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Equation for Nonlinear optical propagation beyond the paraxial approximation

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Abstract

The beam propagation in optics is not only a fundamental but a practical problem. The commonly used approach is the paraxial approximation. It is natural in some situations such as the catastrophic beam collapse in self-focusing media to go beyond the paraxial approximation. Indeed since the late eighties and now more recently the problem of going beyond the paraxial approximation has been revisited numerically and analytically by several groups. In most of these approaches the refractive index variation associated with Kerr nonlinearity is incorporated but they do not take into account the vectorial effects and consequently fail to satisfy the divergence equation. More recently there have been attempts to incorporate the vectorial nature by considering the interaction between propagation and polarization. In particular the interaction between propagation and polarization was considered in a guiding structure for the description of intrafiber geometric rotation of polarization. Recently Crosignani et al. have proposed a different approach based on the coupled mode theory to deal with the problem of nonparaxial propagation.

The purpose and motivation of this work is to examine the general equation for linear and nonlinear optical propagation beyond the paraxial approximation in the context of the coupled mode approach. The complete set of equations incorporating the backward propagating modes are written out. The relation between self-focusing and nonparaxiality is discussed. It is well-known that the model equation for propagation of a laser beam in a nonlinear Kerr media is the nonlinear Schrodinger equation [NLS]. The singularities of NLS equation near the self-focusing region are looked at from the point of view of the general equation for propagation. In particular we attempt to examine the region of validity of NLS and compare the self-focusing region in NLS and the general propagation equation. It is interesting to look at the power in the paraxial and non-paraxial parts.

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1 Introduction

The Helmholtz equation which is the wave equation for the electric field $\mathbf{E}$ can be written as

$$\nabla^2\mathbf{E} + 2\nabla(\mathbf{E}\cdot\nabla\ln n) + k_0^2 n^2\mathbf{E} = 0. \tag{1}$$

It is well-known that Eq. 1 can be readily deduced from Maxwell's equation for a media with a refractive index depending on space, $n(\mathbf{r},z) = n_1 + \delta n$, where $\mathbf{r} = (x,y)$ describes the plane transverse to the beam propagation direction z, and n_1 is the unperturbed part of the refractive index δn being a representative of the perturbation. A reasonable and commonly used approximation is the paraxial approximation. In the paraxial approximation the dependence of the electric field on the transverse and longitudinal coordinates is separated out for a general monochromatic wave as [1]

$$\mathbf{E}(\mathbf{r},z,t) = \mathbf{F}(\mathbf{r},z)\exp i(\omega t - kz). \tag{2}$$

If one plugs Eq. 2 into Eq. 1 and assumes that the perturbation of refractive index is small, i.e. $\delta n \ll 1$ one arrives at the Fock-Leontovich wave equation:

$$(\frac{\partial}{\partial z} + \frac{\mathrm{i}}{2\mathrm{k}}\nabla_\perp^2)\mathbf{F} = -\mathrm{i}\frac{\mathrm{k}}{\mathrm{n_1}}\delta\mathrm{n}\mathbf{F}. \tag{3}$$

A particular form for perturbation of refractive index δn is usually chosen, namely

$$\delta n = n_2|\mathbf{E}|^2 = n_2|\mathbf{F}|^2. \tag{4}$$

This form of variation of the refractive is associated with Kerr nonlinearity. Substituting Eq. 4 into Eq. 3 one arrives at

$$(\frac{\partial}{\partial z} + \frac{\mathrm{i}}{2\mathrm{k}}\nabla_\perp^2)\mathbf{F} + \mathrm{i}\mathrm{n}_2\frac{\mathrm{k}}{\mathrm{n_1}}|\mathbf{F}|^2\mathbf{F} = 0. \tag{5}$$

Eq. 5 is nothing but of the form of the nonlinear Schrodinger equation [NLS][4],

$$-i\frac{\partial\psi}{\partial z} + \Delta_\perp\psi + |\psi|^2\psi = 0. \tag{6}$$

Eq. 6 is the model equation [i.e. NLS] for propagation of laser beam with a Kerr nonlinearity, with $\psi(x,y,z)$ being the electric field envelope, z the distance in the direction of the beam propagation and $\Delta_\perp = \nabla_\perp^2$ is the two dimensional Laplacian in the transverse (x,y) plane.

[1] In Eq. 2 we use $\mathbf{F}$ for the envelope function rather than $\mathbf{A}$ as is usual [see for e.g. [15]] in order to avoid confusion with the vector potential. For most part we try to adhere to the usual conventions in order to make it convenient for the reader.

It is important to note that in the derivation of Eq. 3 from Eq. 1 three approximations in the form of assumptions have been made, namely

1. Paraxial approximation.

2. Slowly varying approximation of the vector field along the direction of propagation, i.e. dropping of the second derivative of the vector field along the direction of propagation [z-axis], $\partial^2\mathbf{F}/\partial z^2$.

3. Scalar approximation, which means that the term in the Helmholtz equation, i.e. Eq. 1, which mixes the polarization, namely the term $2\nabla(\mathbf{E}\cdot\nabla\ln n)$ is neglected. Once this term is dropped the divergence equation $\nabla\cdot(n^2\mathbf{E}) = 0$ is no longer satisfied, as is explicit from the derivation of the Helmholtz equation from the Maxwell's equations. An immediate consequence of dropping the polarization mixing term is that the solutions no longer satisfy the divergence equation $\nabla\cdot(n^2\mathbf{E}) = 0$. One of the places where these points were reported in literature is [3]

The NLS equation, which as mentioned before is the model equation for a laser beam propagation in a media with a Kerr nonlinearity, was used by Kelley [1] to predict the possibility of catastrophic self-focusing of optical beams whose power is above a threshold value, a prediction confirmed by experiments [2]. However in view of the approximations, see items 1 through 3 above, used in arriving at NLS from the complete wave equation, namely the Helmholtz equation, one does not expect the NLS equation to model successfully the advanced stages of the self-focusing. Indeed starting with the work of Feit and Fleck [5] several authors [4, 5, 6, 7, 8, 9] have argued that the approximations used in arriving at NLS should be relaxed in order to get rid of the unphysical singularity formation predicted by the NLS.

A different and attractive approach to study the laser beam propagation which is able to describe the propagation in presence of any tensorial refractive index variation at any order in the smallness of the parameter λ/w is based on the coupled mode theory [10, 11, 12, 13, 14, 15]. The coupled mode approach is able to deal with both linear and nonlinear beam propagation. Some of the distinct advantages of the coupled mode approach are:

1. It is inherently first order in $\partial/\partial z$ without the need of invoking the slowly varying approximation as is done in the approaches based on the Helmholtz equation ,as mentioned above.

2. It automatically takes into account the vectorial effects associated with the polarization mixing term $2\nabla(\mathbf{E}\cdot\nabla\ln n)$.

3. One motivation for using the coupled mode approach is that it is more suited to quantization which is needed for studying quantum effects. This is one of our motivations for considering it.

The layout of this paper is as follows. In the next section we discuss the coupled mode theory approach to the nonparaxial equation for linear and nonlinear optical beam propagation. Section three deals with the incorporation of the backward propagating modes. Section four contains a discussion of questions and possible solutions. Concluding remarks are given in section five.

2 Coupled Mode Theory approach to Nonparaxial equation for Linear and Nonlinear optical beam propagation

As is usual in coupled mode theory we split the quantities [i.e. fields and derivatives (operators)] into transverse and longitudinal parts. In particular the propagating field can be expressed as a superposition of transverse radiation modes of the unperturbed refractive index n_1 and its evolution can be looked at in the presence of the perturbation associated with δn in the framework of the coupled mode theory. A choice for the continuum set of orthogonally polarized normalized modes of the unbound space or radiation modes is [14]

$$\begin{aligned}\mathbf{E}(\xi,1;\mathbf{r}) &= N_1\exp(-i\,\xi\cdot\mathbf{r})\left[\hat{x}-\frac{\xi_x}{\beta_\xi}\hat{z}\right]\\ \mathbf{E}(\xi,2;\mathbf{r}) &= N_2\exp(-i\,\xi\cdot\mathbf{r})\left[\frac{\xi_x\xi_y}{\beta_\xi}\hat{x}-(\beta_\xi+\frac{\xi_x^2}{\beta_\xi})\hat{y}+\xi_y\hat{z}\right].\end{aligned} \tag{7}$$

The magnetic field associated with the electric field in 7 is given by

$$\mathbf{H}(\xi,\sigma;\mathbf{r}) = (1/\omega\mu_0)(\xi+\beta_\xi\hat{z})\times\mathbf{E}(\xi,\sigma;\mathbf{r}),\ \sigma = 1,2. \tag{8}$$

We note that ξ is the propagation vector associated with the transverse plane defined by the vector $\mathbf{r}$. Similarly β_ξ is the propagation component associated with the direction of propagation, namely the z direction, and is given by

$$\begin{aligned}\beta_\xi &= (k^2-\xi_x^2-\xi_y^2)^{1/2},\\ &= (k^2-\xi^2)^{1/2},\ 0\le\xi\le k.\end{aligned} \tag{9}$$

The normalization factors N_1 and N_2 are given by

$$\begin{aligned}N_1(\xi) &= (1/2\pi)[\beta_\xi\omega\mu_0/(\beta_\xi^2+\xi_x^2)]^{1/2},\\ N_2(\xi) &= (1/2\pi)[\beta_\xi\omega\epsilon_1/(\beta_\xi^2+\xi_x^2)]^{1/2},\\ N_2(\xi) &= N_1(\xi)/k\end{aligned} \tag{10}$$

We recall that $k = (\omega/c)n_1 = k_0 n_1$. It is straightforward to see that the modes given in 7 are orthogonal i.e. and satisfy the orthonormalization condition

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\mathbf{r}\hat{z}\cdot\mathbf{E}(\xi,\sigma;\mathbf{r})\times\mathbf{H}^*(\xi',\sigma';\mathbf{r}) = \delta^2(\xi-\xi')\delta_{\sigma,\sigma'} \quad \sigma,\sigma' = 1,2 \tag{11}$$

with normalization factors given as in 10.

The electric field can be expanded in terms of the modes as

$$\begin{aligned}\mathbf{E}(\mathbf{r},z,t) &= \sum_\sigma \int\int d\xi \mathbf{E}(\xi,\sigma;\mathbf{r})[c^+(\xi,\sigma;z)\exp(i\omega t - i\beta_\xi z) \\ &\quad + c^-(\xi,\sigma;z)\exp(i\omega t + i\beta_\xi z)]\end{aligned} \tag{12}$$

Let us consider only the forward propagating mode c^+ for the moment ignoring its coupling with the backward propagating mode c^-. Denoting c^+ by c Eq. 12 simplifies to

$$\mathbf{E}(\mathbf{r},z,t) = \sum_\sigma \int\int d\xi \mathbf{E}(\xi,\sigma;\mathbf{r})c(\xi,\sigma;z)\exp(i\omega t - i\beta_\xi z). \tag{13}$$

On the other hand we may write $\mathbf{E}(\mathbf{r},z,t)$ in terms of the envelope function $\mathbf{F}$ as

$$\begin{aligned}\mathbf{E}(\mathbf{r},z,t) &\equiv \exp(i\omega t - ikz)\mathbf{F}(\mathbf{r},z) \\ &\equiv \sum_\sigma \exp(i\omega t - ikz)\mathbf{F}^\sigma(\mathbf{r},z)\end{aligned} \tag{14}$$

We can immediately see from Eqs. 13 and 14 a relation between the envelope function and the expansion coefficients c, since

$$\begin{aligned}\mathbf{E}(\mathbf{r},z,t) &= \sum_\sigma \exp(i\omega t - ikz)\int\int d\xi \mathbf{E}(\xi,\sigma;\mathbf{r}) \\ &\quad \exp[-i(\beta_\xi z - kz)]c(\xi,\sigma;z) \\ &\equiv \exp(i\omega t - ikz)\mathbf{F}(\mathbf{r},z) \\ &\equiv \sum_\sigma \exp(i\omega t - ikz)\mathbf{F}^\sigma(\mathbf{r},z)\end{aligned} \tag{15}$$

holds as is clear from the above discussion. We may thus write

$$\mathbf{F}^\sigma(\mathbf{r},z) = \int\int d\xi \mathbf{E}(\xi,\sigma;\mathbf{r})\exp[-i(\beta_\xi z - kz)]c(\xi,\sigma;z). \tag{16}$$

Eq. 16 [and Eq. 20 below, when the forward and backward modes are considered] are important equations since they state the essence of the coupled mode theory. For it is from the these equations that we understand the relation between the envelope function and the expansion coefficients, c's. Moreover we can see from these equations that in the coupled mode approach why we need not use the slowly varying approximation to ignore the second derivative.

The evolution equation for the c coefficient can be written as

$$\begin{aligned}\frac{d}{dz}c(\xi,\sigma;z) &= -i\omega\epsilon_0 n_1 \sum_{\sigma'}\int\int d\xi' \int\int d\mathbf{r}' \\ &\quad \exp[i(\beta_\xi - \beta_{\xi'})z]c(\xi',\sigma';z)\mathbf{E}^*(\xi,\sigma;\mathbf{r}')[\delta n(\mathbf{r}',z):\mathbf{E}(\xi',\sigma';\mathbf{r}')].\end{aligned} \tag{17}$$

We can convert Eq. 17 to an evolution equation for the envelope function as follows. Multiplying Eq. 17 by $\mathbf{E}(\xi,\sigma;\mathbf{r})\exp[-i(\beta_\xi z - kz)]$, integrating over ξ, and substituting the resulting expression into the z derivative of Eq. 16, as explained in the next section, we obtain

$$\begin{aligned}L\mathbf{F}^\sigma(\mathbf{r}',z) &= -i\omega\epsilon_0 n_1 \int\int d\mathbf{r}' \int\int d\xi \\ &\quad \mathbf{E}(\xi,\sigma;\mathbf{r})\mathbf{E}^*(\xi,\sigma;\mathbf{r}')[\delta n(\mathbf{r}',z):\mathbf{F}(\mathbf{r}',z)], \\ L &\equiv L^+, \\ L^+ &\equiv \frac{\partial}{\partial z} + \frac{i}{2k}\nabla_\perp^2 - \frac{i}{8k^3}\nabla_\perp^4 + \ldots\end{aligned} \tag{18}$$

In order to express the right-hand side of the above equation 18 in differential form we can expand the tensors $\mathbf{E}(\xi,\sigma;\mathbf{r})\mathbf{E}^*(\xi,\sigma;\mathbf{r}')$ as a power series in the ξ_x and ξ_y. Using the expression of the nonlinear refractive index δn which is valid in the presence of the optical Kerr effect [13, 15] and assuming for simplicity that $F_z \sim -(i/k)\nabla_\perp\cdot\mathbf{F}_\perp$,$F_y = 0$, $F_y(z=0) = 0$ one obtains the following evolution equation for $u = (n_2/n_1)^{1/2}F_x$,

$$\begin{aligned}&\left[\frac{\partial}{\partial Z} + \frac{i}{2}\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right) - \frac{i}{8}\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)^2\right]u + i|u|^2 u = \\ &-\frac{i}{2}\left(\frac{\partial^2}{\partial X^2} - \frac{\partial^2}{\partial Y^2}\right)(|u|^2 u) - \frac{i}{3}\left|\frac{\partial u}{\partial X}\right|^2 u \\ &-\frac{i}{3}\left(|u|^2\frac{\partial^2 u}{\partial X^2} - u^2\frac{\partial^2 u^*}{\partial X^2}\right) - \frac{i}{3}\frac{\partial}{\partial X}\left(|u|^2\frac{\partial}{\partial X}u\right),\end{aligned} \tag{19}$$

where $(X,Y,Z) = k(x,y,z)$.

3 Incorporation of the Backward Propagating Modes

In the presence of both the forward and backward modes present we may readily generalize Eq. 16 to read

$$\mathbf{F}^{\sigma,\pm}(\mathbf{r},z) = \int\int d\xi \mathbf{E}(\xi,\sigma;\mathbf{r})\exp[\mp i(\beta_\xi z - kz)]c^\pm(\xi,\sigma;z). \tag{20}$$

The evolution equations for the c coefficients are:

$$\begin{aligned}\frac{d}{dz}c^+(\xi,\sigma;z) &= \sum_{\sigma'}\int\int d\xi'[K^{+,+}(\xi,\sigma;\xi',\sigma';z) \\ &\quad \exp[i(\beta_\xi - \beta_{\xi'})z]c^+(\xi',\sigma';z) \\ &\quad + K^{+,-}(\xi,\sigma;\xi',\sigma';z)\exp[i(\beta_\xi + \beta_{\xi'})z]c^-(\xi',\sigma';z)]\end{aligned} \tag{21}$$

and

$$\begin{aligned}\frac{d}{dz}c^-(\xi,\sigma;z) &= \sum_{\sigma'}\int\int d\xi'[K^{-,+}(\xi,\sigma;\xi',\sigma';z)\\ &\exp[-i(\beta_\xi+\beta_{\xi'})z]c^+(\xi',\sigma';z)\\ &+K^{-,-}(\xi,\sigma;\xi',\sigma';z)\exp[-i(\beta_\xi-\beta_{\xi'})z]c^-(\xi',\sigma';z)] \qquad (22)\end{aligned}$$

with the condensed notation defined as follows

$$\begin{aligned}K^{p,q}(\xi,\sigma;\xi',\sigma';z) &= pK(\xi,\sigma;\xi',\sigma';z)+qk(\xi,\sigma;\xi',\sigma';z), \quad p,q=+,-,\\ K(\xi,\sigma;\xi',\sigma';z) &= -i\omega\epsilon_0 n_1\int\int d\mathbf{r}\mathbf{E}_t^*[\delta n(\mathbf{r},z):\mathbf{E}_t(\xi',\sigma';\mathbf{r})],\\ k(\xi,\sigma;\xi',\sigma';z) &= -i\omega\epsilon_0 n_1\int\int d\mathbf{r}\mathbf{E}_z^*[\delta n(\mathbf{r},z):\mathbf{E}_z(\xi',\sigma';\mathbf{r})],\\ \mathbf{T}:\mathbf{V} &\equiv T_{ij}V_j. \qquad (23)\end{aligned}$$

In order to obtain the evolution equations for the envelope function, as mentioned above, we first differentiate Eq. 20 with respect to z, to obtain,

$$\begin{aligned}\frac{\partial}{\partial z}\mathbf{F}^{\sigma,\pm}(\mathbf{r},z) &= \int\int d\xi\mathbf{E}(\xi,\sigma;\mathbf{r})\exp[\mp i(\beta_\xi z-kz)]\frac{d}{dz}c(\xi,\sigma;z)\\ &\mp i(\beta_\xi-k)\mathbf{F}^{\sigma,\pm}(\mathbf{r},z),\\ \beta_\xi &= (k^2-\xi^2)^{1/2}=k(1-\xi^2/k^2)^{1/2},\\ (\beta_\xi-k) &\approx (k[1-\frac{1}{2}\frac{\xi^2}{k^2}-\frac{1}{8}(\frac{\xi^2}{k^2})^2+....]-k),\\ \xi &\equiv -i\nabla_\perp,\\ L^\pm &\equiv \frac{\partial}{\partial z}\pm\frac{i}{2k}\nabla_\perp^2\mp\frac{i}{8k^3}\nabla_\perp^4+.... \qquad (24)\end{aligned}$$

Multiplying Eq. 21 by $\mathbf{E}(\xi,\sigma;\mathbf{r})\exp[-i(\beta_\xi z-kz)]$ and integrating over ξ and substituting in Eq. 24 for $\frac{d}{dz}c^+(\xi,\sigma;z)$ using the expression for $\frac{d}{dz}c^+(\xi,\sigma;z)$ given in Eq. 21, we obtain for the evolution equation for the forward propagating mode,

$$\begin{aligned}L^+\mathbf{F}^{\sigma,+} &= \sum_{\sigma'}\int\int d\xi\mathbf{E}(\xi,\sigma;\mathbf{r})\int\int d\xi'[K^{+,+}(\xi,\sigma;\xi',\sigma';z)\\ &\exp[-i(\beta_{\xi'}-k)z]c^+(\xi',\sigma';z)\\ &+K^{+,-}(\xi,\sigma;\xi',\sigma';z)\exp[i(\beta_{\xi'}+k)z]c^-(\xi',\sigma';z)]. \qquad (25)\end{aligned}$$

To obtain the evolution equation for the backward propagating mode we multiply Eq. 22 by $\mathbf{E}(\xi,\sigma;\mathbf{r})\exp[i(\beta_\xi z-kz)]$, integrate over ξ and substitute for $\frac{d}{dz}c^-(\xi,\sigma;z)$ in Eq. 24 using Eq. 22,

$$\begin{aligned}L^-\mathbf{F}^{\sigma,-} &= \sum_{\sigma'}\int\int d\xi\mathbf{E}(\xi,\sigma;\mathbf{r})\int\int d\xi'[K^{-,+}(\xi,\sigma;\xi',\sigma';z)\\ &\exp[-i(\beta_{\xi'}+k)z]c^+(\xi',\sigma';z)\\ &+K^{-,-}(\xi,\sigma;\xi',\sigma';z)\exp[i(\beta_{\xi'}-k)z]c^-(\xi',\sigma';z)]. \qquad (26)\end{aligned}$$

Eqs. 25 and 26 are a set of nonlinear coupled differential equations.

We may exploit various approximations based on specific situations to to simplify Eqs. 25 and 26. For example, we may ignore the longitudinal component of the field, i.e. $\mathbf{E}_z$. The relationship between the envelope function and the c coefficients in conjunction with the definitions given in Eqs. 23 allows us to write Eqs. 25 and 26 as

$$\begin{aligned}L^+\mathbf{F}^{\sigma,+} &= -i\omega\epsilon_0 n_1\sum_{\sigma'}\int\int d\mathbf{r}'\int\int d\xi\mathbf{E}(\xi,\sigma;\mathbf{r})\{\mathbf{E}^*(\xi,\sigma;\mathbf{r}')\\ &[\delta n(\mathbf{r}',z):\mathbf{F}^{\sigma',+}(\mathbf{r}';z)]\\ &+\mathbf{E}^*(\xi,\sigma;\mathbf{r}')\exp[2ikz][\delta n(\mathbf{r}',z):\mathbf{F}^{\sigma',-}(\mathbf{r}',z)]\}. \qquad (27)\end{aligned}$$

and

$$\begin{aligned}L^-\mathbf{F}^{\sigma,-} &= i\omega\epsilon_0 n_1\sum_{\sigma'}\int\int d\mathbf{r}'\int\int d\xi\mathbf{E}(\xi,\sigma;\mathbf{r})\{\mathbf{E}^*(\xi,\sigma;\mathbf{r}')\\ &[\delta n(\mathbf{r}',z):\mathbf{F}^{\sigma',-}(\mathbf{r}';z)]\\ &+\mathbf{E}^*(\xi,\sigma;\mathbf{r}')\exp[-2ikz][\delta n(\mathbf{r}',z):\mathbf{F}^{\sigma',+}(\mathbf{r}',z)]\}. \qquad (28)\end{aligned}$$

respectively. We note that in the absence of the coupling to the backward propagating mode, Eq. 27 reduces to Eq. 18, as expected. This provides a check on our calculation. It is important to keep in mind that it is not permissible to neglect the derivatives of c^- even though the amplitude coefficient itself does not grow to appreciable values [11].

We thus see that based on the coupled mode theory the "simplest" equation which describes the linear and nonlinear optical propagation beyond the paraxial approximation is Eq. 19. The incorporation of the backward propagating mode leads to a more complicated system of equations. Based on the nonlinear equation, viz Eq. 19 for the beam propagation beyond the paraxial approximation one may address some physically relevant questions and possible solutions to these. We briefly discuss these in the next section.

4 Questions and Possible Solutions

The following questions naturally arise in the context of beam propagation using methods beyond the paraxial approximation and which take into account the vectorial effects:

1. Can a small beam nonparaxiality arrest self-focusing of optical beams in a Kerr media?
2. What is the effect on the self-focusing of optical beams when one uses vectorial nonparaxial theory instead of the scalar nonparaxial theory?
3. What can we say about the power balance equation resulting from the nonlinear beam propagation equation?

To discuss the above points we note that Fibich [4] argued on the basis of a modified NLS, viz,

$$\varepsilon\frac{\partial^2\psi}{\partial z^2} - i\frac{\partial\psi}{\partial z} + \Delta_\perp\psi + |\psi|^2\psi = 0, \tag{29}$$

where

$$\varepsilon = \left(\frac{\lambda}{4\pi r_0}\right)^2, \tag{30}$$

that nonparaxiality arrests self-focusing when the beam width becomes comparable to its wavelength. Furthermore the analysis reported in [4] claimed that a series of focusing-defocusing cycles of decreasing magnitude follows, ending with a final defocusing stage. Starting from the vector wave equation Chi and Guo [8] used an order-of-magnitude analysis to give a model for nonparaxial propagation. They [8] claimed that linearly polarized circular input beam will become elliptic in the self-focusing process and that their model leads to noncatastrophic self-focusing with less maximum on-axis intensity than that of Feit and Fleck [5] and Akhmediev and co-workers [6, 7].

Eq. 19 is a generalization of the standard parabolic equation that describes the paraxial propagation in the following ways:

- It includes the nonparaxial terms up to second order in the ratio of the wavelength to the characteristic dimensions of the beam, which can be taken to be the beam waist w, i.e. the ratio λ/w. It is clear from the expansion in Eq. 24 that in the context of the present formalism we can keep the nonparaxial terms to any order in λ/w.
- Vectorial effects are automatically taken into account.

Furthermore it is first order in $\partial/\partial z$, which has been accomplished without using the slowly varying approximation. We note that models of both Fibich [4] and Chi and Guo [8] contain second derivative in the z variable. It is worth noting that Eq. 19 bears some resemblance to the model equation given by Chi and Guo [8]. Our initial analysis of Eq. 19 indicates:

- The unphysical singularity of NLS disappears as expected. In other words the self-focusing is noncatastrophic.
- An initial linearly polarized circular beam is changed to an elliptically polarized one during the self-focusing process. This is expected, since cross-coupling between components of vector field leads to anisotropic behavior. The anisotropies are the result of polarization mixing.

Simulation results in [5] show abrupt power loss at the self-foci and more gradual power loss in between that eventually lead to cessation of self-focusing. We expect that the power balance equation based on the complete treatment such as in coupled mode theory would account for all power transfers so that self-focusing is arrested. Our initial rough calculation confirms this expectation. Moreover coupled mode theory provides a nice picture for power transfer [11].

5 Conclusions

We have presented, as a first step, a discussion of the the general equation for linear and nonlinear optical propagation beyond the paraxial approximation. In this note we have examined the problem of beam propagation using the coupled mode formalism rather than the usual treatment based on the Helmholtz equation. The coupled mode approach has recently been also advocated in [15], however they do not include the backward propagating modes. We have included the backward propagating modes. Initial analysis of the nonlinear equation for beam propagation suggests that the unphysical singularity of the NLS encountered in the self-focusing region is absent. Moreover since the vectorial effects are automatically included, we expect the polarization to change during the self-focusing process. Elementary reasoning suggests that an initial linearly polarized circular beam is changed to an elliptically polarized one during the self-focusing process.

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