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GREEN-SCHWARZ SUPERSTRINGS IN THE GENERALIZED HARMONICAL NEWMAN-PENROZE FORMALISM

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Green-Schwarz Superstring Theory in D=4 and D=10 is formulated in an extended space including additional spinor coordinates of the Cartan moving frame. These spinor coordinates realized in the form of the Lorentz harmonics are used for constructing of covariant irreducible generators of fermionic æ-symmetry. The generalizations of this Lorentz harmonical superstring formulation to the case of super-p-branes with p>1 (including supermembranes p=2) are discussed shortly.

23 refs.

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Теория суперструн Грина-Шварца в D=4 и D=10 сформулирована в расширенном пространстве, дополненном спинорными координатами подвижного репера. Они реализованы как лоренцевы гармоники и используются для построения ковариантных неприводимых генераторов фермионной æ-симметрии. Кратко обсуждается обобщение таких Лоренц-гармонических формулировок суперструн на случай супер-р-бран с р>1 (включая супермембраны с р=2).

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1. Recently new approache for solving the problem of the covariant description of æ-symmetry in superparticles and superstring theories [1] has been suggested [2-13] (see also the list of refs in [12,13]). One of them suggests an extension of the dynamical variable space adding auxiliary fields which can be realized as harmonics [13,2-4,8-10], twistors [14,5-7] or vectors [12,15]. In the framework of this approach the BRST covariant quantization of the whole set of null super-p-branes, including massless superparticles (p=0), null superstrings (p=1) and null supermembranes (p=2), has been fulfilled in the space-time with D=4.

The central point of the approach [8-10] consists in introducing the spinor variables of the Cartan moving frame or the so-called Newman-Penrose diade basis [16] (formed by the commuting Weyl spinors τ_α, o_α or, equivalently, by the Lorentz harmonics v_α^-, v_α^+ [8]) in the target space of null super-p-brane theory and using them for the construction of the action functional.

The moving frame approach is effective not only for null super-p-brane theory. Previously it has been effectively applied in the Yang-Mills, supergravity [17] and bosonic string [18] theories. Thus the use of the harmonic realization of the Cartan vector frame, seems to be conceptually important for a relevant formulation of superstring theory. This originates in the fact that the problem of the covariant description of æ-symmetry has the natural and simplest solution in the case when auxiliary spinor reper variables are present in the target space and the twistor-like action functional is used [8,9].

In accordance with this point of view it is important to transform the Green-Schwarz superstring theory into an classically equivalent one defined for the extended target space containing the spinor moving frame coordinates.

Here we present such new formulations for D=10, N=2B and D=4, N=1 superstring theories. They contain two sets of reper variables.The first one comprises the reper variables of the external space-time realized in the form of (spinorial) Lorentz harmonics. The second set contains the zweibeins of superstring world sheet. We show that the problem of the construction of the covariant irreducible æ-symmetry generator finds its simplest solution in these formulations. We stress that the tensionless limit of these formulation given by null superstring action [9] may be easily obtained by putting $b=0$ and $a=0$ in the action functional (1).

2. The action for D=10, N=2B and D=4, N=1 superstrings may be presented in the unified form using the light-like vectors $u_m^{[\pm 2]}$ [1] which are constructed using the spinorial Lorentz harmonic variables for D=4 [8-10] and for D=10 [19]

$$S_{D,N} \equiv \int d\tau\, d\sigma\, \mathcal{L}(\tau,\sigma) = S_{D,N(b)} + S_{WZ,D,N} \quad , \tag{1}$$

$$S_{D,N(b)} = -\frac{1}{2}\int d\tau\, d\sigma \Big[\rho^{[+2]\mu}\, \omega_\mu^m\, u_m^{[-2]} + b\, \varphi^{[-2]\mu}\, \omega_\mu^m\, u_m^{[+2]}$$

$$+ c\, b\, \alpha'\, \varepsilon_{\mu\nu}\, \rho^{[+2]\mu}\, \varphi^{[-2]\nu} \Big]$$

Here c and b are dimensionless constants; μ=0,1 are the vector index of the world sheet;

$$\omega_\mu^m \equiv \partial_\mu x^m - i \left[\partial_\mu \theta^\alpha\, \sigma_{m\alpha\dot\alpha}\, \bar\theta^{\dot\alpha} - \theta^\alpha\, \sigma_{m\alpha\dot\alpha}\, \partial_\mu \bar\theta^{\dot\alpha} \right]$$

when D=4, N=1 with m=0,1,2,3; α=1,2; $\dot\alpha$=1,2 and $\sigma_{m\alpha\dot\alpha}$ are the relativistic Pauli matrices;

$$\omega_\mu^m \equiv \partial_\mu x^m - i \left[\partial_\mu \theta^{\hat\alpha 1}\, \sigma_{m\hat\alpha\hat\beta}\, \theta^{\hat\beta 1} + \partial_\mu \theta^{\hat\alpha 2}\, \sigma_{m\hat\alpha\hat\beta}\, \theta^{\hat\beta 2} \right]$$

when D=10, N=2B with m=0,...,9; $\hat\alpha$=1,...,16 and $\sigma_{m\hat\alpha\hat\beta}$ are 16×16 D=10 σ-matrices defined in [3]. The coordinates of D=10, N=2B and D=4, N=1 superspaces are denoted by means of the symbols

$$z^M = (x^m,\ \theta^\alpha,\ \bar\theta^{\dot\alpha} \equiv \overline{(\theta^\alpha)})\ \ and\ \ z^M = (X^m, \theta^{\hat\alpha 1}, \theta^{\hat\alpha 2}) \equiv (X^m, \theta^{\hat\alpha I})$$

respectively.

The vectors $u_m^{[\pm 2]}$ which satisfy the conditions of light-likeness

$$u_m^{[-2]}\, u^{[-2]m} = 0 = u_m^{[+2]}\, u^{[+2]m} \tag{2}$$

and normalization

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1) Here and below the expressions in the square brackets above a variable symbol indicate its weight w under SO(1,1) group gauge transformations. This SO(1,1) group is a subgroup of the Lorentz group SO(1,D-1) but also may be considered as structure group of the superstring world sheet in the framework of the presented formulation of superstring theory. When D=4, the weight w may be presented in terms of the charges q_L and q_R [8-10] under the chiral complex subgroups $U_L(1)$ and $U_R(1)$ of SL(2,C) group by the relation $w = \frac{(q_R - q_L)}{2}$.

$$u_m^{(-2)}\, u^{(+2)m} = 2 \qquad (3)$$

are built of the corresponding spinor Lorentz harmonics [8,19] as

$$D=4 : \; u^{(-2)m} \equiv u^{(+|-)m} = v^{\alpha-}\sigma^m_{\alpha\dot\alpha}\bar v^{\dot\alpha+}, \quad u^{(+2)m} \equiv u^{(-|+)m} = v^{\alpha+}\sigma^m_{\alpha\dot\alpha}\bar v^{\dot\alpha-}, \qquad (4a)$$

$$D=10 : \; u_m^{(-2)} = \frac{1}{8}\, u^-_{\hat\alpha A}\, \tilde\sigma^{\hat\alpha\hat\beta}_m\, u^-_{\hat\beta A} \quad , \quad u_m^{(+2)} = \frac{1}{8}\, u^+_{\hat\alpha A}\, \tilde\sigma^{\hat\alpha\hat\beta}_m\, u^+_{\hat\beta A} \quad . \qquad (4b)$$

On the shell defined by the motion equations

$$\frac{\delta S_{D,N}}{\delta\rho^{(+2)\mu}} = 0 = \frac{\delta S_{D,N}}{\delta\varphi^{(-2)\mu}}$$

for the reper coefficients $e_f{}^\mu$ or densities $\tilde e_f{}^\mu = e_f{}^\mu / det(e_g{}^\nu)$

$$\rho^{(+2)\mu} = \frac{1}{2}\,(\alpha')^{-1/2}\left[\hat e_0^\mu - \tilde e_1^\mu\right] \quad and \quad b\varphi^{(-2)\mu} = \frac{1}{2}\,(\alpha')^{-1/2}\left[\hat e_0^\mu + \tilde e_1^\mu\right]$$

of the superstring world sheet the action $S_{D,N,(b)}$ transforms into the action from Ref. [15]

$$\tilde S_{D,N(b)} = \frac{1}{2c\alpha'}\int d\tau\, d\sigma\, \varepsilon^{\mu\nu}\, \omega^m_\mu\, \omega^n_\nu\, u_m^{(-2)}\, u_n^{(+2)} ,$$

where the auxiliary vectors

$$n^0_m = \frac{1}{2}(u_m^{(+2)} + u_m^{(-2)}) \equiv u_m^{(0)}, \qquad n^1_m = \frac{1}{2}(u_m^{(+2)} - u_m^{(-2)}) \equiv u_m^{(1)},$$

comprize the Lorentz harmonics according to Eq.(4).

The vector moving frame formulation [15] is equivalent to the Nambu-Goto one (for N=0) and to the Green-Schwarz one (for N=1,2), after adding the corresponding Wess-Zumino terms into the action. Therefore it is evident that at the classical level the formulations (1) are equivalent to the corresponding Green-Schwarz theories.

The Lorentz harmonic variables [8-10] used in Eqs.(4a) are restricted by the normalization (or harmonicity) conditions

$$\Xi^{(4)} \equiv v^{\alpha-}\, v^+_\alpha - 1 = 0, \qquad \Xi^{(4)} \equiv \bar v_{\dot\alpha}\, \bar v^{\dot\alpha+} - 1 = 0 \qquad (5a)$$

The invariance of the action (1) under the gauge transformations from $[U(1)]^c = U_L(1)\times U_R(1) = SO(1,1)\times SO(2)$ subgroup of $SL(2,\mathbb{C})$ together with the harmonicity conditions (5a) permits to identify the space of harmonics $\{v^\pm_\alpha, v^\pm_{\dot\alpha}\}$ with the coset space $SL(2,\mathbb{C})/[U(1)]^c = SO(1,3)/SO(1,1)\times SO(2)$. Four independent combinations of the Lorentz harmonics $v^\pm_\alpha$ together with the variables ρ^τ and φ^τ parameterize 6 independent components of the two light-like vectors

$$\mathcal{P}_m \pm \frac{1}{\alpha'} \omega_{\sigma m} + \ldots \equiv L_{\pm}$$

which present the superstring theory constraints.

In Eqs.(4b) $U^{a}_{\hat{\alpha}} \equiv \left[u^{+}_{\hat{\alpha}A}, \; u^{-}_{\hat{\alpha}\dot{A}} \right]$ (where A=1,...,8; $\dot{A}$=1,...,8 are the indices of **8**(s) and **8**(c) spinor representations of SO(8) group) are the Lorentz harmonic variables for the case D=10 [19]. These variables are restricted by the 1261 harmonicity conditions

$$\Xi^{(-2)}_{m_1 \ldots m_5} = \frac{1}{8} u^{-}_{\hat{\alpha}\dot{A}} \tilde{\sigma}^{\hat{\alpha}\hat{\beta}}_{m_1 \ldots m_5} u^{-}_{\hat{\beta}\dot{A}} = 0,$$

$$\Xi^{(+2)}_{m_1 \ldots m_5} = \frac{1}{8} u^{+}_{\hat{\alpha}A} \tilde{\sigma}^{\hat{\alpha}\hat{\beta}}_{m_1 \ldots m_5} u^{+}_{\hat{\beta}A} = 0,$$

$$\Xi^{(l)}_{m_1 \ldots m_5} = \frac{1}{8} u^{+}_{\hat{\alpha}A} \gamma^{l}_{A\dot{A}} \tilde{\sigma}^{\hat{\alpha}\hat{\beta}}_{m_1 \ldots m_5} u^{-}_{\hat{\beta}\dot{A}} , \tag{5b}$$

$$\Xi^{(10)} = \frac{1}{128} u^{-}_{\hat{\alpha}\dot{A}} \tilde{\sigma}^{\hat{\alpha}\hat{\beta}}_{m} u^{-}_{\hat{\beta}\dot{A}} u^{+}_{\hat{\alpha}A} \tilde{\sigma}^{m\hat{\alpha}\hat{\beta}} u^{+}_{\hat{\beta}A} - 1 = 0 \tag{5c}$$

imposing only 210+1=211 independent restrictions on the Lorentz harmonics [19]. Due to the invariance of the action (1) under the transformations from SO(1,1) $\times$ SO(8) gauge group and the harmonicity conditions (5b,c) the number of the independent variables among the 256 components of $U^{a}_{\hat{\alpha}}$ equals to 16(=256-211-28-1). In accordance with this fact the harmonics $U^{a}_{\hat{\alpha}}$ may be considered as the coordinates of the coset space SO(1,9)/SO(1,1)$\times$SO(8). Together with the variables ρ^{τ} and φ^{τ} the 16 independent combinations of the harmonic variables $U^{a}_{\hat{\alpha}}$ parameterize the 18 independent components of the two light-like vectors $L_{\pm}$ in D=10.

In this case the light-likeness conditions (2) and the orthogonality of the vectors $u^{(\pm 2)}_{m}$ and $u^{(l)}_{m}$

$$u^{(l)}_{m} \equiv \frac{1}{8} u^{+}_{\hat{\alpha}A} \gamma^{l}_{A\dot{A}} \tilde{\sigma}^{\hat{\alpha}\hat{\beta}}_{m} u^{-}_{\hat{\beta}\dot{A}} \tag{4c}$$

are the consequences of the well-known identity : $\tilde{\sigma}^{(\hat{\alpha}\hat{\beta}}_{m} \tilde{\sigma}^{\hat{\gamma})\hat{\delta}m} \equiv 0$ [3], in contrast to the normalization conditions (3) which coincide with Eq.(5c) and present one of the harmonicity conditions (the relation $u^{(l)}_{m} u^{(j)m} = - \delta^{lj}$ is a consequence of the relation (3)).

In Eq.(1) $S_{WZ,D,N}$ denote the Wess-Zumino term for the corresponding superstring [1]. It has the form

$$S_{WZ\ D=4,N=1} = \frac{i}{\alpha'}\, a \int d\tau\, d\sigma\, \varepsilon^{\mu\nu}\, \partial_\mu x^m \left[\partial_\nu \theta^\alpha \sigma_{m\alpha\dot\alpha} \bar\theta^{\dot\alpha} - \theta^\alpha \sigma_{m\alpha\dot\alpha} \partial_\nu \bar\theta^{\dot\alpha} \right] \qquad (6a)$$

for D=4 , N=1 and

$$S_{WZ,D=10,N=2B} = \frac{i}{\alpha'}\, a \int d\tau d\sigma \left[\varepsilon^{\mu\nu}\, \omega_\mu{}^m \left[\partial_\nu \theta^{\hat\alpha 1} \sigma_{m\hat\alpha\hat\beta} \theta^{\hat\beta 1} - \partial_\nu \theta^{\hat\alpha 2} \sigma_{m\hat\alpha\hat\beta} \theta^{\hat\beta 2} \right] + \right.$$

$$\left. + \varepsilon^{\mu\nu}\, \partial_\mu \theta^{\hat\alpha 1}\, \sigma_{m\hat\alpha\hat\beta}\, \theta^{\hat\beta 1}\, \partial_\nu \theta^{\hat\gamma 2}\, \sigma^m{}_{\hat\gamma\hat\delta}\, \theta^{\hat\delta 2} \right] \qquad (6b)$$

for D=10 , N=2B . The presence of this term with $a=-1/c$ in the action (I) leads to the invariance of this action under the æ-symmetry transformations generated by the following covariant and irreducible first class constraints. For D=4 they has the form

$$\mathbf{D=4}: \quad D^-(\sigma) \equiv v^{\alpha -} D_\alpha(\sigma) - \frac{4i}{c\alpha'} \frac{\partial_\sigma \bar\theta^{\dot\alpha}\, \bar v_{\dot\alpha}{}^+}{\rho^{(-1+)\tau}}\, \bar\nabla^{(-2I0)} + \frac{4i}{c\alpha'} \partial_\sigma \bar\theta^{\dot\alpha}\, \bar v_{\dot\alpha}{}^-\, P^{(+1-)}_{(\rho)\tau},$$

$$D^+(\sigma) \equiv \overline{\left[D^-(\sigma) \right]} \quad , \qquad (7a)$$

where

$$D_\alpha(\sigma) \equiv -\pi_\alpha + i \left[P_m + \frac{1}{c\alpha'} \omega_{\sigma m} \right] \sigma^m_{\alpha\dot\alpha}\, \bar\theta^{\dot\alpha}$$

are the primary Grassmann constraints which coincide with the ones from the original formulation of the Green-Schwarz superstring;

$$P_{\mathfrak{M}} \equiv \left[P_m,\ \pi_\alpha,\ \bar\pi_{\dot\alpha},\ P^+_\alpha,\ P^-_{\dot\alpha},\ P^{(+1-)}_{(\rho)\mu},\ P^{(-1+)}_{(\varphi)\mu} \right] \equiv -(-I)^{\mathfrak{M}}\, \partial L / \partial \dot z^{\mathfrak{M}}$$

are the momentum densities canonically conjugated to the the target space coordinates

$$z^{\mathfrak{M}} \equiv (x^m,\ \theta^\alpha,\ \bar\theta^{\dot\alpha},\ v^-_\alpha,\ \bar v^+_{\dot\alpha},\ \rho^{(-1+)\mu},\ \varphi^{(+1-)\mu})$$

i.e. $[z^{\mathfrak{M}}(\sigma), P_{\mathfrak{N}}(\sigma')]_P = -\delta^{\mathfrak{M}}_{\mathfrak{N}} \delta(\sigma - \sigma')$, and finally $\bar\nabla^{(-2I0)} \equiv \bar v^{\dot\alpha -} P_{\dot\alpha}$.

For D=10 the covariant and irreducible æ-symmetry generators has the following form

$$\mathbf{D=10}: \quad D^1{}_A(\sigma) \equiv u^{\hat\alpha -}_A D^1_{\hat\alpha}(\sigma) - \frac{16i}{c\alpha' \rho^{(+2)\tau}} \gamma^l_{A\dot A}\, u^-_{\hat\beta \dot A}\, \partial_\sigma \theta^{\hat\beta 1} \nabla^{(+2)l} +$$

$$+ \frac{4i}{c\alpha'} u^+_{\hat\beta A}\, \partial_\sigma \theta^{\hat\beta 1} P^{(-2)}_{(\rho)\tau}, \qquad (7b)$$

$$D^{2+}_{\hat{\alpha}}(\sigma) \equiv u^{\hat{\alpha}+}_{\hat{\alpha}} D^{2}_{\hat{\alpha}}(\sigma) - \frac{16i}{c\alpha' b\varphi^{[-2]}\tau} \tilde{\gamma}^{i}_{\hat{\alpha}A} \; u^{-}_{\hat{\beta}A} \partial_\sigma \theta^{\hat{\beta}2} \nabla^{[-2]i} - \frac{4i}{cb\alpha'} \; u^{-}_{\hat{\beta}\hat{\alpha}} \partial_\sigma \theta^{\hat{\beta}2} P^{[+2]}_{(\varphi)\tau},$$

In Eq.(7b) $D^{I}_{\hat{\alpha}}(\sigma) \equiv \left[-\pi^{I}_{\hat{\alpha}} + i\left\{ P_m - (-1)^{I} \frac{1}{c\alpha'} \left[\partial_\sigma X_m - i\partial_\mu \theta^{\hat{\gamma}I} \sigma_{m\hat{\gamma}\hat{\delta}} \theta^{\hat{\delta}I} \right] \right\} \sigma^{m}_{\hat{\alpha}\hat{\beta}} \theta^{\hat{\beta}I} \right]$;

the components of matrix which is inverse to $U_{\hat{\alpha}}{}^{a} \equiv \left[u^{+}_{\hat{\alpha}A} \; u^{-}_{\hat{\alpha}\hat{A}} \right]$ are denoted by $\left[u^{-\hat{\alpha}}_{A} \; u^{+\hat{\alpha}}_{\hat{A}} \right] \equiv U^{-1\hat{\alpha}}_{a}$ [19]. Unlike the case D=4, it is impossible to express the components of $U^{-1\hat{\alpha}}_{a}$ in terms of $u^{+}_{\hat{\alpha}A}$ and $u^{-}_{\hat{\alpha}\hat{A}}$ in a simple covariant way. Therefore the variables $u^{-\hat{\alpha}}_{A}$ $u^{+\hat{\alpha}}_{\hat{A}}$ are to be considered as the independent harmonics. At the same time the 256 inversibility conditions for the matrices U and U^{-1} $\left[U^{-1\hat{\alpha}}_{a} U_{\hat{\alpha}}{}^{b} = \delta^{b}_{a} \right]$ are to be added to the set of harmonicity conditions (5c,d) :

$$\Xi_{AB} \equiv u^{-\hat{\alpha}}_{A} u^{+}_{\hat{\alpha}B} - \delta_{AB} = 0 , \qquad \Xi_{\hat{A}\hat{B}} \equiv u^{+\hat{\alpha}}_{\hat{A}} u^{-}_{\hat{\alpha}\hat{B}} - \delta_{\hat{A}\hat{B}} = 0$$

$$\Xi_{A\hat{A}} \equiv u^{-\hat{\alpha}}_{A} u^{-}_{\hat{\alpha}\hat{A}} = 0, \qquad \Xi_{\hat{A}A} \equiv u^{+\hat{\alpha}}_{\hat{A}} u^{+}_{\hat{\alpha}A} = 0 \tag{5d}$$

The momentum densities

$$P_{\mathbb{M}} \equiv \left[P_m \; \pi^{1}_{\hat{\alpha}}, \; \pi^{2}_{\hat{\alpha}}, \; P^{\hat{\alpha}-}_{A}, \; P^{\hat{\alpha}+}_{\hat{A}}, \; P^{+}_{\hat{\alpha}A}, \; P^{-}_{\hat{\alpha}\hat{A}}, \; P^{[-2]}_{(\rho)\mu}, \; P^{[+2]}_{(\varphi)\mu} \right]$$

entering in the relations (7b) are the canonically conjugated ones for the target space coordinates

$$z^{\mathbb{M}} \equiv (X^m, \; \theta^{\hat{\alpha}1}, \; \theta^{\hat{\alpha}2}, \; u^{+}_{\hat{\alpha}A}, \; u^{-}_{\hat{\alpha}\hat{A}} \; u^{-\hat{\alpha}}_{A} \; u^{+\hat{\alpha}}_{\hat{A}}, \; \rho^{[+2]\mu}, \; \varphi^{[-2]\mu});$$

$\nabla^{[-2]i} \equiv u^{-}_{\hat{\alpha}\hat{A}} \tilde{\gamma}^{i}_{\hat{A}A} P^{\hat{\alpha}-}_{A} - u^{-\hat{\alpha}}_{A} \gamma^{i}_{A\hat{A}} P^{-}_{\hat{\alpha}\hat{A}}$, $\nabla^{[+2]i} \equiv u^{+}_{\hat{\alpha}A} \gamma^{i}_{A\hat{A}} P^{\hat{\alpha}+}_{\hat{A}} - u^{+\hat{\alpha}}_{\hat{A}} \tilde{\gamma}^{i}_{\hat{A}A} P^{+}_{\hat{\alpha}A}$,

$\tilde{\gamma}^{i}_{\hat{A}A} = \left[\gamma^{i}_{A\hat{A}} \right]^{T}$, and, finally $\gamma^{i}_{A\hat{A}}$ are the D=8 "σ-matrices" defined in [1].

Note that unlike superstring a massless superparticle [8] and

null super-p-branes [9][1)] are described by the single light-like vector which is the momentum density P_m . As a consequence, their actions are characterized by more higher (in comparison with superstrings) gauge symmetries including $SO(1,1)\times SO(2)\ltimes K_2$ (for D=4) and $SO(1,1)\times SO(8)\ltimes K_8$ (for D=10), where K_2 and K_8 are the Lorentz boosts for the case D=4 and D=10, generated by the operators [8,9,19]

$$\text{D=4:}\quad \nabla^{(01-2)}+\ldots\equiv v^{\alpha-}P^{-}_{\alpha}+\ldots,\quad \bar{\nabla}^{(+210)}+\ldots\equiv \bar{v}^{\hat{\alpha}+}P^{+}_{\hat{\alpha}}+\ldots \quad \textit{and}$$

$$\text{D=10:}\quad \nabla^{(-2)l}+\ldots\equiv u^{-}_{\alpha\dot{\alpha}}\tilde{\gamma}^{l}_{\dot{\alpha}\hat{\alpha}}P^{\hat{\alpha}-}_{\hat{\alpha}}-u^{-\hat{\alpha}}_{\hat{\alpha}}\gamma^{l}_{\hat{\alpha}\dot{\alpha}}P^{-}_{\alpha\dot{\alpha}}+\ldots \tag{8}$$

Owing to this observation, the harmonics $v^{\pm}_{\alpha}$ and $u^{+}_{\alpha\hat{\alpha}}, u^{-}_{\alpha\dot{\alpha}}$ may be considered as the coordinates of the coset spaces $SO(1,3)/SO(1,1)\ltimes SO(2)\ltimes K_2$ (D=4) *and* $SO(1,9)/SO(1,1)\times SO(8)\ltimes K_8$ (D=10) respectively. The number of this coset coordinates, namely 2 (for D=4) and 8 (for D=10) together with 1 coordinate of the target space ρ^{τ} exactly equals to the number of independent components of the light-like vector P_m (3 or 9 respectively). Thus the demand of the Lorentz boost symmetry considered in [19,20] for the definition of Lorentz harmonics is admissible only for the cases of superparticles [8] and null super p-branes [9], but it is not correct demand for superstring.

Now we investigate the covariant BRST-BFV quantization of the new formulations of superstrings (1) using the approach developed in [9,10].

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1) The action functional for null superstring (p=1) in D=4 [9] and D=10 is given by Eq.(1) with b=0 and the Wess-Zumino term being absent (i.e. α=0 in Eqs.(6)). It is known [21] that null superstring has the invariance under the fermionic æ-symmetry without the inclusion of the Wess-Zumino terms in the action. We stress that in the case of N=2 superstring the structure of æ-symmetry distincts drastically from the one corresponding to null-superstring (b=0) since it's æ-symmetry generators $D^{1-}_{\hat{\alpha}}(\sigma)\equiv u^{\hat{\alpha}-}_{\hat{\alpha}}D^{1}_{\hat{\alpha}}(\sigma)+\ldots$ and $D^{2-}_{\hat{\alpha}}(\sigma)\equiv u^{\hat{\alpha}-}_{\hat{\alpha}}D^{2}_{\hat{\alpha}}(\sigma)+\ldots$ are all in the (s)-spinorial representation of SO(8) group. Thus, in distinction with bosonic string and N=1 superstring case, the tensionless limit of N=2 superstring is singular not only in quantum [22], but also in classical theory.

3. The natural generalization of the action functional $S_{D,N(b)}$ (1) for extended objects with p>1 has the form

$$S^{(p)}_{D,N(b)} = \int d\xi^{p+1} \left[- (\alpha')^{-1/2} \tilde{e}^{\mu}_{b} \omega^{m}_{\mu} u^{(b)}_{m} + c \, (\det \tilde{e}^{\mu}_{b})^{1/p} \right], \qquad (9)$$

where $\tilde{e}^{\mu}_{b} \equiv (\tilde{e}^{\mu}_{0}, \ldots, \tilde{e}^{\mu}_{p})$ are the inverse reper density coefficients for p-brane world sheet. By means of $u^{(b)}_{m}$ the subset of (p+1) vectors from the orthonormal frame set $u^{(n)}_{m} \equiv (u^{(0)}_{m}, u^{(1)}_{m}, \ldots, u^{(D-1)}_{m}) \equiv (u^{(b)}_{m}, u^{(a)}_{m})$ (see [2,15,18]) is defined. These vectors must be constructed using the corresponding spinor Lorentz harmonic variables. For the case of superstring

$$\tilde{e}^{\mu}_{b} = (\alpha')^{1/2} \left[(\rho^{(+2)\mu} + b \varphi^{(-2)\mu}), \; (- \rho^{(+2)\mu} + b \, \varphi^{(-2)\mu}) \right],$$

$$u^{(b)}_{m} = (u^{(0)}_{m}, u^{(D-1)}_{m}) = \left[\frac{1}{2} (u^{(+2)}_{m} + u^{(-2)}_{m}), \frac{1}{2} (u^{(+2)}_{m} - u^{(-2)}_{m}) \right]$$

and the light-like vectors $u^{(\mp 2)}_{m}$ are defined by Eqs.(4) for D=4 and D=10 cases.

It is easy to see that for p>1 the gauge group of the action (9) contains the subgroup SO(1,p)× SO(D-p-1) instead of SO(1,1)× SO(D-2) one which appears in the superstring case. Consequently we need spinorial Lorentz harmonic variables parameterizing the coset

$$SO(1,D-1)/SO(1,p) \times SO(D-p-1)$$

for an adequate description of super-p-branes in D dimensions. The additional Wess-Zumino terms for the actions (9) have well known form, [23] and their existence impose rigid restrictions on the admissible values of the triades (D,p,N).

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