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Lattice results on the electroweak phase transition in four dimension

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Abstract

The present baryon asymmetry of the universe has finally been determined at the finite temperature electroweak phase transition. The strength of this transition plays a crutial role. The perturbative approach breaks down for the physically relevant $m_H > 70$ GeV Higgs-boson mass region. Four-dimensional lattice simulations are carried out in order to give a systematic and controllable solution to the problem. For smaller Higgs-boson masses ($m_H < 70$ GeV) the phase transition is of first order and its strengh rapidly decreases as m_H increases. For larger Higgs-boson masses lattices with asymmetric lattice spacing is used. Perturbative relationship between the lattice asymmetry and the anisotropy parameters of the action is presented. For $m_H \approx 80$ GeV the phase transition is much weaker than the perturbative prediction, the results are even consistent with the no first order phase transition scenario on the $\approx 1-\sigma$ level. This fact is a sign, that the phase transition ends at $m_H \approx 80$ GeV.

1 Introduction

In the early universe at high temperatures the electroweak gauge symmetry has been restored. It is conceivable that the observed baryon asymmetry can be understood within the standard model [1]. Clearly, the study of a "electroweak baryogenesis" requires a detailed knowledge of the transition from the symmetric to the broken phase. In order to provide the observed baryon asymmetry, one needs a departure from thermal equilibrium, thus a sufficiently strong first order phase transition via bubble nucleation. As it will be shown in the minimal standard model the phase transition is too week and the electrowek baryogenesis can be excluded.

Sect. 2 presents the results of the lattice approach with isotropic lattice couplings. They are compared with the perturbative results.

Sect. 3 contains a perturbative analysis of the lattice asymmetry resulting in anisotropies in the lattice action. Since the phase transition is quite weak the different lattice spacings in time and space directions make it possible to fix the high critical temperature and to use a large spatial volume in the meantime. Result of the anisotropic Monte-Carlo simulations are presented. It is shown that the thermodynamical quantities, such as latent heat and interface tension, are much smaller than the perturbative predictions. This result can be interpreted as a sign that the phase transition ends at $m_H \approx 80$ GeV

Sect. 4 gives a summary.

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talk given at Eötvös Conference in Science: Strong and Electroweak Matter (SEWM 97), Eger, Hungary, 21-25 May 1997.

2 Lattice results on isotropic lattices

Perturbative studies show that in the realistic Higgs mass range ($m_H > 70\ GeV$) the perturbative approach predicts $\mathcal{O}(100\%)$ corrections [3,2] for the relevant quantities (e.g. interface tension, latent heat or correction to the course of the phase transition). A nice illustration (c.f. Figure 1) for that is the interface tension as a function of the Higgs-boson mass in different orders of the perturbation theory. Some of the nonperturbative estimates (e.g. magnetic mass [3]) suggest a weaker, others (e.g. vacuum-condensate [30]) a stronger first order phase transition than the perturbative approach. A systematic and fully controllable way to solve the problem is the use of lattice simulations.

A particularly popular way to study the problem is to perform a dimensional reduction in perturbation theory. One starts with the original theory (e.g. Standard Model) and integrates out the heavy degrees of freedom perturbatively. The obtained theory is a three-dimensional bosonic one (e.g. SU(2)-Higgs or SU(2)×U(1)-Higgs model). The temperature dependent parameters of this theory are determined by the matching conditions between the full theory and the reduced one. Static thermodynamical properties, mass spectrum and other related features have been studied both analytically and numerically for these three-dimensional models [5,6,7,8,9,10,11,12,13,14,15,16,17]. According to these results the electroweak phase transition is of first order for small Higgs-boson masses; however, it turns out to be an analytic cross-over above $m_H \approx 67$ GeV (critical Higgs-boson mass value for the SU(2)-Higgs model).

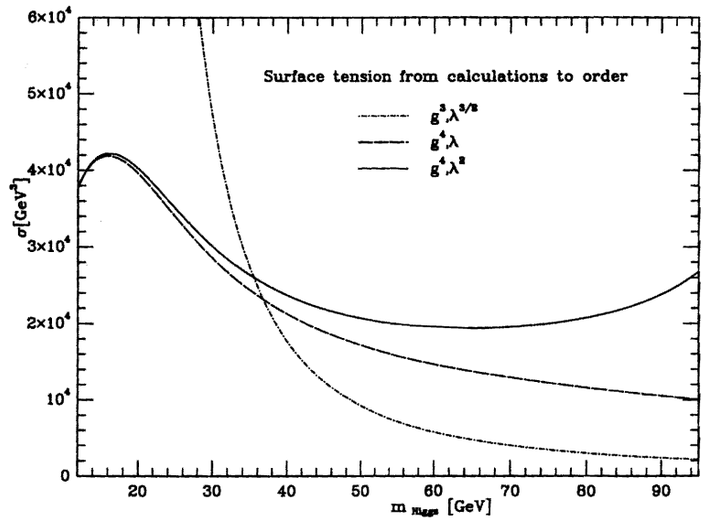

Figure 1: Surface tension from the different potentials as a function of m_H.

Another possibility is to study the full four-dimensional theory by lattice simulations. Since fermions always have nonzero Matsubara frequencies, the perturbative treatment of these, at high temperatures very massive, modes could be satisfactory. Thus, the starting point of the lattice analyses is the $SU(2)$-Higgs model, which contains the essential features of the standard model of electroweak interactions. The four-dimensional $SU(2)$-Higgs model at finite T is defined by the following action

$$S = \int_0^\beta d\tau \int d^3x \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\phi)^\dagger(D_\mu\phi) - \frac{1}{2}m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2\right] , \qquad (1)$$

where D_μ and $F^a_{\mu\nu}$ are the usual covariant derivative and the Yang-Mills field strength, respectively. $\beta = 1/T$, and the τ integration is over periodic bosonic fields. This model has been studied on the lattice by [18]. In the last three years our group (DESY-Electroweak collaboration) presented a series of papers [19,20,21,22,23,24,25,26,27] in order to clarify the details of the phase transition on four-dimensional lattices. Our work has been done on computers at HLRZ Jülich (CRAY-T90) and DESY-Ifh, Zeuthen (APE-Quadrics). This presentation summarizes some of the

results. Some other issues are covered by the talk of J. Hein [28]. Readers, who are interested in the details of the analysis, can found them in the original papers.

The simulations have been performed for four set of parameters ($m_W = 80$ GeV): $m_H \approx 18\ GeV, g^2_{bare} = 0.5$; $m_H \approx 35\ GeV, g^2_{bare} = 0.5$; $m_H \approx 49\ GeV, g^2_{bare} = 0.5$ and $m_H \approx 80\ GeV, g^2_{bare} = 0.5$. A moderate renormalisation of the gauge-coupling (g_R), approximately 5% has been found. Comparing the lattice results with the perturbative predictions these renormalization effects are taken into account.

In this chapter results only for the first three Higgs-boson masses are presented. $m_H \approx 80\ GeV$ needs a special treatment by means of lattices with anisotropic couplings. This method and the results will be discussed later.

The latent heat has been calculated from the discontinuity of the energy density ($\Delta\epsilon$). The necessary partial derivatives along the *lines of constant physics* have been determined by use of the one-loop renormalisation group equations, whereas the critical points have been given by the inspection of the gauge-invariant effective potential [29] and determined nonperturbatively, too. The obtained values are: $(\Delta\epsilon/T_c^4) = 1.68(17)$ (for $m_H \approx 18$ GeV), $(\Delta\epsilon/T_c^4) = 0.240(34)$ (for $m_H \approx 35$ GeV) and $(\Delta\epsilon/T_c^4) = 0.125(19)$ (for $m_H \approx 49$ GeV).

The interface tension is one of the most important quantities determining the course of the electroweak phase transition. It has been determined using the two-coupling method. The obtained results are: $(\sigma/T_c^3) = 0.84(16)$ (for $m_H \approx 18$ GeV), $(\sigma/T_c^3) = 0.065(10)$ (for $m_H \approx 35$ GeV) and $(\sigma/T_c^3) = 0.008(2)$ (for $m_H \approx 49$ GeV). Another possibility [22] is to measure the interface tension by use of the finite volume tunneling between the symmetric and the broken phase. This phenomenon leads to the splitting of the ground state, which gives the interface tension by $E_0 = C\exp(-L_t L^2_{xy}\sigma)$. Measuring correlation lengths of $\mathcal{O}(250)$ the final result reads: $(\sigma/T_c^3) = 0.053(5)$ (for $m_H \approx 35$ GeV).

The comparison for φ_c/T_c, $\Delta Q/T_c^4$, σ/T_c^3 and T_c/m_H is shown in Figure 2 ($m_H \approx 18, 49$ GeV). The dots with error bars represent the perturbative result at one-loop (g^3) and at two-loop (g^4) level. For each quantity the dashed lines show the region allowed by the statistical error, whereas the dotted lines include the systematics, too. The values of g and m_H/m_W are those of [21].

For $m_H \approx 18$ GeV (cf. Figure 2) both the one-loop and the two-loop results are in good agreement with the lattice data. For $m_H \approx 49$ GeV the two-loop results agree definitely better with the Monte Carlo data, except for the surface tension. The two plots may be interpreted in the following way. For small Higgs-boson masses the perturbative approach is in very good shape, already the one-loop approximation gives a reliable result. As m_H grows, the higher order contributions become

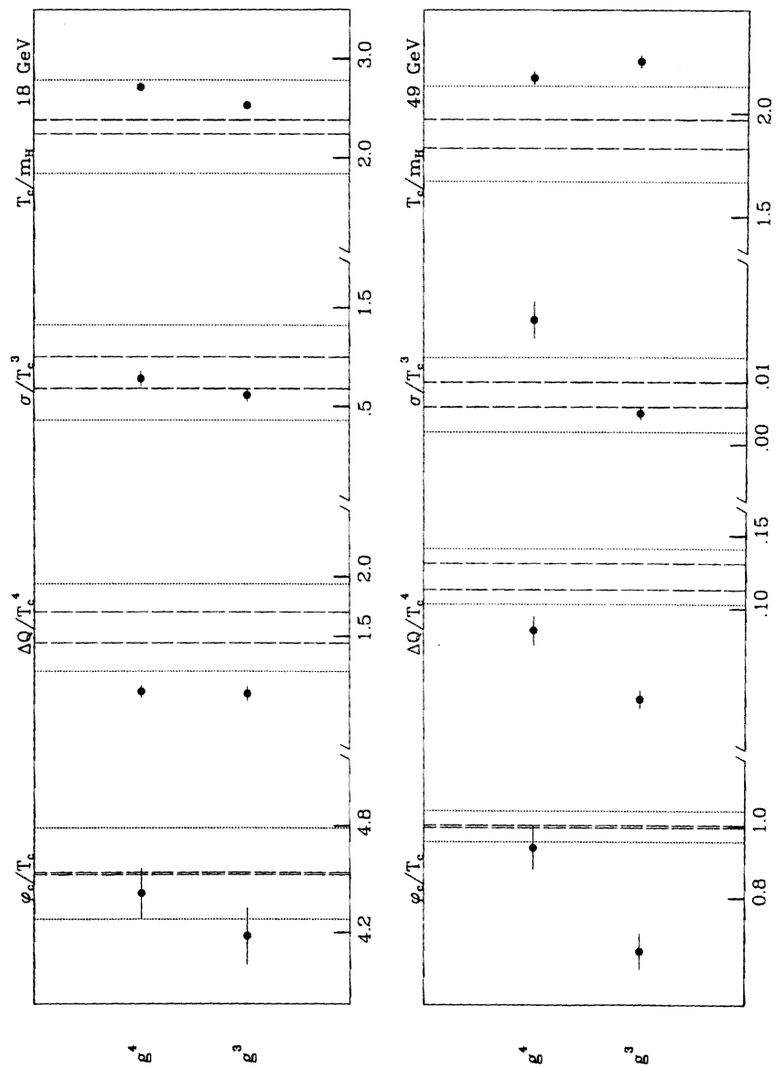

Figure 2: Perturbative and lattice results for $m_H \approx 18$ GeV and $m_H \approx 49$ GeV.

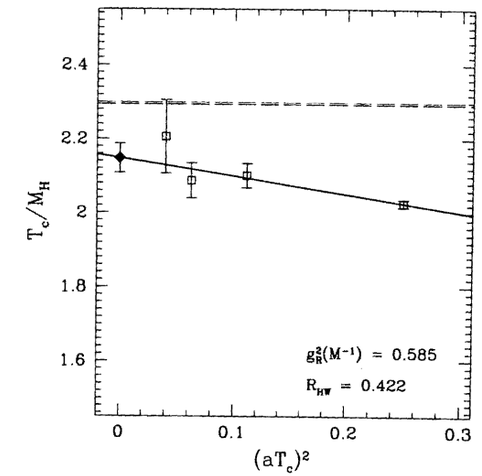

Figure 3: Critical temperature as a function of the lattice spacing.

more and more important, yet a two-loop calculation is still satisfactory for $m_H \approx 49$ GeV. In this range of parameters the non-perturbative features of the symmetric phase are not important enough to destroy the perturbative picture.

The comparison for $m_H \approx 35$ GeV is presented for T_c/m_H because for this quantity very precise results exist and an extrapolation to zero lattice spacing is also possible (Figure 3). As it can be seen there is approximately three standard deviation discrepancy between the lattice and perturbative result. Note, that the result of the three-dimensional technique predicts a T_c/m_H much closer to the perturbative result. The lattice artefacts of the four-dimensional approach are expected to be proportional to a^2, whereas in the three-dimensional method they are proportional to a.

3 Electroweak theory on asymmetric lattices

For larger M_H (e.g. $M_H = 80$ GeV) the phase transition gets weaker, the lowest excitations have masses small compared to the temperature, T. From this feature one expects that a finite temperature simulation on isotropic lattice would need several hundred lattice points in the spatial directions even for $L_t = 2$ temporal extension. These kinds of lattice

sizes are out of the scope of the present numerical resources.

In order to solve this problem we will use the simple idea that finite temperature field theory can be conveniently studied on asymmetric lattices, i.e. lattices with different spacings in temporal (a_t) and spatial (a_s) directions. This method solves the two-scale problem in a natural way [31]. Another advantage is, well-known and often used in QCD, that this formulation makes an independent variation of the temperature (T) and volume (V) possible. The perturbative corrections to the coupling anisotropies are known in QCD (see refs. [32,33]).

For simplicity, we use equal lattice spacings in the three spatial directions ($a_i = a_s,\ i = 1,2,3$) and another spacing in the temporal direction ($a_4 = a_t$). The asymmetry of the lattice spacings is characterized by the asymmetry factor $\xi = a_s/a_t$. The different lattice spacings can be ensured by different coupling strengths in the action for time-like and space-like directions. The action reads

$$\begin{aligned} S[U,\varphi] \;=\; & \beta_s \sum_{sp}\left(1-\frac{1}{2}\mathrm{Tr}\,U_{pl}\right)+\beta_t\sum_{tp}\left(1-\frac{1}{2}\mathrm{Tr}\,U_{pl}\right) \\ & +\sum_x\left\{\frac{1}{2}\mathrm{Tr}\,(\varphi_x^+\varphi_x)+\lambda\left[\frac{1}{2}\mathrm{Tr}\,(\varphi_x^+\varphi_x)-1\right]^2\right. \\ & \left.-\kappa_s\sum_{\mu=1}^{3}\mathrm{Tr}\,(\varphi_{x+\hat{\mu}}^+U_{x,\mu}\,\varphi_x)-\kappa_t\mathrm{Tr}\,(\varphi_{x+\hat{4}}^+U_{x,4}\,\varphi_x)\right\}, \quad (2) \end{aligned}$$

where $U_{x,\mu}$ denotes the SU(2) gauge link variable, U_{sp} and U_{tp} the path-ordered product of the four $U_{x,\mu}$ around a space-space or space-time plaquette, respectively. The symbol φ_x stands for the Higgs field, which is also written as $\varphi_x = \rho_x \cdot \alpha_x$, with $\rho_x \in \mathbf{R}^+$ and $\alpha_x \in \mathrm{SU}(2)$.

The anisotropies

$$\gamma_\beta^2 = \frac{\beta_t}{\beta_s} \quad , \quad \gamma_\kappa^2 = \frac{\kappa_t}{\kappa_s} \qquad (3)$$

are functions of the asymmetry ξ. On the tree-level the coupling anisotropies are equal to the lattice spacing asymmetry; however, they receive quantum corrections in higher orders of the loop-expansion

$$\gamma_\beta^2 = \xi^2\left[1+c_\beta(\xi)g^2+b_\beta(\xi)\lambda+\mathcal{O}(g^4,\lambda^2)\right], \qquad (4)$$

$$\gamma_\kappa^2 = \xi^2\left[1+c_\kappa(\xi)g^2+b_\kappa(\xi)\lambda+\mathcal{O}(g^4,\lambda^2)\right]. \qquad (5)$$

Here formal double expansion in g^2 and λ has been performed. In this double expansion we use the formal power counting $\lambda \sim g^2$. It is useful to introduce the hopping parameter $\kappa^2 = \kappa_s\kappa_t$ and $\beta^2 = \beta_s\beta_t$. In general, the determination of $\gamma_\beta(\xi)$ and $\gamma_\kappa(\xi)$ should be done non-perturbatively.

This can be achieved by requiring that the Higgs- and W-boson correlation lengths in physical units are the same in the different directions. This idea can be applied in perturbation theory as well (see e.g. [33]), and we will follow this method in our analysis too. It is believed that this procedure ensures a rotationally invariant effective action [33,34].

The main goal of a perturbative approach is to perform a one-loop analysis of the theory defined by eq. (2). This means first the determination of the mass-counterterm. One wants to tune the bare parameters in a way that the one-loop renormalized masses are finite in the continuum limit (however, their values in lattice units vanish $a_sM_{ren} = 0$ for $a_s \to 0$). At the same time the vacuum expectation value of the scalar field will be also zero in lattice units ($a_sv = 0$ for $a \to 0$) , i.e. we are at the phase transition point between the spontaneously broken Higgs phase and the SU(2) symmetric phase. The condition is fulfilled by an appropriate choice of the hopping parameter (critical hopping parameter). The ratios of the couplings (γ_β and γ_κ) are still free parameters and can be fixed by two additional conditions. We demand rotational (Lorenz) invariance for the scalar and vector propagators on the one-loop level. This ensures that the propagators with one-loop corrections have the same form in the z and t directions. Clearly, arbitrary couplings for different directions in eq. (2) would not lead to such rotationally invariant two-point functions.

The calculation is rather tedious and only the results are presented. The critical hopping parameter is given by

$$\kappa_c = \frac{\xi}{2(3+\xi^2)}+\frac{1}{(3+\xi^2)^2}\left[6\xi J_1(\xi,0)-\frac{\xi^2}{(3+\xi^2)}\right]\lambda_c+\frac{9\xi J_1(\xi,0)}{16(3+\xi^2)^2}g^2, \qquad (6)$$

where the notation

$$J_n(\xi,ma_s) = a_s^{4-2n}\int_k \frac{1}{(m^2+\hat{k}^2)^n} \qquad (7)$$

for the dimensionless lattice integrals on asymmetric lattices is used.

One can demand rotational invariance in the continuum limit, $a_s, a_t \to 0$ at fixed $\xi = a_s/a_t$. The corrections to the anisotropies in the kinetic parts of the tree level propagators should be cancelled by the kinetic parts of the self-energies. This requires the knowledge of the wave function correction term in our theory. The functions $c_\beta(\xi)$ and $c_\kappa(\xi)$ of eq. (4) are plotted in Figure 4.

There are several important features of the result, which should be mentioned.

a. Masses in the propagators: a consistent perturbative procedure on the lattice determines the bare parameters, for which the renormalized masses vanish, cf. eq. (6). With these bare couplings other quantities,

e.g. asymmetry parameters, are determined. However, using the one-loop renormalized masses ($a_s M_H = a_s M_W = 0$) in the propagators instead of the bare ones leads to changes in the results, which are higher order in g^2 and λ. Therefore, all our results are given by the integrals with renormalized masses.

b. g^2 and λ corrections: In Figure 4 we have given only $c_\beta(\xi)$ and $c_\kappa(\xi)$. The functions $b_\beta(\xi)$ and $b_\kappa(\xi)$ vanish, thus there are no corrections of $\mathcal{O}(\lambda)$ to the anisotropy parameters. It is easy to understand this result qualitatively, since only graphs with two or more scalar self-interaction vertices have non-trivial dependence on the external momentum. This feature is connected with the well-known fact that the Φ^4 theory does not have any wave function correction in first order in the scalar self-coupling. It is worth mentioning that there is only one type of two-loop graph (the setting-sun) which should be combined with the one-loop graphs, in order to obtain the whole $\mathcal{O}(\lambda^2)$ correction.

c. Pure gauge theory: Some graphs contributing to the anisotropies are identical with those of the pure gauge theory. Evaluating these diagrams one reproduces the result of ref. [32] (the function $c_\beta(\xi)$ of the present paper corresponds to $c_\tau(\xi) - c_\sigma(\xi)$ of ref. [32]). The most important contribution comes from the self-energy graph with vector four-coupling. Inclusion of the scalar particles gives only small changes. The relative difference between the $c_\beta(\xi)$ functions for the pure SU(2) theory and for the SU(2)-Higgs model is typically a few %.

d. Quantum corrections to the hopping parameter: the contributions to the hopping parameter come from graphs with external scalar legs. This correction has the same sign and order of magnitude than that of the gauge anisotropy parameter; however it is somewhat smaller. It is possible to combine the anisotropies $c'_\beta(\xi) = c_\beta(\xi) - c_\kappa(\xi)$. For this choice in the gauge sector and with $\gamma_\kappa = \xi$ the rotational invariance can be restored on the one-loop level, choosing the appropriate value for the lattice spacing asymmetry a_s/a_t. Thus, the masses in both directions will be the same. However, the obtained lattice spacing asymmetry will then slightly differ from the original ξ; one gets $a_s/a_t = \xi(1-g^2 c_\kappa(\xi)/2) + \mathcal{O}(g^4, \lambda^2)$.

It should be emphasized that the truly non-perturbative analysis of several thermodynamical quantities on lattices with anisotropic couplings ought to include the non-perturbative determination of the anisotropy parameters. This can be achieved by the study of the correlation functions and/or static potentials given by Wilson-loops.

This analysis has been done at $m_H = 72(5)$ GeV and $g_R^2 = 0.577(15)$ (c.f. Figure 5). We performed $T = 0$ simulations at $\kappa = 0.10662$ and at fixed $\gamma_\kappa = 4$ but at three different γ_β values. We have chosen $\gamma_\beta = 4$ and $\gamma_\beta = 3.919$, as the tree-level and one-loop predictions, respectively. The third choice of the γ_β parameter was 3.8, a value which makes an

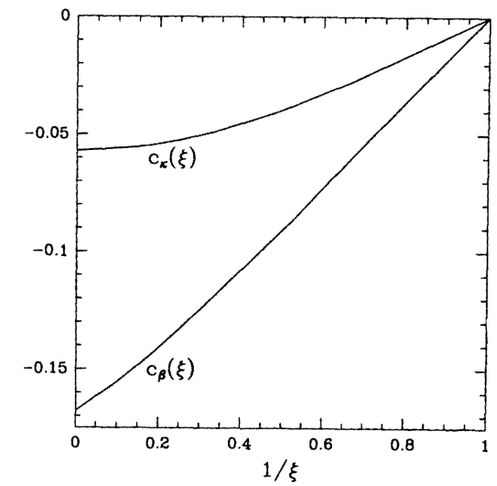

Figure 4: $c_\beta(\xi)$ and $c_\kappa(\xi)$ as functions of $1/\xi$.

interpolation for the restoration of the rotational invariance possible.

The asymmetry parameter has been measured in three channels: Higgs, W and static potential. E.g. the asymmetry parameter in the Higgs channel is defined by $\xi_H = m_H(z)/m_H(t)$, where the masses are determined in lattice units.

At $\gamma_\beta = 4$ the maximal asymmetry parameter is found in the W channel and the minimal one in the Higgs channel. At $\gamma_\beta = 3.8$ the situation is just opposite. The simulations at the perturbatively predicted $\gamma_\beta = 3.919$ point show that for all channels the obtained asymmetries are consistent with each other and with the perturbative value $\xi^P = 4.052$. Combining all of the nine asymmetry results one gets a rather small error ellipse for the anisotropy-asymmetry plane. This result, which is in complete agreement with the perturbative one, is shown in the insert of Figure 5.

The agreement is not surprising. At $T = 0$ the electroweak theory is weak and can be treated perturbatively. Since the anisotropy parameters are the lattice counterterms, which are completely determined at zero temperature, one can safely use the perturbative results for the definition of the theory even in the highly nonperturbative finite temperature regime.

After confirming the perturbative result for the anisotropy parameters one should calculate the thermodynamically interesting quantities at $m_H \approx 80$ GeV. As it is mentioned earlier in this mass region the phase transition is rather weak. This is the basic reason for using $\xi \approx 4$, which reduces the lattice volume by almost two orders of magnitude. The simulation parameters correspond to $m_H = 78(4)$ GeV pole mass and $g_R^2 = 0.539(16)$ renormalized gauge coupling.

We have determined the critical temperature for two temporal extentions. The results are $T_c/m_H = 1.86(2)$ and $1.8(2)$ for $L_t = 2$ and 3, respectivcely. These numbers are quite smaller than the prediction of the continuum perturbative approach; however, lattice perturbation theory shows that T_c/m_H increases with the temporal extention [21].

We have seen in our previous analysis that the most robust way to determine the interface tension is to use the so-called two-coupling method. The lattice is an elongated one ($L_t = 2, 3 \; L_z \gg L_x, L_y$). After enforcing an interface pair perpendicular to the z-direction by dividing the lattice volume in symmetric and Higgs phases with ($\kappa_1 < \kappa_c$ for the one half of the lattice and $\kappa_2 > \kappa_c$ for the other half of the lattice) the related additional free energy gives an estimate for the interface energy. The final answer is obtained in the $\Delta\kappa = \kappa_2 - \kappa_1 \to 0$ limit. Our result is $\sigma/T_c^3 = 0.0006(4)$, which was determined by two different volumes: $V = 2 \cdot 24^2 \cdot 196$ and $V = 2 \cdot 36^2 \cdot 256$.

The jumps of different order parameters (e.g. plaquette, link and length of the Higgs field) with the partial derivatives along the lines of constant physics give the latent heat. The lattice volume we used was $V = 2 \cdot 24^2 \cdot 96$. For the normalized latent heat one obtains $\Delta\epsilon/T_c^4 = 0.0033(27)$.

The values of the thermodynamical quantities (σ/T_c^3 and $\Delta\epsilon/T_c^4$) for $m_H \approx 80$ GeV are substantially smaller than their perturbative values. They are even consistent with a no first order phase transition scenario on the approximately 1-σ level. The fact that this result deviates from those of the three-dimensional investigations (for which this Higgs-boson mass region is definitely in the analytic cross-over regime) should be clarified in the future.

4 Conclusions

Perturbative studies show, that the electroweak phase transition can not be described above $m_H = 70$ GeV by perturbative methods. One systematic way to solve the problem is to perform lattice simulations. There is an intensive work to do Monte-Carlo simulations, in both three and four dimensions.

The good quantitative agreement between the perturbative and lattice results found for Higgs mass values $m_H \approx 18$ GeV and $m_H \approx 49$

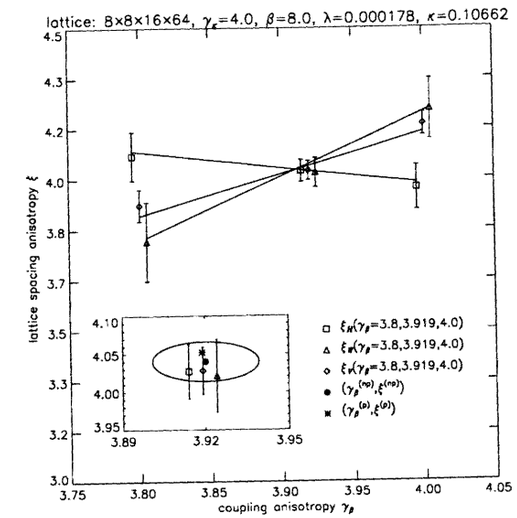

Figure 5: ξ-evaluation at $m_H = 72(5)$ GeV and $g_R^2 = 0.577(15)$.

GeV is interpreted as evidence for the correctness of the present understanding of the electroweak phase transition. Non-perturbative effects present in the symmetric phase are neglected by perturbation theory, but they should contribute to the lattice results. These effects should be more pronounced for larger Higgs-boson masses. Thus, an important problem is to perform lattice simulations in the physically relevant parameter region ($m_H > 70$ GeV) and to understand the above mentioned non-perturbative effects. This understanding is essential for a clear answer concerning the observed baryon asymmetry of the universe.

We have performed a systematic analysis to answer the above question by means of lattices with anisotropic couplings. We have defined the theory in an unambiguous way, by determining the perturbative relations between the lattice couplings and the lattice spacing asymmetry. It has been shown that these relations are correct in a nonperturbative sense, too. Using this technique we have determined the thermodynamical quantities (σ/T_c^3 and $\Delta\epsilon/T_c^4$) for $m_H \approx 80$ GeV. The results are substantially smaller than their perturbative values. They are even consistent with a no first order phase transition scenario on the approximately 1-σ level.

5 Acknowledgement

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