-Hawking wave function.

IV. THE RELATION BETWEEN THE UNIVERSE FIELD AND THE QUANTUM POTENTIAL

Next, we want to show that there is an interesting relation between ψ_c and Q_{3G}. By applying the Schwartz inequality:

$$\int|f|^2dx\int|g|^2dx\geq|\int f^*gdx|^2, \tag{66}$$

where f, g are complex functions and integrals are assumed to converge, we have the following inequality

$$\begin{aligned}&-\int(\psi_c-<\psi_c>)^2R^2d\psi_c\int R\frac{\partial^2R}{\partial\psi_c^2}d\psi_c\\&=\int(\psi_c-<\psi_c>)^2R^2d\psi_c\int(\frac{\partial R}{\partial\psi_c})^2d\psi_c\end{aligned}$$

$$\geq |\int(\psi_c - <\psi_c>)R\frac{\partial R}{\partial\psi_c}d\psi_c|^2$$
$$= \frac{1}{4}, \tag{67}$$

where we have used $\int R^2 d\psi_c = \int |\Psi|^2 d\psi_c = 1$. Thus we get

$$(\Delta\psi_c)^2 \int (\frac{\partial R}{\partial\psi_c})^2 d\psi_c \geq \frac{1}{4}, \tag{68}$$

where $(\Delta\psi_c)^2 \equiv \int(\psi_c - <\psi_c>)^2 R^2 d\psi_c$. Using (23), we have the desired relation:

$$(\Delta\psi_c)^2 < Q_{3G} > \geq \frac{1}{8}. \tag{69}$$

We shall calculate the concrete expression of $< Q_{3G} >$ under the Gaussian ansatz (28). Using $\frac{\partial R}{\partial\psi_c} = -D[\psi_c - \eta]R$, we have

$$\begin{aligned} < Q_{3G} > &= \frac{1}{2}\int_{-\infty}^{\infty} d\psi_c (\frac{\partial R}{\partial\psi_c})^2 \\ &= \frac{1}{2}D^2 \int_{-\infty}^{\infty} d\psi_c [\psi_c - \eta]^2 R^2 \\ &= \frac{1}{2}D^2[<\psi_c^2> - 2\eta <\psi_c> + \eta^2]. \end{aligned} \tag{70}$$

From $<\psi_c^2> = \frac{1}{2D} + \eta^2$ and $<\psi_c> = \eta$, we get

$$< Q_{3G} > = \frac{1}{4}D. \tag{71}$$

Thus, under the Gaussian ansatz (28), we have

$$(\Delta\psi_c)^2 < Q_{3G} > = \frac{1}{8}. \tag{72}$$

In the case in which $r = -i$, from (58) and (59), the asymptotic behavior of $< Q_{3G} >$ is given by

$$< Q_{3G} > = \frac{1}{8}\exp(-\frac{2}{3\Lambda}) \ll 1 \quad \text{for } \sigma \to 0, \tag{73}$$

$$< Q_{3G} > = \frac{1}{8}(\Lambda\sigma - 1)^{1/2} \gg 1 \quad \text{for } \sigma \to \infty. \tag{74}$$

From $(\Delta\psi_c)^2 = \frac{1}{2D}$, we have

$$(\Delta\psi_c)^2 = \exp(\frac{2}{3\Lambda}) \gg 1 \quad \text{for } \sigma \to 0, \tag{75}$$

$$(\Delta\psi_c)^2 = (\Lambda\sigma - 1)^{-1/2} \ll 1 \quad \text{for } \sigma \to \infty. \tag{76}$$

(75) and (76) show that the uncertainty of the universe field become small for large scales($\sigma \to \infty$) and become large for small scales ($\sigma \to 0$) (For detailed discussion about uncertainty relation in a third-quantized universe, see Ref.[13]).

V. SUMMARY AND DISCUSSION

We have studied quantum potential interpretation of third-quantized de Sitter minisuperspace model by using a Schrödinger picture in the field representation. We got the modified Wheeler-DeWitt equation (26) with a "gravitational quantum force" term. This "gravitational quantum force" term represents all the characteristic effects of third-quantized gravity. In quantum potential interpretation, trajectories of the universe field $\psi_c(\sigma)$ on the configuration space $\psi_c(\sigma)$ for all σ are obtained by integrating the guidance formula (34). We had the general solution (35) to this formula under the Gaussian ansatz (28) for $\Psi(\sigma, \psi_c)$. $\Psi(\sigma, \psi_c)$ is a quantum mechanical state in third quantization. It was shown that the probability distribution of trajectories of the universe field $\psi_c(\sigma)$ at large scales is strongly peaked at Hartle-Hawking wave function. We note that the general solution (35) to the guidance formula (34) is a solution to the modified WDW equation (26)(see Appendix). We have solved eq.(34) numerically, and it was shown that the universe field at small scales deviates from Hartle-Hawking wave function. It was also shown that there is an interesting relation (69) between fluctuations of the universe field and the gravitational quantum potential in third quantization.

ACKNOWLEDGMENTS

I am thankful to members of the high energy group of Physics Department, Kobe University for valuable discussions. I also would like to thank Professor M. Ida for helpful

comments on the manuscript.

APPENDIX

In this appendix, we describe the relation between the modified WDW equation(26) and the guidance formula (34). When we differentiate eq.(34) by σ, we have

$$\frac{d^2\psi_c}{d\sigma^2} = -I_{,\sigma}[\psi_c - \eta] - I[\frac{d\psi_c}{d\sigma} - \eta_{,\sigma}] + P_{,\sigma}. \tag{77}$$

By using eq.(41), eq.(34),eq.(43), and eq.(44), eq.(77) becomes

$$\frac{d^2\psi_c}{d\sigma^2} = -U\psi_c + D^2[\psi_c - \eta]. \tag{78}$$

This equation(78) is the modified WDW equation(26). Thus the solution (35) to eq.(34) is a solution to eq.(26).

REFERENCES

[1] B. S. DeWitt, Phys. Rev. **160** (1967) 1113.

[2] V. A. Rubakov, Phys. Lett. B **214** (1988) 503; M. McGuigan, Phys. Rev. D **38** (1988) 3031; Phys. Rev. D **39** (1989) 2229; A. Hosoya and M. Morikawa, Phys. Rev. D **39** (1989) 1123; S. B. Giddings and A. Strominger, Nucl. Phys. B **321** (1989) 481; W. Fischler, I. Klebanov, J. Polchinski and L. Susskind, Nucl. Phys. B **327** (1989) 157; V. A. Rubakov and P. G. Tinyakov, Nucl. Phys. B **342** (1990) 430; M. J. Duncan, Nucl. Phys. B **361** (1991) 695. T. Horiguchi, Preprint KOBE-TH-93-02.

[3] Conceptual Problems of Quantum Gravity, ed. A. Ashtekar, J. Stachel, (The Center for Einstein Studies, Boston University, 1991); J. B. Hartle in Quantum Cosmology and Baby Universe, ed. S. Coleman, J. B. Hartle, T. Piran and S. Weinberg (World Scientific, 1991); F. J. Tipler, Phys. Rep. **137** (1986) 231; J. J. Halliwell, Phys. Rev. D **39** (1989) 2912; T. Padmanabhan, Phys. Rev. D **39** (1989) 2924; S. Wada, Mod. Phys. Lett. A **3** (1988) 645; Mod. Phys. Lett. A **3** (1988) 929.

[4] D. Bohm, Phys. Rev. **85** (1952) 166; **85** (1952) 180.

[5] D. Bohm and B. J. Hiley, Phys. Rep. **6** (1987) 323; D. Bohm, B. J. Hiley and P. N. Kaloyerou, Phys. Rep. **6** (1987) 349.

[6] C. Philippidis, C. Dewdney and B. J. Hiley, Nuovo Cimento **52 B** (1979) 15; R. D. Prosser, Int. J. Theoret. Phys. **15**, 169, 181 (1976); C. Dewdney and B. J. Hiley, Found. of Phys. **12** (1982) 27; P. R. Holland, Phys. Rep. **224** (1993) 95.

[7] J. C. Vink, Nucl. Phys. B **369** (1992) 707. T. Horiguchi, Preprint KOBE-TH-93-04; KOBE-TH-93-07.

[8] A. H. Guth, Phys. Rev. D **23** (1981) 347; K. Sato, Mon. Not. R. Astron. Soc. 195 (1981) 467; D. Kazanas, astrophys. J. 241 (1980) L 59; A. D. Linde, Phys. Lett. B **108** (1982) 389; A. Albrecht and P. Steinhadt, Phys. Rev. Lett. **48** (1982) 1220.

[9] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28** (1983) 2960.

[10] S. W. Hawking, Nucl. Phys. B **239** (1984) 257.

[11] A. Vilenkin, Phys. Rev. D **33** (1986) 3560.

[12] A. Vilenkin, Phys. Rev. D **37** (1988) 888.

[13] T. Horiguchi, Preprint KOBE-TH-93-05, to appear in Phys. Rev. D (see eq.(36) in this preprint).

[14] M. Abramowitz and I. A. Stegun, Hand book of mathematical functions (Dover, New York, 1972).

FIGURES

Fig.1 Solid curves represent numerical solutions ψ_c to the guidance formula (34), with $\Lambda = 0.1$ in Planck unit. Dashed curve represents Hartle-Hawking wave function(47).

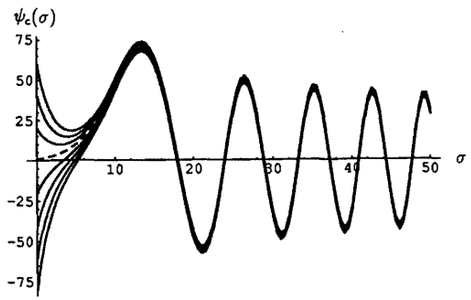

Fig.1