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# BEAM LOADING OF ACCELERATING CAVITIES AND OPTIMIZATION OF RF-GENERATORS OPERATION

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ТЕЛЕГИН Ю.Н. Нагрузка пучком ускоряющих резонаторов и оптимизация режимов работы ВЧ - генераторов: Препринт ХФТИ 92-28. Харьков: ХФТИ, 1992. - 11 с.

В рамках метода эквивалентных схем рассмотрены эффекты нагрузки пучком ускоряющих резонаторов в стационарном и переходном режимах. Показано, что в случае накопителя-растяжителя (НР) целесообразно использовать рассогласованный режим работы ВЧ-генераторов с оптимизированными значениями коэффициента связи $\beta$ и угла расстройки $\psi$. Переходные эффекты, возникающие при инжекции интенсивного сгруппированного пучка в НР, могут быть в значительной степени скомпенсированы введением программированного скачка фазы сигнала ВЧ-возбуждения.

Рис. 6, список лит. - 5 назв.

Steady-state and transient beam loading of accelerating cavities is treated using the simple equivalent circuit model. It is shown that mismatched operation of RF-generators with optimal values of the coupling factor $\beta$ and the cavity tuning angle $\phi$ is preferable for a pulse stretcher ring. Transient beam loading effects ,arising at injection of a high-intensity bunched beam into an empty ring, can be essentially reduced by applying the synthesized correction to the RF-drive signal ("phase jump" technique).

6 figs., 5 refs.

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## INTRODUCTION

The RF-system of the pulse stretcher and storage ring (PSSR) which is under design at Kharkov [1] has to support the necessary accelerating voltage parameters in different operation modes at minimal power inputs. According to these requirements the basic design of the RF-system is outlined and various operation conditions for RF-power generators are considered. For the machine with a high circulating beam current, such as PSSR is, beam loading of the accelerating cavities has to be taken into account. Beam loading effects can be treated for two basic cases:

-steady-state beam loading;

-transient beam loading.

The former is valid for routine storage ring operation and was treated by many authors (see ref. [2]). The latter case concerns synchrotron and pulse stretcher operation. In our case transient beam loading takes place at injection of a high current electron beam from the linac into the empty ring, when circulating beam current rapidly increases from zero to its peak value.

In the present paper both steady-state and transient beam loading of the PSSR accelerating cavities are considered using the simple equivalent circuit model. Claims on feedback loops controlling RF-voltage parameters are put forward in order to ensure the effective operation of high power RF-amplifiers.

## STEADY STATE BEAM LOADING

Consider a cavity with the unloaded Q-value $Q_0$ and the shunt impedance R loaded with a bunched beam with average current I. In the vicinity of the resonance frequency $\omega_0$ the cavity with a high Q-value can be substituted by the parallel RLC-circuit. The equivalent circuit of the cavity, coupled through the transmission line without losses to the current generator $\bar{I}_g$, as seen from the accelerating gap, is presented in fig.1. Interaction between the transmission line and the cavity is defined by the coupling coefficient $\beta = Q_0/Q_{ext}$, where $Q_{ext}$ is the external Q-value. The loaded Q-value $Q_l$ is related to $\beta$ and $Q_0$ by the expression:

$$Q_l = Q_0/(1 + \beta) \qquad (1)$$

The relativistic bunched beam is presented in the circuit with the current generator $I_b = \bar{I}_b \exp(i\phi_s)$ ($I_b$-the component of the beam current at the fundamental frequency, for point-like bunches $I_b = 2I$; $\phi_s$-the synchronous phase of the bunch), switched parallel to the accelerating gap.

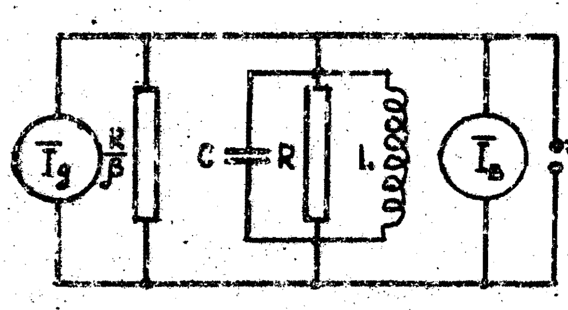


Fig.1 The equivalent circuit.

The phasor diagram for the accelerating gap voltage and current is presented in fig.2. From the analysis of the equivalent circuit (fig.1) and the phasor diagram Wilson has derived the following equations for voltage components $V_g$ and $V_b$ [2]:

$$\bar{V}_g = 2(P_{in}R\beta)^{1/2}\cos\psi/(1+\beta) \qquad (2)$$

$$\bar{V}_b = IR\cos\psi/(1+\beta), \qquad (3)$$

where $P_{in}$ is the cavity input power providing the accelerating gap voltage $V_c$, and $\psi$ is the cavity tuning angle, defined by the following expression:

$$\tan\psi = -2Q_0/(1+\beta)x(\omega-\omega_0)/\omega_0 = -2Q_1\delta\omega/\omega_0, \qquad (4)$$

$\omega$ being the generator frequency. From the analysis of the phasor diagram the relation follows:

$$P_{in} = \frac{V_c^2(1+\beta)^2}{4R\beta}\left\{\left[1+\frac{IR}{V_c(1+\beta)}\cos\phi_s\right]^2 + \left[\tan\psi+\frac{IR}{V_c(1+\beta)}\sin\phi_s\right]^2\right\} \qquad (5)$$

Notation is defined in the caption of fig.2. Relation (5) describes generator performance in the most general case, i.e. under mismatched conditions and for an arbitrary tuning angle $\phi$. By

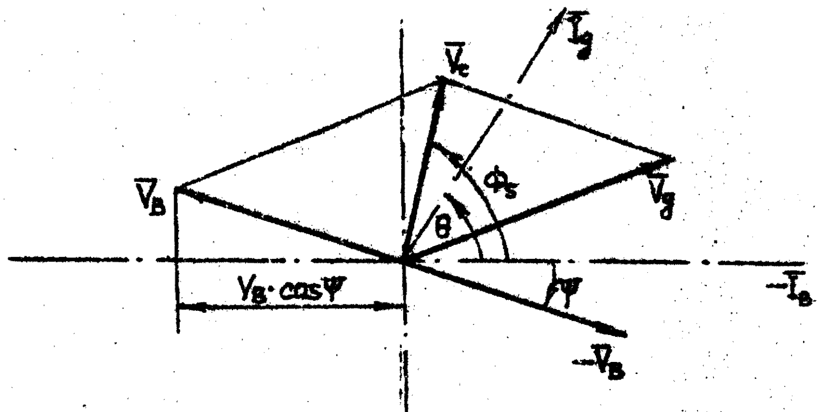


Fig.2. The phasor diagram for the accelerating gap currents and voltages (steady state beam loading). The notation is:

$\bar{V}_g(\bar{I}_g)$ – the generator voltage (current);
$\bar{V}_b(\bar{I}_b)$ – the beam-induced voltage (current);
$\bar{V}_c$ – the total cavity voltage;
$\phi_s$ – the synchronous bunch phase;
$\psi$ – the cavity tuning angle (the phase angle between $\bar{I}_g$ and $\bar{V}_g$ or between $\bar{I}_b$ and $\bar{V}_b$);
$\theta$ – the beam loading angle (the phase angle between $\bar{I}_g$ and $-\bar{I}_b$)

a proper adjustment of parameters $\beta$ and $\phi$ one can set the relation between the RF-power and the beam current.

In RF-systems of electron storage rings the coupling factor $\beta$ is usually chosen so as to ensure matching between a generator and a cavity for a nominal beam current $I_o$. For $I=I_o$ we have:

$$P_{in} = P_c + P_o \quad (6a)$$

$$P_c = I_o V_o \cos\phi_s \quad (6b)$$

$$P_o = V_o^2/R \quad (6c)$$

$$\beta = (P_o + P_c)/P_o = 1 + I_o R \cos\phi_s / V_o \quad (7)$$

$$\tan\phi = -\frac{\beta - 1}{\beta + 1} \tan\phi_s \quad (8)$$

Using (7), we can rewrite relation (5) in the following form:

$$\frac{P_{in}}{P_o} = \frac{\beta-1}{4\beta} \{ [\frac{\beta+1}{\beta-1} + \frac{I}{I_o}]^2 + [\frac{\beta+1}{\beta-1} \tan\phi + \frac{I}{I_o} \tan\phi_s]^2 \} \quad (9)$$

Setting $\phi$ is made by one of the two following ways:

1. The tuning angle $\phi$ is adjusted according to (8) and is fixed. Relation (9) turns into

$$\frac{P_{in}}{P_o} = \frac{\beta-1}{4\beta} \{ [\frac{\beta+1}{\beta-1} + \frac{I}{I_o}]^2 + \tan\phi_s [\frac{I}{I_o} - 1]^2 \} \quad (10)$$

2. The resonance frequency of a cavity is changed synchronously with beam current variations so as to compensate the reactive beam loading component. In this case, expression (9) is reduced to

$$\frac{P_{in}}{P_o} = \frac{\beta-1}{4\beta} [ \frac{\beta+1}{\beta-1} + \frac{I}{I_o} ]^2 \quad (11)$$

The $P_{in}/P_o$ values versus current load $I/I_o$ calculated for PSSR design parameters in the pulse stretching mode ($E_o$=3.0 GeV, $\phi_s$=72°, $I_o$=0.14 A) are presented in fig.3. Shunt impedance of the accelerating cavity was taken 22 MΩ. The figure can be treated in the following way.

The generator is matched to a load for a peak beam current $I=I_o$ injected into the ring (right end-point in the figure). During slow beam extraction a circulating beam current decreases down to zero (in the figure - from right to left). For supporting $V_o$= const a cavity input power has to be changed according to the dependencies presented with crosses (fixed $\phi$) or x's (reactive beam loading compensation). It is clearly seen that in the first case

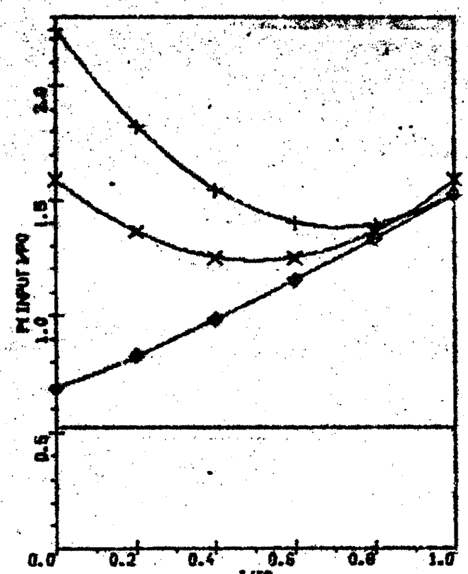


Fig.3. Cavity input power versus beam current ( $I_o$= 0.14 A, $\Phi_s$=72°, $P_o$=21.6 kW). Notation: + – matched operation (at $I=I_{max}$), no compensation; – matched operation with reactive beam loading compensation; x – mismatched operation with $\beta_{opt}$ and $\Phi_{opt}$ (minimal peak power regime).

the power supplied to the cavity at the end of extraction (maximal mismatch) is essentially higher than that in the second case. In addition to it $P_{in}(I=0) > P_{in}(I=I_o)$, so it imposes more stringent limits on the stored beam current and requires incorporation of high power circulators in the RF-power transmission line.

The tuning system for the PSSR accelerating cavities with mechanical tuners provides the reactive beam loading compensation only for long-time beam current variations and cavity temperature drifts, so it is reasonable to optimize RF-generator operation using the basic relation (5). The main optimization requirement is to decrease to minimum the peak value of cavity input power, supporting $V_o$=const during extraction, that is realized under the condition:

$$P_{in}(I=0) = P_{in}(I=I_{max}) \qquad (12)$$

This relation together with the main requirement gives the following optimal values $\beta_{opt}$ and $\phi_{opt}$:

$$\beta_{opt}=([I_{max}R/(2V_c)+\cos\phi_s]^2+\sin^2\phi_s)^2 \qquad (13)$$

$$\phi_{opt}=-\arctan[I_{max}R/[2V_c(1+\beta)\sin\phi_s]+\cot\phi_s) \qquad (14)$$

The input power modulation dependency for mismatched operation with the optimal $\beta$ and $\phi$ is also shown in fig.3. It is evident that this operation regime requires lower peak power levels than operation with $\phi$ fixed at the matching point $I=I_{max}$.

The mismatched operation with $\beta_{opt}$ and $\phi_{opt}$ has the additional advantage: it does not require increasing an input power by a jump at injection. It should be also noted that the cavity tuning loop in the pulse stretching mode has to be insensitive to periodic changes in beam loading.

## TRANSIENT BEAM LOADING

At injection of a bunched beam into an empty ring, in high-Q RF-cavities oscillations of the cavity voltage occur, the damping time of which is proportional to a loaded Q-value of the "beam-cavity" system. This so-called "transient beam loading" must be properly treated and the cavity voltage variations must be damped in order to avoid significant effects on the beam. The analysis of transient beam loading and its possible cures with the fast feedback technology in proton circular accelerators are given in refs. [3,4].

Consider the transient regime of a cavity at injection using the equivalent circuit of fig.1. The phasor diagram in fig.4 represents the circuit behavior at injection. The starting oscillation amplitude $V_{so}$ can be obtained from the condition that the gap voltage cannot change instantly, i.e. immediately after injection we have:

$$V_g = V_c + V_{so} = V_g + V_b + V_{so} \qquad (15)$$

In time, the vector $V_s(t)$, which represents the instant value of the oscillation amplitude at the moment t, is decreasing exponentially with the time constant $\tau_c$ and is turning around:

$$V_s(t)= V_{so}\exp(-t/\tau_c)\ \exp[i(\theta+\delta\theta)], \quad (16)$$

where:

$$\tau_c= 2Q_l/\omega_c, \quad (17)$$

and the phase angle $\delta\theta$ depends on $\tau_c$ and the cavity tuning angle $\phi$:

$$\delta\theta= \delta\omega\ t= t/\tau_c\ \tan\phi \quad (18)$$

The instant gap voltage $\bar{V}_r(t)$ is defined by:

$$\bar{V}_r(t)= \bar{V}_c+ \bar{V}_s(t) \quad (19)$$

The above formulas can be extended to the case of two-turn injection. The corresponding expressions are given in Appendix.

Using this approach we have calculated the gap voltage amplitude and phase at injection for two operation regimes of RF-generators: matched (at $I=I_{max}$) and mismatched with $\beta_{opt}$ and $\psi_{opt}$. The results obtained are presented in fig.5. They show that for the first regime gap voltage perturbations are a little more pronounced than for the second. For the designed current I=.14 A the amplitude deviation from its steady state value is about 23%, the phase jump at injection amounts to 56.3°.

To maintain $\bar{V}_r$ constant during injection within the limits $\delta V_r/V_r=\pm2\%$ and $\delta\phi_s=\pm2^o$ using feedback technology, one has to attain the delay time in the feedback loops less than 150 ns. It seems more practical to apply the synthesized correction signal to the RF-drive system, thus compensating cavity voltage perturbations.

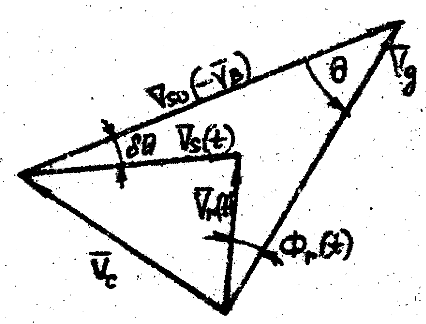


Fig.4. The phasor diagram for the accelerating gap voltages (transient beam loading at injection).

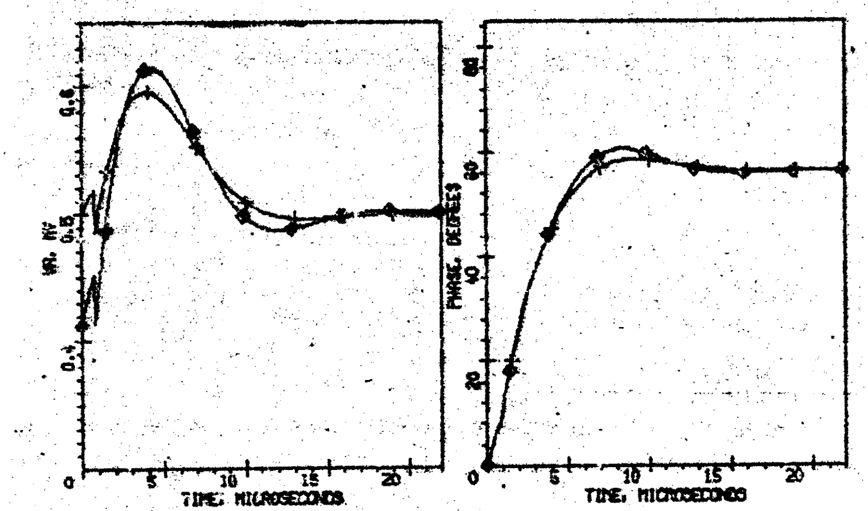


Fig.5. The variations of the gap voltage parameters at injection ($I_0$=0.14 A, $\phi_s$=72$^\circ$). Notations: + – matched operation, – mismatched operation with $\beta_{opt}$ and $\phi_{opt}$.

The calculations and the analysis indicate that the phase jump in the RF-drive will be sufficient to attain the goal. The amplitude deviations during injection result in a negligible change of the beam energy spread (~1 MeV) which is less than the initial energy spread of the injected beam ($\pm$3 MeV). The technique of introducing a phase jump at injection together with an offset in tuning of cavities has been used in the RF-system of the SLC damping ring [5]. In our case, only a phase jump is sufficient, if the mismatched operation with $\beta_{opt}$ and $\phi_{opt}$ takes place.

## CONCLUSIONS

A simple treatment of cavity beam loading for a pulse stretcher ring indicates that the mismatched regime for RF-power generators with optimized values of the coupling factor and the cavity tuning angle is preferable. Transient beam loading effects at injection can be essentially reduced by introduction of a phase jump into a RF-drive signal.

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## APPENDIX

The phasor diagram of accelerating gap voltages for two-turn injection is presented in fig.A1. Indices (1) and (2) correspond to voltage values before and after beginning of the second turn.

First of all we shall find the relation between $V_c^{(1)}$ and $V_c^{(2)}$. From the phasor diagram we find:

$$V_c^{(1)2} = V_g^2 + V_b^{(1)2} - 2V_g V_b^{(1)} \cos\theta$$

$$V_c^{(2)2} = V_g^2 + V_b^{(2)2} - 2V_g V_b^{(2)} \cos\theta$$

Considering that

$$V_b^{(2)} = 2V_b^{(1)} = V_b \ ,$$

we come to the relation

$$V_c^{(1)2} = 1/2\,(V_c^{(2)2} + V_g^{2} - V_b^{2}/2) \qquad \text{(A1)}$$

The values $V_g$ and $V_b$ are calculated using formulas (2) and (3), $V_c^{(2)}$ is a fixed value. The angle $\theta$ can be found from the following expression:

$$\theta = \arcsin\{[V_c^{(2)}/V_g]\,\sin[\pi-(\phi_s-\phi)]\} \qquad \text{(A2)}$$

During the time interval before the beginning of the second turn ($0<t<t_1$), where $t_1$ is the rotation period for bunches in the ring), the instant amplitude value can be calculated from the equation:

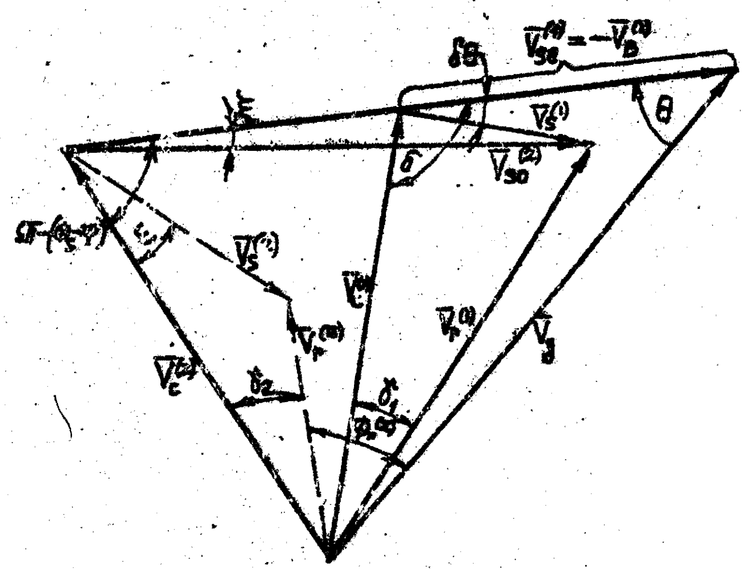

Fig. A1. The phasor diagram for two-turn injection. Notations are:
$V_b^{(1)}$; $V_b^{(2)}$–the beam induced voltage (steady state value);
$V_s^{(1)}$; $V_s^{(2)}$–the amplitude of the gap voltage oscillations;
$V_{so}^{(1)}$; $V_{so}^{(2)}$–the starting oscillation amplitude;
$V_c^{(1)}$; $V_c^{(2)}$–the total gap voltage (steady state value);
$V_r^{(1)}$; $V_r^{(2)}$–the instant value of the gap voltage amplitude.

$$V_r^{(1)2} = V_o^{(1)2} + V_s^{(1)2} - 2V_o^{(1)} V_s^{(1)} \cos(\delta - \delta\theta_1), \qquad (A3)$$

where

$$\delta = \arcsin(V_s/V_o^{(1)} \sin\theta) \qquad (A4)$$

$$V_s^{(1)} = V_{so}^{(1)} \exp(-t/\tau_o) = V_b/2 \exp(-t/\tau_o) \qquad (A5)$$

The cavity time constant $\tau_o$ and the phase shift $\delta\theta_1$ are calculated using equations (17) and (18), correspondingly. The instant phase value is defined by the expression:

$$\phi_r^{(1)} = \pi - (\theta + \delta) - \gamma_1, \qquad (A6)$$

where

$$\gamma_1 = \arcsin[V_s^{(1)}/V_p^{(1)} \sin(\delta - \delta\theta_1)] \qquad (A7)$$

On the second turn $(t > t_1)$, the instant values of the gap voltage and the phase can be obtained from the following relations:

$$V_r^{(2)2} = V_s^{(2)2} + V_o^{(2)2} - 2V_s^{(2)} V_o^{(2)} \cos\varepsilon, \qquad (A8)$$

where

$$V_s^{(2)} = V_{so}^{(2)} \exp[(t-t_1)/\tau_o] = [V_s^{(1)2} + V_b^2/4 - -V_s^{(1)} V_b \cos(\pi - \delta\theta_1)]^{1/2} x \exp[(t-t_1)/\tau_o] \qquad (A9)$$

$$\varepsilon = [\pi - (\phi - \psi)] - \zeta - \delta\theta_2 \qquad (A10)$$

$$\zeta = \arcsin[V_s^{(1)}/V_{so}^{(2)} \sin(\pi - \delta\theta_2)] \qquad (A11)$$

$$\delta\theta_2 = (t-t_1)/\tau_o \tan\psi \qquad (A12)$$

and

$$\phi_p^{(2)} = (\phi_s - \psi - \theta) - \gamma_2, \qquad (A13)$$

where

$$\gamma_2 = \arcsin[V_s^{(2)}/V_p^{(2)} \sin\varepsilon] \qquad (A14)$$

The $V_s^{(1)}$ and $\delta\theta_1$ values are calculated using formulas (A5) and (18), correspondingly, putting $t = t_1$.

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