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# SUPERFIELD DESCRIPTION OF N=2 EXTENDED ONE-DIMENSIONAL SUPERSYMMETRIC QUANTUM MECHANICS

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N =2 superfields are used to construct the action for the one-dimensional N =2 extended supersymmetric quantum mechanics.

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С использованием N=2 суперполей построено действие для одномерной N=2 расширенной суперсимметричной квантовой механики.

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## 1. Introduction

Just as one-dimensional N=1 Supersymmetric Quantum Mechanics (SQM) /1/ links into a unity two Hamiltonians with the same or nearly the same spectra, so N-extended one-dimensional SQM unites together several, namely, N+1, quantum-mechanical Hamiltonians in such a way that their eigenfunctions appear related to each other by a multiple Darboux transformation /2/. The N =2 case turns out to be closely connected with the inverse scattering problem /3/. This close connection between the two-fold Darboux transformation and the inverse scattering problem was also indicated by several authors /4,5/, though without considering extended supersymmetry.

The Hamiltonian of the N-extended one-dimensional SQM was derived by the use of N=1 superfields /6/, so that the invariance of the theory with respect to the extended supersymmetry transformations is valid only at certain relationships between different superpotentials entering into this Hamiltonian. As to the extended supersymmetry, this approach is fully similar to a component approach to a nonextended supersymmetry. One of the adventages of the superfield approach using extended superfields is the automatic invariance of the expressions obtained with respect to the extended supersymmetry transformations. This enables one to describe most general invariant Lagrangians.

The aim of the present work is to give such a superfield description for the N=2 (in the sense that there are 2 complex or 4 real supercharges) extended one-dimensional SQM. In Section 2 we deal with the N-extended supersymmetry algebra and construct irreducible superfields. The general form of the Lagrangian and the Hamiltonian coinciding in the form with that of /6/ are deduced in Section 3. In section 4 we discuss the possibility of extending the present description to the case of higher N.

## 2. Superfields in (1,4) - superspace

The N-extended supersymmetry algebra has the form :

$$\{Q_i, Q_k\} = \delta_{ik} H$$

$$[H, Q_i] = 0; \quad i = 1, 2, \ldots, 2N$$

and may be realized in the (1, 2N) superspace with one boson coor-

dinate t (further playing the role of time) and 2N Grassmann real coordinates, which can be united by pairs in N complex Grassmann coordinates $\theta^\alpha$. Note that this algebra is sometimes called as 2N-extended algebra, i.e., according to this terminology we are constructing the superfield description of the N =4 extended SQM.

As a result of supersymmetry transformations, the superspace coordinates change as

$$\delta\theta^\alpha = \epsilon^\alpha; \quad \delta\bar{\theta}_\alpha = \bar{\epsilon}_\alpha; \quad \delta t = \frac{i}{2}(\bar{\epsilon}_\alpha\theta^\alpha + \epsilon^\alpha\bar{\theta}_\alpha). \tag{1}$$

The covariant derivatives hav the form :

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{i}{2}\bar{\theta}_\alpha\frac{\partial}{\partial t}; \quad \bar{\mathcal{D}}^\alpha = -\frac{\partial}{\partial\bar{\theta}_\alpha} + \frac{i}{2}\theta^\alpha\frac{\partial}{\partial t}. \tag{2}$$

In the (1,4)-superspace we may consider different superfields which realize the representations of the N=2 extended supersymmetry. One of them is the real scalar superfield

$$\begin{aligned}\Phi(t,\theta) &= A(t) + \theta^\alpha\psi_\alpha(t) - \bar{\theta}_\alpha\bar{\psi}^\alpha(t) + (\theta\theta - \bar{\theta}\bar{\theta})F(t) + \\ &+ i(\theta\theta + \bar{\theta}\bar{\theta})G(t) + \theta^\alpha\bar{\theta}_\beta B^\beta_\alpha(t) + \theta\theta\bar{\theta}_\alpha\bar{\chi}^\alpha(t) - \\ &- \bar{\theta}\bar{\theta}\theta^\alpha\bar{\chi}_\alpha(t) + \theta\theta\bar{\theta}\bar{\theta}D(t).\end{aligned} \tag{3}$$

For brevity, we introduced the following notation: $\theta\theta = \theta^\alpha\theta_\alpha$ with the upper and lower positions of $\alpha$ defined by the formulae $\theta^\alpha = \varepsilon^{\alpha\beta}\theta_\beta$ and $\theta_\beta = \theta^\alpha\varepsilon_{\alpha\beta}$, where $\varepsilon_{\alpha\beta}$ is the antisymmetric tensor and $\varepsilon_{12} = \varepsilon^{12} = 1$ ; the functions $A(t)$, $B^\beta_\alpha(t)$, $F(t)$, $G(t)$, $D(t)$, are bosonic, while $\psi_\alpha(t)$ and $\chi_\alpha(t)$ are the Grassmann functions of time t . In the SQM construction they will describe bosonic and fermionic degrees of freesom, respectively. Therefore, in order to describe the one-dimensional SQM, it is necessary that among the boson functions of time only one should be dynamic, the remaining functions being auxialiary.

The superfield (3) is reducible. To extract the irreducible part from it, we consider the auxiliary chiral field $\Lambda(t,\theta)$ which satisfies the conditions $\bar{\mathcal{D}}_\alpha\Lambda = 0$ and realizes the irreducible representation of the N=2 supersymmetry

$$\Lambda(t,\theta) = M(\tau_L) + \theta^\alpha \psi_\alpha(\tau_L) + \theta\theta N(\tau_L),$$

$M(\tau_L)$ and $N(\tau_L)$ are the boson complex functions of the variable $\tau_L = t - \frac{i}{2}\bar{\theta}_\alpha \theta^\alpha$; $N = F + iG$; $M = A + iC$. In going from the chiral basis $(\tau_L, \theta)$ to the central basis $(t, \theta, \bar{\theta})$ we write the superfield $\Lambda(t,\theta)$ as

$$\Lambda(t,\theta) = A(t) + iC(t) + 2\theta^\alpha \psi_\alpha(t) + 2\theta\theta[F(t) + iG(t)] +$$
$$+ \frac{i}{2}\theta^\alpha \bar{\theta}_\alpha [\dot{A}(t) + i\dot{C}(t) + 2\theta^\beta \dot{\psi}_\beta(t)] +$$
$$+ \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}[\ddot{A}(t) + i\ddot{C}(t)].$$

The real part of this superfield has the form (3) but with fewer independent functions of time. Besides, it comparises only the time derivative of C(t) and it is, therefore, natural to introduce a new function $B(t) = -2\dot{C}(t)$. Thus we come to the real superfield

$$\Phi(t,\theta) = A(t) + \theta^\alpha \psi_\alpha(t) - \bar{\theta}_\alpha \bar{\psi}^\alpha(t) + \theta\theta(F + iG) -$$
$$\bar{\theta}\bar{\theta}(F - iG) + \theta^\alpha \bar{\theta}_\alpha B + i\theta^\alpha \bar{\theta}_\alpha (\theta^\beta \dot{\psi}_\beta(t) + \bar{\theta}_\beta \dot{\bar{\psi}}^\beta(t)) + \quad (4)$$
$$+ \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\ddot{A}(t)$$

with the following supersymmetry transformation laws for its components

$$\delta A(t) = \epsilon^\alpha \psi_\alpha(t) - \bar{\epsilon}_\alpha \bar{\psi}^\alpha(t),$$
$$\delta \psi_\alpha(t) = i\bar{\epsilon}_\alpha \dot{A}(t) + 2\epsilon_\alpha (F + iG) + \bar{\epsilon}_\alpha B,$$
$$\delta(F + iG) = i\bar{\epsilon}_\alpha \dot{\psi}^\alpha, \quad (5)$$
$$\delta B = i(\epsilon^\alpha \dot{\psi}_\alpha + \bar{\epsilon}_\alpha \dot{\bar{\psi}}^\alpha).$$

In comparison with Expression (3), the superfield (4) comparises the boson functions of time half as many. Note that if made an analogous procedure in the (1,2) superspace corresponding to Witten's SQM /1/, we should again come to the most general real super-

field /7/. Another, more convement way to derive the Expression (4) for the irreducible real superfield is to consider some constraints on this superfield. Such constraints are

$$\{\varepsilon_{ac} \mathcal{D}_b \bar{\mathcal{D}}^c + \varepsilon_{bc} \mathcal{D}_a \bar{\mathcal{D}}^c\} \Phi = 0 \tag{6}$$

Indeed, the solution of these constraints is superfield (4).

3. Lagrangian of one-dimensional SQM

The only Lagrangian for the field $\Phi(t,\theta)$ (4) which has no higher derivatives and may be represented as an integral over all the Grassmann coordinates, has the form

$$\mathcal{L}_0 = -2\int d^2\theta \, d^2\bar{\theta} \, V(\Phi(t,\theta)), \tag{7a}$$

where $V(\Phi)$ is an arbitrary real function. Besides, in accordance with the transformations laws (5), the Lagrangian can also include some additional invariant terms with the arbitrary numerical parameters $f$, $g$, $b$:

$$\mathcal{L}_1 = f\,F(t) + g\,G(t) + b\,B(t) \tag{7b}$$

which are the analogs of the F term in 4-dimensional supersymmetry.

After integrating over the Grassmann variables in formula (7a) and adding together the obtained result and Expression (7b), we get the Lagrangian

$$\begin{aligned}\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 = {} & V''(A)\left(\tfrac{1}{2}\dot{A}^2 + 2F^2 + 2G^2 + \tfrac{1}{2}B^2 - \tfrac{i}{2}\psi^\alpha\dot{\bar{\psi}}_\alpha - \tfrac{i}{2}\bar{\psi}_\alpha\dot{\psi}^\alpha\right) \\ & + \tfrac{1}{2}V'''(A)\left\{B\psi^\alpha\bar{\psi}_\alpha + F(\bar{\psi}\bar{\psi} - \psi\psi) + i\,G(\bar{\psi}\bar{\psi} + \psi\psi)\right\} \\ & - \tfrac{1}{8}V^{IV}(A)\,\psi\psi\bar{\psi}\bar{\psi} + bB + fF + gG,\end{aligned}$$

in which B, F, G variables are auxiliary. $V''(A)$, $V'''(A)$ and $V^{IV}(A)$ denote, respectively, the second, trird and fourth derivatives of the function $V(A)$ with respect to its argument.

The B, F, G variables may be determinad from the appropriate equations of motion, and after substituting them again into the Lagrangian we have

$$\mathcal{L} = V''(A)\left\{\tfrac{1}{2}\dot{A}^2 + \tfrac{i}{2}\dot{\bar{\psi}}_\alpha\psi^\alpha - \tfrac{i}{2}\bar{\psi}_\alpha\dot{\psi}^\alpha\right\} - \frac{1}{8V''(A)}\left\{f^2 + g^2 + 4b^2\right\} -$$

$$-\frac{V'''(A)}{8V''(A)}\left\{(f+ig)\bar{\psi}\bar{\psi}+4b\,\psi^{\alpha}\bar{\psi}_{\alpha}-(f-ig)\psi\psi\right\}+$$
$$\frac{1}{16V''(A)}\left\{3(V'''(A))^{2}-2V^{IV}(A)V''(A)\right\}\psi\psi\bar{\psi}\bar{\psi}\,.$$

To bring the kinetic term into the standard form, we replace the variables A and $\psi^{\alpha}$ by new variables $x$ and $\chi$ using the formulae

$$\sqrt{V''(A)}\,\psi^{\alpha}=\chi^{\alpha},\qquad \frac{dx}{dA}=\sqrt{V'(A)}$$

For convenience in the further consideration, we also introduce the function

$$W(\bar{x})=\frac{1}{2}\sqrt{f^{2}+g^{2}+4b^{2}}\,A(x)\equiv-\frac{\lambda}{2}A(x)\,. \tag{8}$$

In term of the introduced quantities, the Lagrangian takes on the form

$$\mathcal{L}=\frac{1}{2}\dot{x}^{2}+\frac{i}{2}\dot{\bar{\chi}}_{\alpha}\chi^{\alpha}-\frac{i}{2}\bar{\chi}_{\alpha}\dot{\chi}^{\alpha}-\frac{1}{2}\left(\frac{dW}{dx}\right)^{2}-$$
$$-\frac{1}{2}\frac{d^{2}W}{dx^{2}}\left\{(f+ig)\bar{\chi}\bar{\chi}-(f-ig)\chi\chi+4b\,\chi^{\alpha}\bar{\chi}_{\alpha}\right\}-$$
$$-\frac{d}{dx}\left(\frac{\frac{d^{2}W}{dx^{2}}}{\frac{dW}{dx}}\right)\chi^{1}\bar{\chi}_{1}\chi^{2}\bar{\chi}_{2}\quad .$$

Making an appropriate linear substitution

$$\varphi^{\alpha}=S^{\alpha}_{\beta}\chi^{\beta}+T^{\alpha\beta}\bar{\chi}_{\beta}$$

it is possible to eliminate the constants and from the Lagrangian and obtain the following final form for the Lagrangian

$$\mathcal{L}=\frac{1}{2}\dot{x}^{2}+\frac{i}{2}\dot{\bar{\varphi}}_{\alpha}\varphi^{\alpha}-\frac{i}{2}\bar{\varphi}_{\alpha}\dot{\varphi}^{\alpha}-\frac{1}{2}\left(\frac{dW}{dx}\right)^{2}-$$
$$-\bar{\varphi}_{\alpha}\varphi^{\alpha}\frac{d^{2}W}{dx^{2}}-\bar{\varphi}_{1}\varphi^{1}\bar{\varphi}_{2}\varphi^{2}\frac{d}{dx}\left(\frac{\frac{d^{2}W}{dx}}{\frac{dW}{dx}}\right)\,. \tag{9}$$

In this notation, the Lagrangian (9) fully coincides with that of the N=2 extended supersymmetric mechanics from /6/. By virtue of Expression (8), the superpotential $W(x)$ in (9) coinsides with the zero superfield component $\Phi(t,\theta)$ (4).

## 4. Conclusion

In our analysis the possibility of the superfield description of the N=2 extended one-dimensional supersymmetric quantum mechanics is essentially based on the existence of the $\Phi(t,\theta)$ superfield (4), which comprises only time derivative of the one bosonic variable which then becomes the dynamic variable .

However, for N=3 the situation is complicated by the presence of the time derivative of B(t) in the superfield. So, the possibility of the superfield description in this case needs further investigations.

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СУПЕРПОЛЕВОЕ ОПИСАНИЕ $N=2$ РАСШИРЕННОЙ ОДНОМЕРНОЙ СУПЕРСИММЕТРИЧНОЙ КВАНТОВОЙ МЕХАНИКИ

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