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Talk given at
the 31st International Symposium Ahrenshoop on the Theory of Elementary Particles,
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Status of the electroweak phase transitions

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Abstract: The observed baryon asymmetry of the universe has finally been determined at the finite temperature electroweak phase transition. In order to understand the baryon asymmetry a quantitative description of the electroweak phase transition is needed. In this talk some features of this phase transition are summarized. Particular interest is paid to the analytical and lattice estimates on the sphaleron transition and to the nature of the electroweak phase transition in the standard model and in the minimal supersymmetric standard model. On the one hand due to the large top-quark mass no SM-like Higgs boson can give strong enough phase transition. Thus, in the standard model the electroweak baryogenesis scenario can be excluded. On the other hand there is a small region in the parameter space of the minimal supersymmetric standard model, which might explain the observed baryon asymmetry. Since this small region is predicted perturbatively a lattice confirmation is needed.

1 Introduction

According to the standard picture of the cosmological phase transitions at high temperatures (e.g. in the early universe) the electroweak gauge symmetry has been restored. As the universe expands and supercools there is a phase transition between the high temperature "symmetric" and low temperature "broken" phases. The characteristics of this phase transition (critical temperature: T_c, surface tension: σ, etc.) are clearly of interest. There are evidences that the world is made of exclusively matter, at least on the $10^{13} M_\odot$ scale. The quantitative measure of the baryon asymmetry is the dimensionless ratio of the baryon number density to the entropy density

$$\Delta_B = B/s \approx 5 \cdot 10^{-11}. \tag{1}$$

Since no known mechanism can separate on the above huge scales we have to understand the origin of this asymmetry. There are basically two possibilities. The baryon asymmetry might have been an initial condition of our universe or it could have been generated in the early universe. The second possibility (baryogenesis) is clearly more attractive.

In order produce the observed baryon asymmetry three conditions must be satisfied.

a. baryon number violating processes

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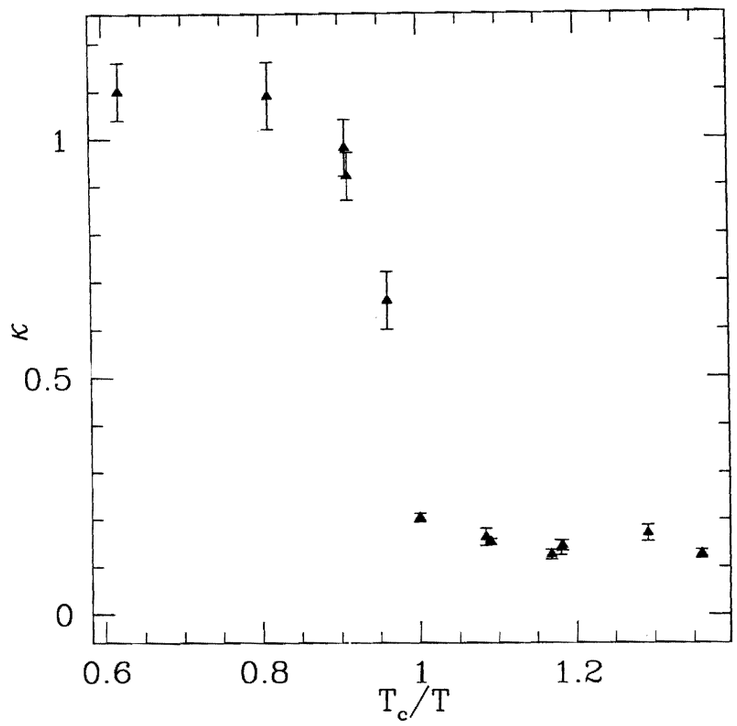

Figure 1: Transition rate normalized by T^4 as a function of the temperature.

b. C and CP violation
c. departure from the thermal equilibrium (the basic question is the strength of the phase transition).
It is easy to understand the role of these conditions.
a. In the lack of baryon number violation the baryoniC charge of the universe were constant. Assuming zero baryon number as an initial condition would result in no baryon asymmetry today.
b. If C or CP were conserved, then the rate of processes producing baryons (a) would be the same as the rate of processes producing antibaryons. No net baryon number could have been generated.
c. In thermal equilibrium the universe stationary, it has no time dependence. If the initial condition were zero baryon number, it would remain zero forever. An attractive proposal is to understand the observed baryon asymmetry as produced at the finite temperature electroweak phase transition [1]. Section 2 will discuss the first (a) Section 3 the last (c) condition in this scenario. Section 4 contains the conclusion.

2 Sphaleron rate at finite temperature

The basic source of the baryon number violating processes in the standard model is the electroweak anomaly for baryon number, which demands that

$$\Delta B \sim g^2 \int dt d^3x \mathrm{tr} F\tilde{F}, \tag{2}$$

where g is the weak coupling and F is the field strength of the weak gauge field. This anomaly equation relates the produced baryon number to topological transitions of the weak gauge fields. The vacua with

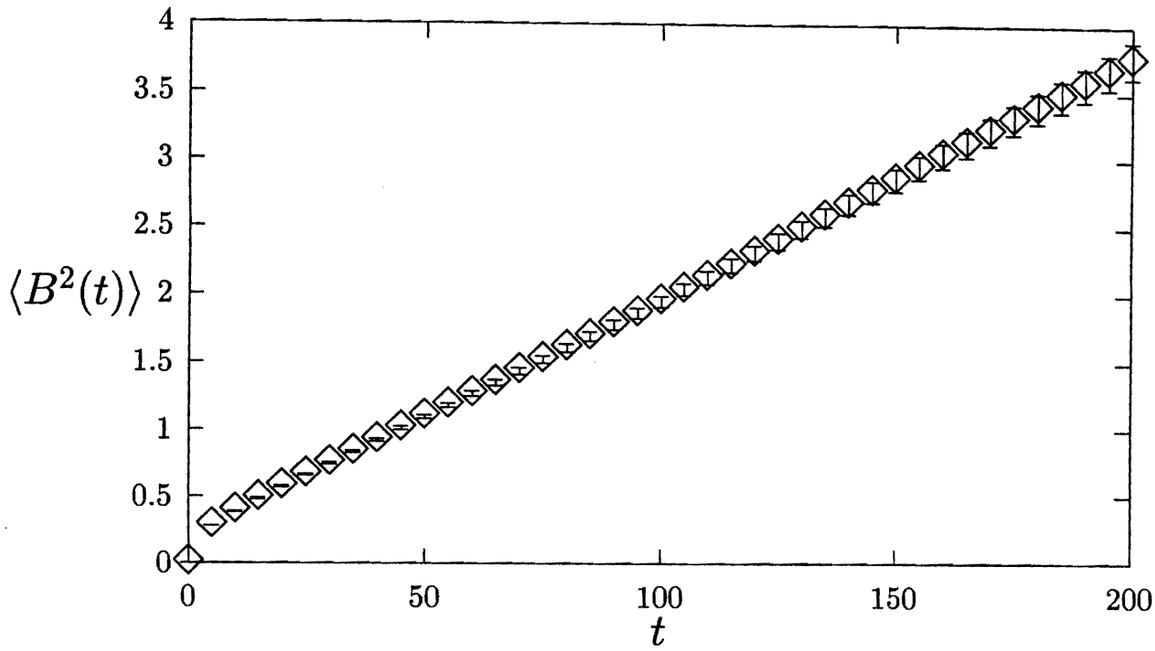

Figure 2: Diffusion of N_{CS}.

different topological numbers (N_{CS}) are separated by a potential barrier.

Note, that the above anomaly equation holds not only for baryons but for leptons, too. Thus, baryon and lepton numbers are violated simultaneously ($B + L$ violation).

At zero temperature the transition between the vacua is a tunneling event with an unobservably small probability: $\approx 10^{-170}$. The transition at high temperatures, but below T_c is a thermal jump. The system jumps up to the top of the barrier (a saddle point: sphaleron) and rolls down to the neighbouring vacuum. In the meantime baryon and lepton numbers are violated. The jump to the barrier, therefore the transition rate is suppressed by the Boltzmann factor

$$\Gamma \propto \exp\left[-E_{sph}/T\right], \tag{3}$$

where the sphaleron energy is proportional to the mass of the W-boson $E_{sph} \propto m_W/\alpha_W$ [2].

Rapid baryon violation processes in the broken phase can wash out the asymmetry generated before. Therefore a "minimal" condition for a successful baryogenesis is that these processes should be slower than the expansion rate of the universe. Comparing (3) with the Hubble constant the minimal condition can be written as $\varphi_b/T_b > 1$, where φ_b is the value of the Higgs field at the transition temperature T_b.

Above T_c the exponential suppression of the transition rate is absent. Naive power counting suggest

$$\Gamma = \kappa \cdot (\alpha_w T)^4, \tag{4}$$

where the constant κ is of order 1.

Recently it has been argued [3] that the assumptions used to derive 4 are not valid. The real dependence on α_w is a different one

$$\Gamma = \kappa' \cdot \alpha_w^5 T^4. \tag{5}$$

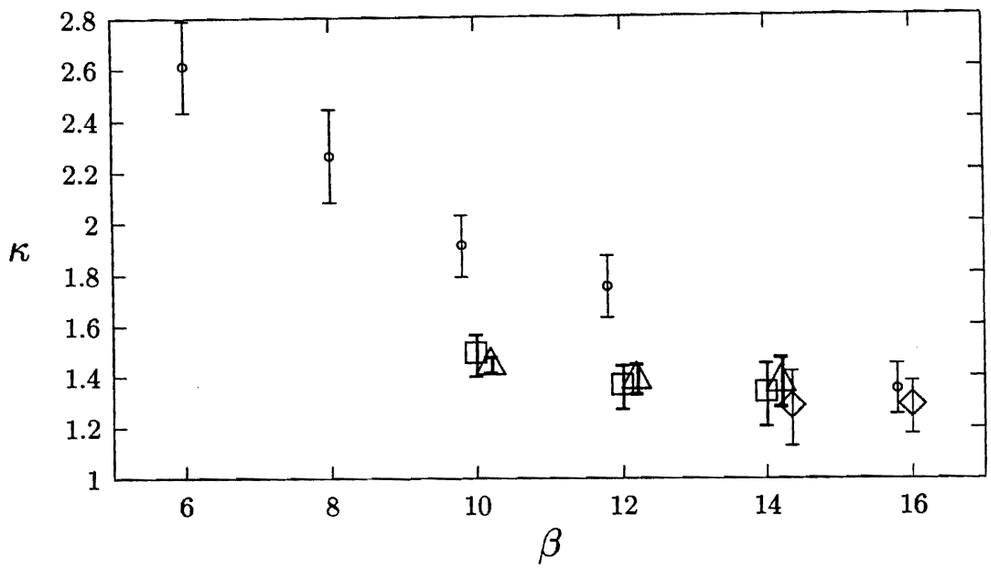

Figure 3: The dimensionless coefficient κ as a function of the inverse lattice spacing.

The origin of this damping is the fact that nearly static magnetic fields can be absorbed by the system, which results in a loss of energy for the magnetic fields of interest. In the symmetric phase the baryon number violating processes are much faster than the expansion rate of the universe.

In recent years there has been considerable activity in order to determine the sphaleron transition rate at both sides of the phase transition. No successful numerical method is known for a full Minkowskian theory; however, important results have been obtained by real time simulations in the classical approximation for the finite temperature theory.

The used procedure contains several steps. The classical theory is formulated in terms of fields and their conjugate momenta on a spatial lattice. The phase-space of the system is then sampled with the statistical weight $\exp(-H/T)$. Starting with some initial configuration from the thermal sample the classical canonical equations give the real-time evolution of the fields. As a function of time one can determine different observables and their averages (such as changes of the topological charge). There are fewer physical degrees of freedom in the theory than the number of the phase space variables used to formulate it. It is a highly non-trivial task to find effective thermalization algorithms for constrained systems. At present there are two good solutions to this problem [4].

In the broken phase numerical simulations [5] indicated an extreme difference between the lattice results and (3) The observed sphaleron transition rate was a bit smaller than in the symmetric phase; however, as it can be seen on Fig. 1 no Boltzmann suppression has been observed. Going down into the broken phase the difference between the analytical and lattice results is several orders of magnitude. It is argued [3] that the reason is the problematic definition of the topological charge on a lattice, which give a systematic error in the rate.

A large scale numerical study for the sphaleron rate in the symmetric phase [6] favoured the naive law

$$\Gamma = \kappa(\alpha_w T)^4. \tag{6}$$

The classical motion of the Chern-Simons number consists of two pieces. The thermal fluctuation, which

linearly diverges with the lattice cutoff, and a random walk between different vacua (see Fig. 2).

$$< B^2(t) >= c + \Gamma V t. \tag{7}$$

A detailed finite size and finite scaling analysis suggested that the above naive formula is correct with a coefficient of $\kappa = 1.09 \pm 0.05$.

According to [3] the observed rate might be only a lattice artifact. The authors reanalysed their data with an improved technique (cooling of lattice field configurations to determine the time evolution of the topological charge) [7].

They introduced, along with the real time t, a cooling time τ. The dynamical variables are functions of them: $p_i(t,\tau), q_i(t,\tau)$. The t evolution is given by the standard equation of motions, while for the evolution in the τ direction an overdamped motion is used

$$\partial_\tau q_i(t,\tau) = -\partial_{q,i} V[q(t,\tau)]. \tag{8}$$

It is easy to see that in the vicinity of a static solution cooling leads to an exponential decay of stable eigenmodes and exponential growth of unstable ones. Moreover, the rate of decay (growth) is exponentially rapid for high frequency mode. The new results has shown some deviation from the older one; however, the $\Gamma = \kappa' \alpha_w^5 T^4$ law of the symmetric phase can not be confirmed.

Fig. 3 shows the result of this analysis (open squares and triangles) and that of [8] (open dots). According to [3] the rate should have a $1/\beta$ behaviour. Clearly, larger measurement samples and careful systematic error analysis is needed to resolve the issue of the lattice spacing dependence of the rate.

A definitely positive outcome of the cooling technique is the new simulation in the broken phase. With the cooling technique no transition between the different vacua has been observed. This result resolves the discrepancy between [5] and eq. (3); however, no definite answer to the numerical value of the transition rate could be given. (Recently the multicanonical technique has been successfully applied to determine the nonperturbative transition rate in the broken phase [9].)

3 The nature of the phase transition

Perturbative studies show that in the realistic Higgs mass range ($m_H > 70\ GeV$) the perturbative approach breaks down [11], it predicts $\mathcal{O}(100\%)$ corrections [10] for the relevant quantities (e.g. interface tension, latent heat or correction to the course of the phase transition).

A nice illustration (c.f. Fig. 4) for that is the interface tension as a function of the Higgs-boson mass in different orders of the perturbation theory. As it can be seen in the physically allowed region (according to the LEP experiments the lower bound for the standard model Higgs-boson mass is relatively large: $m_H > 75$ GeV) the perturbation theory breaks down, its predictions can not be believed.

A popular way to study this basically non-perturbative problem is first to perform a dimensional reduction in perturbation theory. One starts with the original theory (e.g. Standard Model) and integrates out the heavy degrees of freedom perturbatively. The obtained theory is a three-dimensional bosonic one. The temperature dependent parameters of this theory are determined by the matching conditions between the full theory and the reduced one.

The theory has a dimensionful coupling, which fixes the overall scale. The two dimensionless quantities are x and y. One of them is x which determines the properties of the phase transition (connected to the zero temperature features of the original four-dimensional theory)

$$x \sim \frac{1}{8}\frac{m_H^2}{m_W^2} + c_2 \frac{m_{top}^4}{m_W^4}. \tag{9}$$

The other one is y which is used to tune the system in order to find the phase transition point (connected to the temperature of the original four-dimensional theory)

$$y \sim c_1 \frac{T - T_c^{(pert)}}{T_c^{(pert)}}. \tag{10}$$

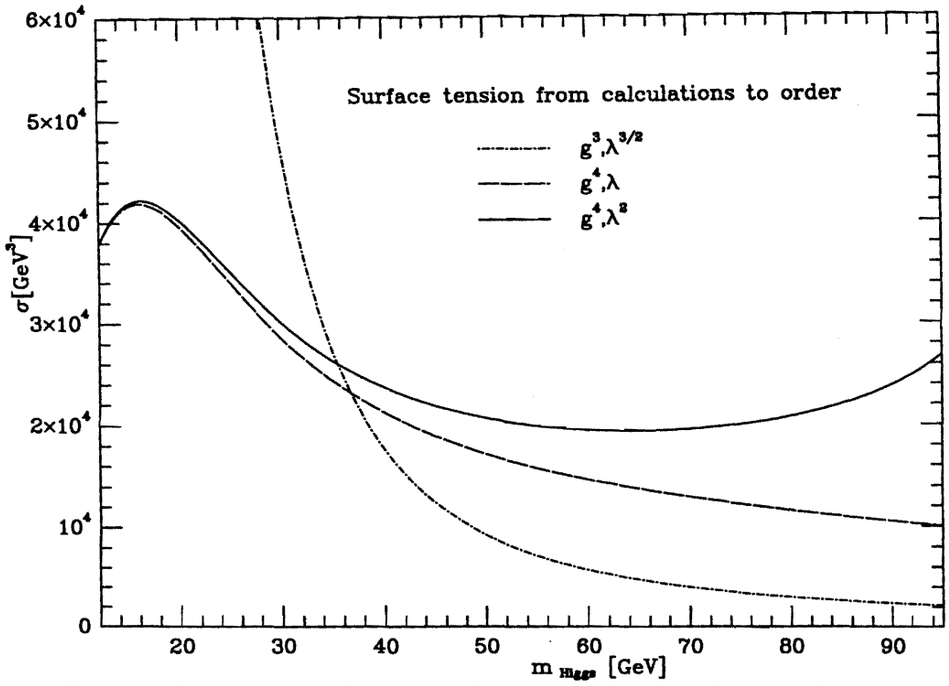

Figure 4: Surface tension from the different potentials as a function of m_H.

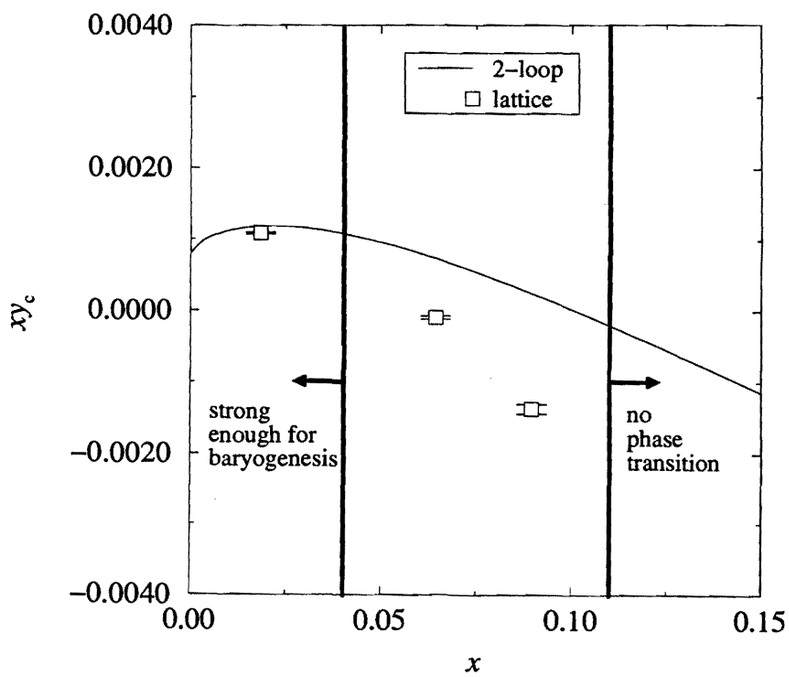

Figure 5: Phase diagram of the three-dimensional SU(2)-Higgs model.

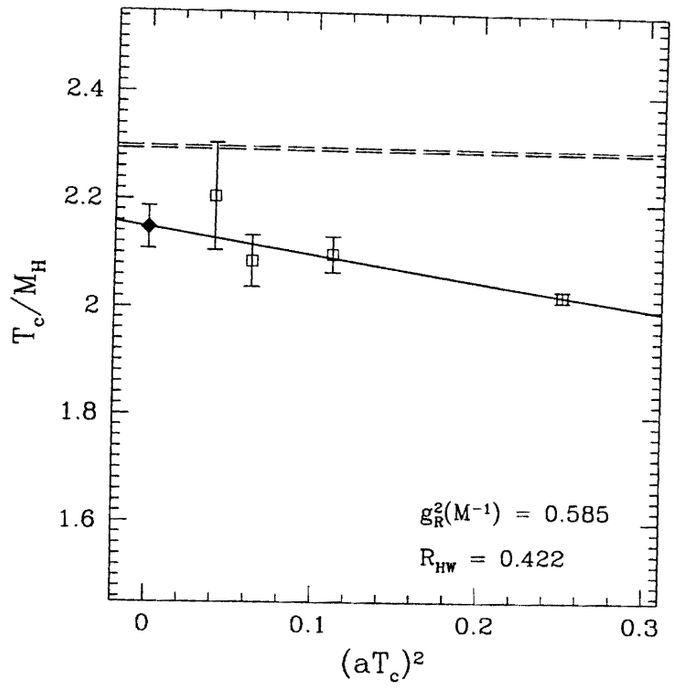

Figure 6: Critical temperature as a function of the lattice spacing.

Here c_1 and c_2 are some constants.

Static thermodynamical properties, mass spectrum and other related features have been studied both analytically and numerically for these three-dimensional models (a recent summary is [12] and see references therein).

According to these results the electroweak phase transition is of first order for small Higgs-boson masses; however, it turns out to be an analytic cross-over above $m_H \approx 67$ GeV (critical Higgs-boson mass value for the SU(2)-Higgs model) [13]. Due to the large mass of the top-quark the phase transition can not fulfill the φ_b/T_b condition for any choice of the Higgs-boson mass.

Fig. 5 shows the phase diagram of the three-dimensional theory. The solid line corresponds to the two-loop perturbation theory and the open symbols represent the result of the lattice simulations. For x values smaller than ~ 0.03 the phase transition is strong enough to satisfy the minimal condition $\varphi_b/T_b > 1$. The phase transition ends around $x \sim 0.1$ (we will discuss the determination of the endpoint). For large enough x values no phase transition can be observed, only a rapid cross-over occurs.

The determination of the endpoint of the finite temperature EWPT, thus a characteristic feature of the phase diagram, can be done by the use of the Lee-Yang zeros of the partition function $\mathcal{Z}$ [13]. One analytically continues $\mathcal{Z}$ to complex values of the couplings by reweighting the available data. Denoting κ_0 the lowest zero of $\mathcal{Z}$, i.e. the zero closest to the real axis, one expects in the vicinity of the endpoint the scaling law $\mathrm{Im}(\kappa_0) = d_1(x)V^{-\nu} + d_2(x)$. In order to pin down the endpoint one looks for a x value for which d_2 vanishes. Again, the change in x can been done by reweighting (or by direct simulations at different x couplings).

Using this Lee-Yang technique the Bielefeld group obtained $x_{end} = 0.0951(16)$, whereas the result of the Leipzig group is $x_{end} = 0.1020(20)$ (the two groups used different lattice spacings in their simulations).

Another possibility in order to understand the non-perturbative features of the electroweak phase transition is to study the full four-dimensional theory by lattice simulations. Since fermions always have

nonzero Matsubara frequencies, the perturbative treatment of these, at high temperatures very massive, modes could be satisfactory. Thus, the starting point of the lattice analyses is the $SU(2)$-Higgs model, which contains the essential features of the standard model of electroweak interactions.

In the last three years our group (DESY-Electroweak collaboration) presented a series of papers (see e.g. [14]) in order to clarify the details of the phase transition on four-dimensional lattices. Our work has been done on computers at HLRZ Jülich (CRAY-T90) and DESY-Ifh, Zeuthen (APE-Quadrics). The simulations have been performed for four set of parameters ($m_W = 80$ GeV): $m_H \approx 18$ GeV, 35 GeV, 49 GeV and 80 GeV.

For the first three masses the usual lattice formulation can be applied, thus identical lattice spacings in the spacial and temporal directions. In these cases a fairly good agreement is found between the lattice results and perturbation theory [16].

As an illustration (see Fig. 6) T_c/m_H is presented for $m_H \approx 35$ GeV. For this quantity very precise results exist and an extrapolation to zero lattice spacing is also possible. We have used $L_t = 2, 3, 4$ and 5 to extrapolate to the continuum limit. As it can be seen there is approximately three standard deviation discrepancy between the lattice and perturbative result. Note, that the result of the three-dimensional technique predicts a T_c/m_H much closer to the perturbative result. The lattice artifacts of the four-dimensional approach are expected to be proportional to a^2, whereas in the three-dimensional method they are proportional to a.

The $m_H \approx 80$ GeV case is much more difficult. The phase transition gets weaker, the lowest excitations have masses small compared to the temperature, T. From this feature one expects that a finite temperature simulation on isotropic lattice would need several hundred lattice points in the spatial directions even for $L_t = 2$ temporal extension.

In order to solve this problem we have used the simple idea that finite temperature field theory can be conveniently studied on asymmetric lattices, i.e. lattices with different spacings in temporal and spacial directions [15]. The resulting action contains anisotropies in the couplings. This action has been studied in perturbation theory and on the lattice.

One wants to tune the bare parameters in a way that the one-loop renormalized masses are finite in the continuum limit (however, their values in lattice units vanish $a_s M_{ren} = 0$ for $a_s \to 0$). At the same time the vacuum expectation value of the scalar field will be also zero in lattice units ($a_s v = 0$ for $a \to 0$) , i.e. we are at the phase transition point between the spontaneously broken Higgs phase and the SU(2) symmetric phase. The condition is fulfilled by an appropriate choice of the parameters (e.g. critical hopping parameter). The ratios of the other couplings are still free parameters and can be fixed by two additional conditions. We demand rotational (Lorenz) invariance for the scalar and vector propagators on the one-loop level. This ensures that the propagators with one-loop corrections have the same form in the z and t directions.

Clearly, arbitrary couplings for different directions would not lead to such rotationally invariant two-point functions. Technically the corrections to the anisotropies in the kinetic parts of the tree level propagators should be cancelled by the kinetic parts of the self-energies. This requires the knowledge of the wave function correction term in our theory. We have carried out both the perturbative and the lattice determinations of anisotropies. They are in complete agreement.

The values of the thermodynamical quantities (σ/T_c^3 and $\Delta\epsilon/T_c^4$) for $m_H \approx 80$ GeV are substantially smaller than their perturbative values. They are even consistent with a no first order phase transition scenario on the approximately 1-σ level. These results can be interpreted as a sign for the endpoint for the finite temperature electroweak phase transition.

Similarly to the standard model case perturbative two-loop results exist for the minimal supersymmetric standard model phase transition [17]. An interesting feature of the result that it opens the baryogenesis window for light stop. Setting sun QCD diagrams (stop-gluon) can give large logarithmic contributions which increase the strength of the phase transition. Typically, setting sun diagrams give contribution proportional to $\varphi^2 \log \varphi$.

In the case of stop-gluon graph the prefactor is proportional to the strong coupling, which resulted in an enhancement effect on φ_b/T_b. In order not to have infrared divergencies for the stop sector the authors restrict themself to $m_U^2 > 0$ sector (negative m_U^2 are related to the existence of colour breaking

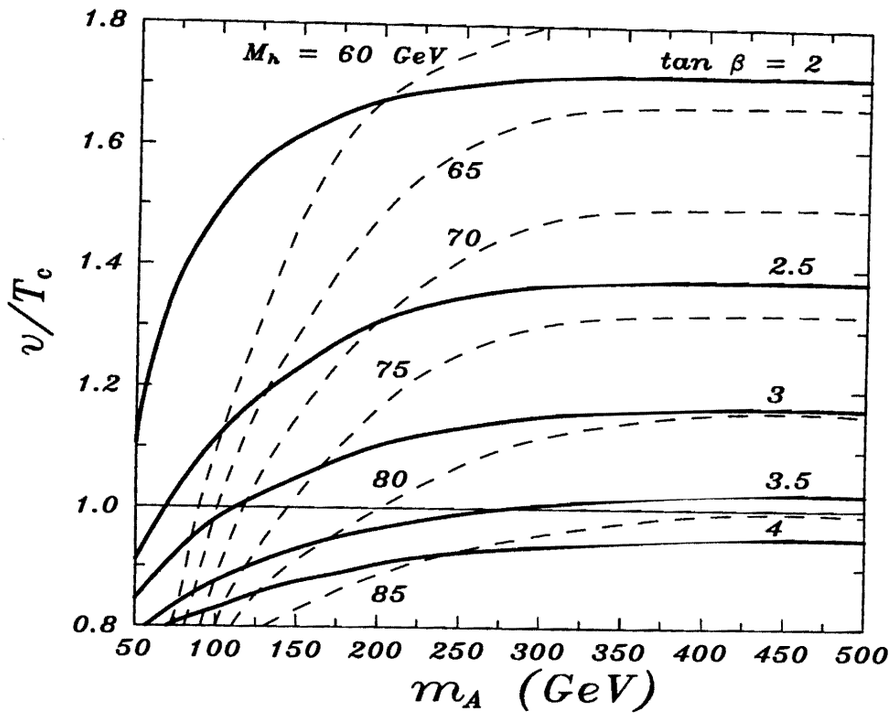

Figure 7: The normalized jump of the Higgs field in the MSSM.

minimum [18]). Fig. 7 shows that the ratio of the jump in the Higgs field divided by the temperature can be larger than one for low values of $\tan\beta$ (solid lines) and large enough m_A. Very low $\tan\beta$ corresponds to a light neutral Higgs (lines of constant Higgs masses are dashed). The LEP limit tells us that this mass must be larger than ≈ 70 GeV. Using this experimental bound the window for baryogenesis can be summarized as: $m_H < 85$ GeV; $2.25 < \tan\beta < 3.60$ and $m_A > 120$ GeV.

The allowed parameter space is very constrained. In particular $m_H < 85$ GeV will be soon tested at LEP. The authors claim that the large correction compared to the one-loop results (approximately 100%) is not a serious problem here and it does not mean the breakdown of the perturbation theory. The reason for that is the fact that the QCD corrections are leading order on the two-loop level; thus the large correction is not a symptom of the perturbative expansion, it is merely the appearance of a new interaction term.

Nevertheless, the setting-sun diagrams have bad infrared behaviour in the standard model. Direct lattice Monte-Carlo approach is probably needed in order to answer the question in the MSSM case, too.

In order to answer the question for the MSSM the three-dimensional approach (dimensional reduction plus lattice Monte-Carlo simulations in three-dimensions) is used, too (see e.g. [12]). The results are qualitatively the same as summarized above. However, these analyses are less reliable precisely in the small m_U^2 region, which is the interesting one for electroweak baryogenesis.

4 Conclusions

Analytical and real time numerical result for sphaleron processes are still not in complete agreement. Above T_c new analytical estimates suggest a $\Gamma \propto \alpha_w^5 T^4$ behaviour for the transition rate, which is not seen unambigously in numerical studies. The present numerical techniques are not sensitive enough to determine the sphaleron transition rate below T_c. Perturbative studies show, that the electroweak phase transition can not be described above $m_H = 70$ GeV by perturbative methods. Lattice results exclude successful baryogenesis in the minimal standard model. The present bounds on the Higgs-boson mass most probably results in a cross-over (no phase transition scenario). The minimal supersymmetric standard model still has some constrained parameter region, which perturbatively predicts a phase transition, strong enough for baryogenesis.

Acknowledgements

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