A Model Independent Analysis of the Electroweak Parameters *

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ABSTRACT

A new framework to study electroweak physics at the one-loop level in general SU(2)_L × U(1)_Y theories is introduced. It separates the 1-loop corrections into two pieces: process specific ones from vertex and box contributions and the universal ones due to contributions to the gauge boson propagators. The latter are parametrized model-independently in terms of four effective form factors, $e^2(q^2)$, $s^2(q^2)$, $g_1^Z(q^2)$ and $g_2^Z(q^2)$ correspondingly to $\gamma\gamma$, $\gamma Z$, $ZZ$ and $WW$ propagators. In addition we introduce one form factor, $g_3^Z(q^2)$, for the $Zb_L\bar{b}_L$ vertex because of its strong dependence on the unknown top quark mass. By assuming only the Standard Model contributions to the process specific corrections except for the $Zb_L\bar{b}_L$ vertex, we determine $g_2^Z(m_Z^2)$ and $s(m_Z^2)$ from the Z parameter measurements, $g_3^Z(0)$ and $s(0)$ from the low energy neutral current experiments, and $g_2^W(0)$ from the W mass measurements. These values are then compared systematically with the predictions of SU(2)_L × U(1)_Y theories. We also study the quantitative significance of the $Zb_L\bar{b}_L$ vertex correction $\delta_4(m_Z^2)$. The preferred ranges of the top quark and Higgs boson masses within the Standard Model are extracted as functions of $m_Z$. The limitations in the theoretical predictions due to the uncertainty in $\alpha(m_Z)$ are critically discussed. Electroweak physics at TRISTAN is also described in this framework which clarifies the role of each measurement.

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1 Introduction

One of the most exciting developments of recent years has been the observation [1] that the electroweak mixing angle $\sin^2 \theta_W$ measured precisely at LEP agrees excellently with the prediction of the supersymmetric (SUSY) SU(5) grand unification theory (GUT). The agreement is so impressive that we can hope in the near future to learn about SUSY particle masses [2,3] with a better measurement of the QCD coupling constant. It has been argued [4] that the uncertainty in the GUT scale particle masses screens any possible effects of SUSY particle threshold corrections to the coupling constant unification condition. The works of refs. [2,3] showed that the non-observation of proton decay effectively constrains the GUT particle contributions to the coupling constant unification and that our hope of learning about the SUSY mass scale from the precision measurements has been revived.

What is exciting about this exercise is that there now seems to be a strong indication that new particles and new interactions may exist at the electroweak scale. They may be produced at the Tevatron, LEP200 and at super colliders. Even prior to their discovery, their effects could be observed in precision experiments through quantum corrections. The effects can be significant if some of the new particles are as light as weak bosons, or if many new particles contribute constructively, or if there exist new strong interactions among them. Even in the absence of such a signal, we can constrain certain new physics possibilities and that we will learn more in the future precision experiments. The purpose of this work is to provide a new framework to confront the electroweak theories with various precision experiments, that allows us to look for new physics beyond the Standard Model in a systematic way at the one-loop level.

In section 2, we briefly review our formalism which can be used for general 4-fermion processes. The S-matrix elements for the neutral and charged currents processes are expressed in terms of the four universal form factors which contain tree-level couplings and radiative effects to the gauge boson propagators, and the process-specific ones which represent the vertex and box corrections. In section 3, we make a systematic analysis according to several steps, using the data of $Z$ parameter measurements, of the low energy neutral current experiments and of the $W$ mass measurements. Validity of the SU(2)$_L \times U(1)_Y$ gauge theories is checked quantitatively, and the $q^2$-dependence of the two of the four universal form factors is compared with the SM predictions. We also make a detailed analysis for the SM. The dependence of the results on $m_W$ and the uncertainty from the QED effective constant $\alpha(m_Z^2)$ are examined quantitatively.

Application of our formalism for the electroweak physics at the TRISTAN energy is discussed in section 4. We summarize our results in section 5.

2 A new framework for 1-loop electroweak physics

2.1 4 charge form factors

Since what we want to learn from the electroweak precision experiments are the possible effects of new physics beyond the SM, whose exact nature is unknown, we would like to analyse the data in a framework which allows interpretations in wider classes of theoretical models. On the other hand the framework cannot be too general, since our ability to identifying effects of new physics from the precision experiments relies on the renormalizability of the electroweak theory which allows us to predict many observables in terms of a few parameters up to finite quantum corrections. Because the SM corrections are precisely known, those experiments which are sensitive to the quantum effects have a chance to identify a signal of physics beyond the SM. We therefore restrict ourselves to models that respect SU(2)$_L \times U(1)_Y$ gauge symmetry which breaks spontaneously down to U(1)$_{B-L}$. In our approach, all new physics contributions that do not respect the spontaneously broken SU(2)$_L \times U(1)_Y$ gauge symmetry can be identified by our inability to fit the data successfully within our framework: these exotic interactions include all non-renormalizable effective interactions among light quarks and leptons that may arise from an exchange of a heavy particle such as a new gauge boson or lepton quark boson, or from new strong interactions that bind common constituents of quarks and leptons.

Our restriction on the electroweak gauge group implies in the tree level that all quarks and leptons couple to the electroweak gauge bosons universally with the same coupling constant as long as they have common electroweak quantum numbers. This universality of the gauge boson coupling to quarks and leptons can in general be violated at the quantum level. It has widely been recognized, however, that this universality of the couplings holds true even in the one-loop level in a wider class of models where new particles affect the precision experiments only via their effects on the electroweak gauge boson propagators [7-14]. This class of effects due to new physics is often called oblique [7,11] or propagator [13] corrections or those satisfying generalised universality [14]. This concept of universality can be generalized to certain vertex corrections with the non-standard weak boson interactions [15]. It is also sometimes useful in theories with intrinsic vertex and box corrections, such as the SUSY-SM, since the propagator corrections are often larger than the vertex/box ones: propagator corrections can be significant either because of a large multiplicity of contributing particles or...
by a presence of a relatively light new particle, whereas the vertex and box corrections depend on a specific combination of new particles that match the quantum number of the process and are suppressed if one of them is heavy. Our framework adopts this distinction between new physics contributions to the gauge boson propagators and the rest, where we allow the most general contributions in the former whereas we consider only the SM contributions to the latter (vertex and box corrections).

The new physics degree of freedom is then expressed in terms of four charge form factors, each associated with one of the four types of the electroweak gauge boson propagators:

\[ \bar{e}^L(q^2) = e^L(1 - \text{Re} \Pi_{\gamma\gamma}(q^2)) \] for the $\gamma\gamma$ propagator, (1a)

\[ \bar{\bar{e}}^L(q^2) = e^L(1 + \frac{1}{\pi} \text{Re} \Pi_{\gamma\gamma}(q^2)) \] for the $\gamma Z$ propagator, (1b)

\[ \bar{g}_Z(q^2) = g_Z(1 - \text{Re} \Pi_{Z\gamma}(q^2)) \] for the $ZZ$ propagator, (1c)

\[ \bar{g}_{WW}(q^2) = g_{WW}(1 - \text{Re} \Pi_{WW}(q^2)) \] for the $WW$ propagator, (1d)

where the hatted couplings $\bar{e}$, $\bar{\bar{e}}$, $\bar{g}_Z$, and $\bar{g}_{WW}$ are the propagator functions renormalized in the $\overline{MS}$ scheme. The gauge boson two-point functions that appear in eq.(1) are defined as

\[ \Pi_{\nu\nu}(q^2) = \Pi_{\nu\nu}^{\overline{MS}}(q^2) - \text{Re} \Pi_{\nu\nu}(q^2) q^2 - m_{\nu}^2, \] (2)

where $m_{\nu}$ is the pole mass of the gauge boson $\nu$ ($m_{\nu} = 0$) and the subscript $\nu$ stands for the transverse part of the vacuum polarization tensor $\Pi_{\nu\nu}(q)$. The propagators are calculated in the 'tHooft-Feynman gauge and the so-called pinch term [8,20,21] of the vertex functions due to diagrams with the weak boson self-couplings are included in the overlined functions $\Pi_{\nu\nu}^{\overline{MS}}(q^2)$.

In addition to these four form factors we have the two weak boson masses $m_{\gamma}$ and $m_{Z}$ as the parameters of the electroweak theory. Since the charge form factors are real continuous functions of $q^2$, we have infinite degrees of free parameters when we use them to parametrize a theory. In practice, however, these charge form factors can be measured accurately enough only at specific $q^2$ ranges; all four of them at $q^2 = 0$ ($q^2 \ll m_{\nu}^2$), and two of them, $\bar{e}(q^2)$ and $\bar{g}_Z(q^2)$, at $q^2 = m_{Z}^2$. Hence, we have just 8 parameters that are measured accurately to test a theory. Among these 8 parameters, three are known precisely; $\alpha$, $G_F$, and $m_{Z}$. Since the gauge boson properties are fixed at
tree level by the three parameters in general models with the SU(2) × U(1) symmetry broken by a vacuum expectation value, we can use the remaining 5 parameters to test the theory at the quantum level; see Table 1. We therefore first determine the 5 parameters, $\hat{F}(m_z^2)$, $\hat{g}_z^2(m_z^2)$, $\hat{F}(0)$, $\hat{g}_z^2(0)$, and $\hat{g}_z(0)$, from precision experiments, and then confront their values with various theoretical predictions.

When the new physics scale is significantly higher than the scale ($\leq m_Z^2$) of precision measurements, we can often neglect new physics contributions to the running of the charge form factors. Among our 5 parameters, the values of $\hat{F}(0)$ and $\hat{g}_z^2(0)$ can then be determined from $\hat{F}(m_z^2)$ and $\hat{g}_z^2(m_z^2)$, respectively, by the SM physics only. The effective number of the free parameters is then 3, which corresponds precisely to that of $S$, $T$, $U$, $\alpha$, $c_1$, $c_2$, $c_3$ [13], or other related triplets of parameters in refs. [12]. When the scale of new physics that couples to gauge boson propagators is near or below the weak boson masses, we may identify its signal as an anomalous running of the charge form factors. This point has been emphasized in refs. [10] in connection with possible existence of the light SUSY particles. The triplet parametrizations are then no longer sufficient to account for new physics degrees of freedom, and we should regard all 5 parameters in the Table 1 as free parameters. Several alternative approaches to the same problem have been proposed in refs. [16–18].

In the minimal SM, all the quantum corrections are determined by just two parameters, $m_t$ and $m_b$, and hence all the charge form factors are determined by their values. We show in Fig. 1 the four charge form factors in the SM. The trajectories are fixed such that they give correct values for the 3 precisely known parameters, $0$, $s$, and $g_\gamma(0)$, as found. One of its crossed channels) as

$$ T_0 = m_b A_1 + J_f, $$

where $J_f$ and $J_f'$ denote the bare currents without the coupling factor: $J_f = \bar{\psi}_f r^+ P_L \gamma_\mu \psi_f$ for $f = e, \mu$, where $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$ are the chiral projectors. All the one-loop corrections appear in the scalar amplitudes $M_{ij}$ which depend on flavor and chirality of the currents and on the invariant momentum transfers $s$ and $t$ of the process.

In the neutral current amplitudes, the photonic corrections attached only to the external fermion lines are gauge invariant in themselves. Therefore we can obtain finite and gauge invariant amplitudes by excluding all the external fermion corrections. We find e.g. for the process $\bar{\psi}_f \psi_f$ the following closed form in the one-loop order

$$ M_{ij}^{NC} = \frac{Q_i Q_j}{s} \left[ e^2(s) + \frac{1}{2} \left( f_+ + f_+(s) - 2 i \Delta_{\gamma}(s) \right) + \frac{1}{2} \left( f_+ + f_+(s) - 2 i \Delta_{\gamma}(s) \right) \right], $$

$$ + \frac{1}{8} \left[ \left( 1 - 4 Q_i Q_j s^2 \right) + \left( 1 - 4 Q_i Q_j s^2 \right) \Delta_{\gamma}(s) \right] \left( f_+ + f_+(s) - 2 i \Delta_{\gamma}(s) \right) \right], $$

$$ + \frac{1}{8} \left[ \left( 1 - 4 Q_i Q_j s^2 \right) + \left( 1 - 4 Q_i Q_j s^2 \right) \Delta_{\gamma}(s) \right] \left( f_+ + f_+(s) - 2 i \Delta_{\gamma}(s) \right) \right], $$

where the hatted couplings $\hat{e}_i = \hat{e}_i = 2\hat{e}_i$ are renormalized in the MS scheme, and three of the four charge form factors of eq.(1), $\hat{e}(s)$, $\hat{f}(s)$ and $\hat{g}_z(s)$ are identified:

$$ \hat{e}_i(s) = e_i + \frac{1}{2} \left( 1 - \text{Re} \tilde{\Pi}_\gamma(s) \right), $$

$$ \hat{f}_i(s) = f_i + \frac{1}{2} \left( 1 - \text{Re} \tilde{\Pi}_\gamma(s) \right), $$

$$ \hat{g}_z(s) = g_z + \frac{1}{2} \left( 1 - \text{Re} \tilde{\Pi}_\gamma(s) \right), $$

$$ \hat{g}_y(s) = g_y + \frac{1}{2} \left( 1 - \text{Re} \tilde{\Pi}_\gamma(s) \right). $$

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Imaginary parts of the propagator correction factors are denoted by $\Delta_{r}(q^{2})$, $\Delta_{z}(q^{2})$, and $\Delta_{zz}(q^{2})$ which are defined as

$$
\Delta_{r}(q^{2}) = \text{Im} \Pi_{rr}^{\pm}(q^{2}), \quad \Delta_{z}(q^{2}) = \text{Im} \Pi_{zz}^{\pm}(q^{2}), \quad \Delta_{zz}(q^{2}) = \text{Im} \Pi_{zz}^{\pm}(m_{Z}^{2}) - \frac{1}{m_{Z}^{2}} \text{Im} \Pi_{zz}^{\pm}(m_{Z}^{2})
$$

(6a)

(6b)

(6c)

The last equation is a consequence of the LEP convention for $m_{Z}$ and $\Gamma_{Z}$. The vertex functions $\Gamma_{r}^{\pm}(s)$ and the box functions $B_{r}^{\pm}(s, t)$ are process specific. We first note that the residues of the $\gamma$ and $Z$ poles are separately physical observables ($\mu$-independent and gauge invariant). At $q^{2} = 0$, we find

$$
\Gamma_{r}^{\pm}(0) = \Gamma_{z}^{\pm}(0) = 0
$$

(7)

for all $f_{a}$, which are ensured by the Abelian and non-Abelian parts of the Ward identities, respectively. The universal residue of the photon pole gives the square of the unit electric charge $e^{2}(0) = \frac{4\pi}{\alpha}$.

The overlines on the vertex functions $\Gamma_{r}^{\pm}(s)$ indicates the removal of the pinch term [8,21]. The vertex functions $\Gamma_{r}^{\pm}(s)$ are proportional to the square of the fermion mass inside the loop, and are non-negligible only for $f_{a} = t_{b}$ in the SM. The functions $\Gamma_{z}^{\pm}(s)$ are vanishing for all $f_{a}$ in the SM, though they appear in extended models such as the minimal SUSY-SM. We shall see that the box functions $B_{r}(s, t)$ are significant only at low energy $NC$ processes and in $\mu$-decay at $s = t = 0$. It is worth noting here that the box contributions to the helicity amplitudes can be expressed in the above simple current product form only when the external fermion masses can be neglected in the loop amplitude. All the vertex and box functions are known precisely in the SM. If we assume no new physics contributions to these process specific ($f_{a}$-dependent) corrections, we can determine the three form factors $e^{2}(q^{2})$, $g^{2}(q^{2})$ and $s^{2}(q^{2})$ from the precision experiments independent of further model assumptions.

For the charged current $(CC)$ process $ij \rightarrow i'j'$, we find similarly

$$
M_{ij}^{CC} = \frac{1}{1 - \frac{m_{W}^{2}}{m_{W}^{2}}}[G_{F}(t) + iG_{F}(t) + \Gamma_{r}^{+} + \Gamma_{r}^{-} + \Gamma_{z}^{+} + \Gamma_{z}^{-} + \Gamma_{zz}^{+} + \Gamma_{zz}^{-} + \Gamma_{zzz}^{+} + \Gamma_{zzz}^{-}](t) + \Gamma_{ij}^{CC}(s, t),
$$

(8)

off the $W$ pole, with an appropriate CKM factor $V_{ij}V_{ij}^{\ast}$. Precise values of the $CC$ matrix elements are needed only at low energies, and we find for the muon decay constant

$$
G_{F} = \frac{\frac{\alpha}{4\pi}}{\frac{\alpha}{4\pi}}
$$

(9)

Here the factor $\delta_{ij}$ denotes the sum of the vertex and the box contributions, whose value is precisely known ($\delta_{ij} = 0.0055$) in the SM. Eq.(9) gives the physical $W$ mass in terms of $G_{F}$ once the $\delta_{ij}$ value is known for a given model. The overline here again indicates the removal of the pinch terms and that its numerical value is significantly (about 25%) smaller than the standard factor [25].

### 2.3 $B_{b}b_{b}$ vertex

The only vertex whose magnitude is not yet known in the SM is the $B_{b}b_{b}$ vertex which depends strongly on the assumed top quark mass [33]. Hence we find it convenient to introduce one extra form factor

$$
\delta_{b}(s) = \Gamma_{b}^{+}(s) + \Gamma_{b}^{-}(s) + \Gamma_{b}^{0}(s)
$$

(10)

in our analysis. A similar strategy has been proposed in ref. [17]. An advantage is that the parameter $\delta_{b}$ allows us to determine the quantitative significance of the $B_{b}b_{b}$ vertex correction [34], independent of the specific SM mechanism. Furthermore, it

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Fig. 2 The $m_{b}$ dependence of the $B_{b}b_{b}$ vertex form factors $\Gamma_{b}^{+}(m_{b}^{2})$, $\Gamma_{b}^{-}(m_{b}^{2})$, $\Gamma_{b}^{0}(m_{b}^{2})$ (solid lines) and $\delta_{b}(m_{b}^{2})$ (long dashed line). $\delta_{b}(m_{b}^{2})$ is calculated including the known 2-loop effects [35-37] for $m_{W} = 100$ GeV and $\alpha = 0.12$ (long dashed line).
allows us to separate the data analysis stage from the evaluation of \( \delta_b \) in a specific model, that includes \( O(\alpha, m^2) \) [35] and \( O(m^4) \) [36,37] two-loop corrections of the SM. The \( m_w \)-dependence of the vertex form factors \( \Gamma_{\mu}^\mu(m_w^2), \Gamma_{\mu}^\mu(m_w^2), \Gamma_{\mu}^\mu(m_w^2) \) and \( \delta_b(m_w^2) \) in the SM are shown in Fig. 2.

2.4 Constraints due to \( \alpha, G_F, m_\mu \)

When the basic three parameters of the models with the SU(2)_L \times U(1)_Y symmetry broken by just one vacuum expectation value are renormalized by the three well-known quantities \( \alpha, G_F \) and \( m_\mu \), all the predictions of the theory are determined at the tree level. It is therefore convenient to introduce three parameters which are proportional to the finite quantum correction effects only. Among the various proposals in the literature [11–13], we find that the \( S, T, U \) parameters of Peskin and Takeuchi [11]

![Graph](image)

**Fig. 3** The SM predictions for the \((S, T, U, \delta_b)\) parameters as defined in eqs (11) as functions of \( m_\mu \) for selected \( m_H \) values. We set \( \alpha = 0.12 \) in the two-loop \( O(\alpha) \) corrections for \( S, T, U \) [42] and \( \delta_b(m_w^2) \) [35].

is most convenient if they are extended to include the SM contributions as well. We define these parameters in terms of our two-point functions with the pinch terms [21], which are related to our change form factors as follows:

\[
S \equiv 16 \frac{e^2}{\alpha} \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(m_w^2)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] \mid_{q^2=0} \frac{4\pi^2(m_\mu^2)}{\alpha(m_\mu^2)} - \frac{16\pi^2}{\alpha(0)} = 4 \frac{e^2(m_\mu^2)}{\alpha(m_\mu^2)} - \frac{16\pi^2}{\alpha(0)}, \tag{11a}
\]

\[
\alpha T \equiv 4\sqrt{2} G_F \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] = 1 + \delta_b - 4\sqrt{2} G_F m_\mu^2, \tag{11b}
\]

\[
U \equiv 16 \pi \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] = \frac{4\pi^2(m_\mu^2)}{\alpha(m_\mu^2)} + \frac{16\pi^2}{\alpha(0)} = \frac{16\pi^2}{\alpha(0)} \tag{11c}
\]

These definitions allow us to express all the charge form factors and hence all experimental observables in terms of the three parameters \( S, T \) and \( U \) without separating the SM contributions to the gauge boson propagators. First, the form factor \( g_{\mu}(0) \) is determined from \( T \) via eq (11b). Second, the form factor \( f_2(m_w^2) \) is determined from \( S \) via eq (11a). And finally the form factor \( g_\mu(0) \) is determined from \( U \) via eq (11c):

\[
\frac{1}{g_{\mu}(0)} = \frac{4\sqrt{2} G_F m_\mu^2}{\alpha(0)} = 1 + \delta_b - \alpha T = \frac{1}{g_{\mu}(0)} + \frac{4\sqrt{2} G_F m_\mu^2}{\alpha(0)} \tag{12a}
\]

\[
f_2(m_w^2) = \frac{1}{2} \left[ \frac{1}{4} \frac{1}{\alpha(0)} \! - \! \frac{S}{16\pi} \right] \tag{12b}
\]

\[
g_{\mu}(0) = \frac{e^2(m_w^2)}{\alpha(m_w^2)} - \frac{1}{16\pi} (S + U) \tag{12c}
\]

The running of these form factors is determined by their defining equations (1) by properly performing the renormalization group improvement:

\[
\frac{\delta f_\mu^2(0)}{f_\mu^2(0)} = \frac{\delta f_\mu^2(0)}{f_\mu^2(m_w^2)} - \frac{\delta f_\mu^2(0)}{f_\mu^2(0)} = \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(m_w^2)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] \tag{13a}
\]

\[
\frac{1}{g_{\mu}(q^2)} - \frac{1}{g_{\mu}(0)} = \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(m_w^2)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] - 2 \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(m_w^2)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] \tag{13b}
\]

\[
\frac{1}{g_{\mu}(q^2)} - \frac{1}{g_{\mu}(0)} = \text{Re} \left[ \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(m_w^2)} \! - \! \frac{\Pi_{\mu\mu}(q^2)}{\Pi_{\mu\mu}(0)} \right] \tag{13c}
\]

All the form factors are thus easily calculable for arbitrary models for fixed \( \alpha, G_F \) and \( m_\mu \). In fact, the SM curves in Figs. 1 are obtained this way. By assuming only the SM contribution to the muon decay vertex and box corrections in \( \delta_b \) and by assuming the SM running of the form factors, especially for \( \delta f_\mu^2 \), we can express all the charge form factors as a power series in the above three parameters. To first order, we find
\[ g_2^2(0) = 0.5456 + 0.00407, \quad (14a) \]
\[ x^2(m_Z^2) = 0.2334 + 0.0036S - 0.00241 \]
\[ g_5^2(0) = 0.4183 - 0.0031S + 0.00441 \]
\[ \delta m_H = 0.2334 + 0.0036S - 0.00241 \]
\[ \delta m_H = 0.4183 - 0.0031S + 0.00441 \]

where we added the shifts due to the uncertainty in the estimated $1/\alpha(m_Z^2)$ value, $\delta_m$, which is as large as ±0.10 [19]. It is clearly seen that $g_2^2(0)$ measures $T$, $x^2(m_Z^2)$ measures a combination of $S$ and $T$, whereas $g_5^2(0)$ measures a combination of all three parameters.

The SM predictions for $S$, $T$, $U$ and $\delta m_H$ are shown in Fig.3 as functions of $m_H$ for selected values of $m_H$ by including all the known two-loop corrections of $O(\alpha)$ [36, 37, 41] and of $O(\alpha^2)$ [35, 42] at $\alpha(m_Z) = 0.12$. From Fig.3, one can see that the parameters $S$ and $T$ show mild sensitivity to $m_H$, but the parameters $U$ and $\delta m_H$ are almost independent of $m_H$.

3 Systematic Analysis

In this section, we make a systematic analysis in the following steps, by systematically strengthening the assumptions underlying the analyses:

1. First, by assuming that the precisely known SM contributions dominate the process specific vertex and box corrections, we determine the universal charge form factors from the precision experiments: $g_2^2(m_Z^2)$ and $x^2(m_Z^2)$ from the $Z$ parameter measurements, $g_5^2(0)$ and $x(0)$ from the low energy neutral current experiments, and $g_5^2(0)$ from the $W$ mass measurements at $pp$ colliders. The $Zb\bar{b}$ vertex form factor $\epsilon(q^2)$ at $q^2 = m_Z^2$ is fitted simultaneously with the data. New physics contributions that do not respect the spontaneously broken $SU(2)\times U(1)$ gauge symmetry can be identified by our inability to fit the data successfully.

2. Once the charge form factors are determined, we can test the running of the two form factors $g_2^2(q^2)$ and $x^2(q^2)$ which are determined both at $q^2 = m_Z^2$ and at $q^2 = 0$. When there exist new particles which are not so heavy ($\sim m_Z$), we may identify its signal as an anomalous running of the charge form factors.

3. By assuming further that the $q^2$-dependence of the charge form factors is governed by the SM physics only, we determine the three universal parameters $S$, $T$ and $U$, together with $\delta m_H$. They are sensitive to radiative effects of heavy physics. Here again the assumption of the underlying $SU(2)\times U(1)$ gauge symmetry is tested by the $\chi^2$-goodness of the fit, and deviation from the SM can be identified from the fitted $S$, $T$, $U$ values.

4. At the final stage, we assume the minimal SM contributions to $S$, $T$, $U$ and $\delta m_H$, and examine its $\chi^2$ goodness of the fit as functions of the two unknown parameters $m_H$ and $m_H$.

At each step, we pay attention to the $\alpha_s$-dependence of our fits, while the uncertainty from the QED effective coupling constant $\alpha(m_Z)$ is examined in the last two steps.

3.1 Determination of the charge form factors from precision experiments

3.1.1 Z boson parameters

The most recent results from experiments at LEP and SLC on the $Z$ boson parameters have been reported in refs. [26, 27]. The $Z$ line-shape parameters are determined at LEP as [27]

\[ m_Z(GeV) = 91.187 \pm 0.007 \]
\[ \Gamma_Z(GeV) = 2.489 \pm 0.007 \]
\[ \sigma^2_x(0) = 41.56 \pm 0.14 \]
\[ R_1 = \sigma_x^2/\sigma_y^2 = 20.763 \pm 0.049 \]
\[ A_{\nu}^b = 0.0158 \pm 0.0018 \]

The other electroweak data that we used in our fit are as follows [26, 27]:

\[ P_L = -0.139 \pm 0.014, \]
\[ A_{4\nu} = 0.10 \pm 0.044 \]
\[ A_{4\nu} = 0.099 \pm 0.006, \]
\[ A_{4\nu} = 0.075 \pm 0.015, \]
\[ R_L = \sigma_x^2/\sigma_y^2 = 2.2033 \pm 0.0027 \]

Significant improvements over the last year have been achieved for many of the above measurements.

These parameters are expressed in terms of the scalar amplitudes (4) as follows. Since most of the formulae are common for LEP/SLC measurements and TRISTAN measurements, we show the expressions at arbitrary $s$. The LEP/SLC results are obtained by setting $s = m_Z^2$.
First, the cross sections for $e^+e^- \rightarrow f\bar{f}$ are given by

$$\sigma_f = \frac{\alpha}{4\pi} \left( \left( |M_{f\ell}^s|^2 - |M_{f\ell}^a|^2 \right) \frac{C_{\ell\ell}}{2} + \left( |M_{f\ell}^s|^2 + |M_{f\ell}^a|^2 \right) \frac{C_{\ell\ell}}{2} \right) \left( 1 + \frac{3}{4} \frac{Q_f}{\pi} \alpha(s) \right).$$

(17)

where the factor $\left( 1 + \frac{3}{4} \frac{Q_f}{\pi} \alpha(s) \right)$ account for the external QED corrections. Since both at LEP/SLC and TRISTAN energies, the SM box contributions are negligibly small, we neglect the $\cos \theta$-dependence of the box correction factors and use their $\cos \theta = 0$ values throughout the analysis. The factors $C_{\ell\ell}$ for quarks contain the external QED corrections for the vector part [29] and for the axial vector part [30], together with the finite mass corrections of the final state fermions:

$$C_{\ell\ell} = 3 \left\{ \frac{\beta_f}{3} \left[ \frac{3}{(s)^{3/2}} \left( \frac{\alpha_s}{\pi} \right)^2 \right] - \frac{12 \left( \frac{\alpha_s}{\pi} \right)^3}{s} \right\},$$

(18a)

$$C_{\ell\ell} = 3 \left\{ \frac{\beta_f}{3} \left[ \frac{3}{(s)^{3/2}} \left( \frac{\alpha_s}{\pi} \right)^2 \right] + \frac{12 \left( \frac{\alpha_s}{\pi} \right)^3}{s} \right\},$$

(18b)

with $\alpha_s = \alpha_s(\sqrt{s})$ and

$$\sqrt{s} = \sqrt{1 - \frac{4m_f^2}{s}}.$$

(19)

where $m_f(\sqrt{s})$ denotes the MS running quark mass as evaluated at the unit-of-mass scale $\sqrt{s}$ [31]. The running masses are calculated in the next-to-leading order [31] for the following pole mass choices:

$$m_u = 1.4 \text{ GeV},$$

(20a)

$$m_d = 4.8 \text{ GeV}.$$  

(20b)

The $O(\alpha_s^2)$ axial parts contain $m_b$ dependence through the function $f(m_b)$ [30]:

$$f(m_b) = 2 \ln \frac{m_b}{m_0} - \frac{47}{12} - \frac{28}{81} \left( \frac{m_b m_t}{m_t} \right)^2 + \frac{0.2107}{m_b} \left( \frac{m_b}{2 m_t} \right)^{1/2}. \quad \text{(21)}$$

The minus sign should be taken in front of $f(m_b)$ in eq.(18b) for $u, c$ quarks, and the plus sign for $d, s, b$ quarks. For charged leptons, these factors are given as

$$C_{\ell\ell} = \beta_{\ell\ell} \left( \frac{3}{2} - \frac{3}{2} \right),$$

(22a)

$$C_{\ell\ell} = \beta_{\ell\ell}.$$  

(22b)

with

$$\beta_{\ell\ell} = \sqrt{1 - \frac{4m_\ell^2}{s}}.$$  

(23)

When we can neglect the masses of the final leptons (for $\ell = e$ and $\mu$), eq.(17) reduces to a simple one:

$$\sigma_f = \frac{\alpha}{4\pi} \left( |M_{f\ell}^s|^2 + |M_{f\ell}^a|^2 \right) \left( 1 + \frac{3}{4} \frac{Q_f}{\pi} \alpha(s) \right).$$

(24)

On the $Z$ pole, the partial widths $\Gamma_f$, the width ratios $R_l$ and $R_b$ are defined from the cross sections (17) and (24) as

$$\Gamma_{f} = \frac{3}{4} \left( |M_{f\ell}^s| + |M_{f\ell}^a| \right)$$

(25a)

$$R_l = \frac{9}{16} \frac{\alpha_s}{\alpha},$$

(25b)

$$R_b = \frac{9}{16} \frac{\alpha_s}{\alpha},$$

(25c)

Next, the Forward-Backward asymmetry is given by

$$A_{FB} = \frac{3}{4} \left( |M_{f\ell}^s| + |M_{f\ell}^a| \right) - \frac{3}{4} \left( |M_{f\ell}^s| + |M_{f\ell}^a| \right),$$

(26)

for leptons, and

$$A_{FB} = \frac{3}{4} \left( |M_{f\ell}^s| + |M_{f\ell}^a| \right) - \frac{3}{4} \left( |M_{f\ell}^s| + |M_{f\ell}^a| \right)$$

(27)

$$\beta_{\ell\ell} = \sqrt{1 - \frac{4m_\ell^2}{s}}, \quad (q = b, c).$$

(28)

for quarks ($q = b, c$). The QCD corrections for the FB asymmetries [32] have not been included in eq.(27). The reported asymmetries from LEP $A_{FB}^{\mu}(LEP)$ and $A_{FB}^{b}(LEP)$ have been corrected for these effects by assuming a linear $\cos \theta$ dependence and $\alpha_s = 0.12$. We therefore calculate the LEP asymmetries by the following formula;

$$A_{FB}^{\mu}(LEP) = A_{FB}^{\mu}(LEP) \frac{1 + k_4 \alpha_s}{1 + k_4 \alpha_s}$$

(28)

with $k_4 = 0.75$ [27]. The QCD correction depends on details of the final charm and bottom quark tagging procedure, and each experiment should give the $\alpha_s$-dependence of the corrected asymmetry value.
Finally, the $\tau$ polarization asymmetry and the Left-Right asymmetry are expressed as

$$P_\tau = \frac{|M_{\tau L}^2| - |M_{\tau R}^2|}{|M_{\tau L}^2| + |M_{\tau R}^2| + |M_{\tau L}^2 - M_{\tau R}^2|},$$

$$A_{LR} = \frac{\sum f |M_{\tau L}^2| + |M_{\tau R}^2| - |M_{\tau L}^2 - M_{\tau R}^2|}{\sum f |M_{\tau L}^2| + |M_{\tau R}^2| - |M_{\tau L}^2 - M_{\tau R}^2|}.$$  

On the $Z$-pole, these asymmetries ($A_{LR}$, $P_\tau$) determine $s^2(m_Z)$ almost independently of $g_2^Z(m_Z)$, $\delta(m_Z)$ and $\alpha$.

In the absence of an accurate quantitative measurement of the QCD coupling constant and for the convenience of the GUT studies, we choose $\alpha \equiv \alpha_s(m_Z)$ as an input parameter of our fit, and present the results as functions of $\alpha$.

One can then either add independent data from direct measurements, or study quantitative consequences of a particular GUT model that predicts $\alpha$.

In order to determine the universal charge form factors $s^2(m_Z)$, $g_2^Z(m_Z)$ and the $ZbLbL$ vertex form factor $\delta(m_Z)$, we further estimate the process specific contributions to the vertex and box diagrams. The SM contributions at $\sqrt{s} = m_t$ to the vertex factors in eq.(4) are listed in Table 2, where the corresponding values for the $ZbLbL$ vertex are also listed together for comparison (see Fig. 2). It should be noted that, on the $Z$-pole, the box contributions are negligible (\(\sim O(\alpha_s^2)\)) in the cross section as compared to the $O(\alpha_s)$ propagator and vertex corrections.

We now assume the SM dominance to the vertex and box corrections except for the $ZbLbL$ vertex, and make a fit in terms of the three parameters $s^2(m_Z)$, $g_2^Z(m_Z)$ and $\delta(m_Z)$. The overall fit to all the $Z$ parameters listed above gives

$$\delta(m_Z) = 0.5546 - 0.031(\alpha_s - 0.12) \pm 0.0017$$

$$s^2(m_Z) = 0.2313 + 0.008(\alpha_s - 0.12) \pm 0.0007$$

$$\rho_{\text{corr}} = \begin{pmatrix} 1 & 0.14 & -0.36 \\ 0.14 & 1 & -0.0662 \\ -0.36 & -0.0662 & 1 \end{pmatrix},$$

\[\chi^2_{\text{min}} = 1.66 + (\alpha_s - 0.103)^2,\]

for a given value of $\alpha_\tau \equiv \alpha_s(m_Z)$; the errors and the correlations are almost independent of $\alpha_s$.

Finally, the $\tau$ polarization asymmetry and the Left-Right asymmetry are expressed as

$$P_\tau = \frac{|M_{\tau L}^2| - |M_{\tau R}^2|}{|M_{\tau L}^2| + |M_{\tau R}^2| + |M_{\tau L}^2 - M_{\tau R}^2|},$$

$$A_{LR} = \frac{\sum f |M_{\tau L}^2| + |M_{\tau R}^2| - |M_{\tau L}^2 - M_{\tau R}^2|}{\sum f |M_{\tau L}^2| + |M_{\tau R}^2| - |M_{\tau L}^2 - M_{\tau R}^2|}.$$
The above results are shown in Fig. 4, along with the SM predictions with all known corrections of the O(s^4) level [36, 37, 38-41] and of the O(a_s) two-loop corrections [35, 42] in perturbative QCD, but without non-perturbative f_0 threshold effects [43].

The SM prediction to \( \beta^0(m_t^2) \) is also sensitive to the hadronic vacuum polarization correction, for which we take \( (\Delta^2 \beta^0)_{\text{had}} = -0.2083/(\alpha = -3.88) \). Its error \( \delta_{\alpha} = \pm 0.0007/\alpha = \pm 0.10 \) leads to a shift in the SM predictions for \( \beta^0(m_t^2) \) by \( \pm 0.00026 \).

The SM predictions in Fig. 4 are obtained by setting \( \delta_{\alpha} = 0 \).

We show in Fig. 4 1-s contours of the fit for three representative \( \alpha_s \) values. It is clearly seen that the Z\( \gamma \)-mixing parameter \( \beta^0(m_t^2) \) is measured rather independent of \( \alpha_s \), while the Z coupling strength \( \gamma Z(m_t^2) \) is negatively correlated with the assumed \( \alpha_s \) value, reflecting its sensitivity to the total Z width. This anti-correlation leads to a preference of larger \( m_t \) in the SM for smaller \( \alpha_s \). The parameter \( \beta^0(m_t^2) \) is relatively insensitive to \( \alpha_s \) because it is measured mainly from the asymmetry parameters that are either completely or almost insensitive to the QCD corrections.

Before leaving the \( Z \) parameters, we would like to give two comments on the measurements of the \( Z \) vertex form factor \( \Delta Z(m_t^2) \) and \( \alpha_s \), which are strongly correlated. As is clearly seen from Fig. 4, the fit to the parameter \( \delta \) depends strongly on \( \alpha_s \), reflecting its sensitivity to \( R_Z \) and \( \Gamma_Z \), more than to \( R_T \) that measures \( \delta Z(m_t^2) \) directly and is rather insensitive to \( \alpha_s \). Because of this sensitivity to \( \alpha_s \), it is not meaningful to quote a bound on \( \delta Z \) or on \( m_t \) from the SM \( Z \) vertex vertex correction, without studying carefully its \( \alpha_s \) dependence. It is worth emphasizing here that there is no evidence of the \( Z \) vertex vertex for \( \alpha_s \geq 0.13 \), as the corresponding parameter for \( \delta \) or \( s \) is about \(-0.003 \). For \( \alpha_s \leq 0.12 \), we can obtain rather stringent upper bound on \( m_t \) [17, 34] that one can read off from Fig. 4, mainly because there is no good evidence for the large \( Z \) vertex vertex effect.

This point has also been emphasized by the LEP electroweak working group [27]. Furthermore, this strong correlation makes the fitted \( \alpha_s \) value depend strongly on the assumed \( \delta \) value. If we allow \( \delta \) and \( \alpha_s \) to be fitted freely by the data, then the result (31) gives \( \Delta Z(m_t^2) = 0.0015 \pm 0.0071 \) and \( \alpha_s (m_t^2) = 0.103 \pm 0.013 \), with \( \rho_{\text{corr}} = -0.85 \). It is therefore necessary to assume the SM contributions to \( \Delta Z(m_t^2) \), and to a lesser extent those to \( \gamma Z(m_t^2) \), in order to measure \( \alpha_s \) from the electroweak \( Z \)-parameters. The result of such an analysis is given in section 4.3 where we study consequences of the minimal SM.

### 3.1.2 Low Energy Neutral Current

We consider in our analysis four types of low energy neutral current experiments. They are the neutrino-nuclei scattering \( (\nu_e, q) \), the neutrino-electron scattering \( (\nu_e, e) \), atomic parity violation (APV), and the polarized electron-deuteron scattering experiments (eO). All of them measure the universal form factors \( F_2(0) \) and \( F_2^2(0) \). Effects due to small but finite momentum transfer in these processes are corrected for by assuming that the running of these form factors are determined by the SM particles only (see Fig. 1), which is an excellent approximation at low energies. Vertex and box corrections are performed by assuming that they are dominated by the SM contributions. For each sector, we first give a model-independent parametrization of the data, and then give our fit in the \( (F_2(0), F_2^2(0)) \) plane.

**\( \nu - q \) scattering**

For the \( \nu_e - q \) data, we used the results of the analysis of ref. [44]. The fitted parameters \( \gamma Z, \delta^2, R_\gamma \) are, however, dependent on the assumed value of the charged quark mass \( (m_q) \) in the slow-rescaling formula for the charged current cross sections. By using the constraint on \( m_q \) from the charged current experiments, \( m_q = 1.54 \pm 0.33 \text{ GeV} \) [44], we can properly take into account the \( m_q \) dependence of the fit.

We thus find a new model-independent parametrization of the \( \nu_e - q \) data [6]:

\[
\begin{align*}
\gamma Z &= 0.2986 \pm 0.0044 \\
\delta^2 &= 0.0307 \pm 0.0047 \\
R_\gamma &= 0.0589 \pm 0.0237 \\
\rho_{\text{corr}} &= 0.92.
\end{align*}
\]

**\( \nu - e \) scattering**

For the \( \nu_e - e \) data, we used the results of CHARM, BNL E374 and CHARM-II [48], which are summarized by Beyer f481 as

\[
\begin{align*}
\gamma \nu_e(0) &= 0.5483 \pm 0.0081 \\
\delta^2(0) &= 0.2392 \pm 0.0143 \\
\rho_{\text{corr}} &= 0.92. \\
\end{align*}
\]

The strong positive correlation is a consequence of the smallness of the error of \( \gamma \nu_e + \delta^2 \) in (32) that measures the total neutral current cross section off isoscalar targets. The above fit is given in Fig. 3 as a 1-s counter.
The quoted quality of the fit is $\chi^2_{min} = 7.7$ for d.o.f. = 14. These effective parameters are obtained from the data by assuming the tree-level formula for the $\nu_e e$ and $\nu_e \ell$ scattering cross sections.

We can hence obtain the electroweak parameters by evaluating the full matrix elements at an average momentum transfer of these experiments, $(-t) \approx m^2_{\chi}$, and then by expressing the above effective parameters in terms of the radiatively corrected cross sections. We reproduce the known results of ref. [49], and find that the only significant correction comes from the neutrino 'charge radius' factor and the $WW$ box contributions. We find

$$g_2^e(0) = 0.5459 \pm 0.0153$$
$$\rho_{\text{corr}} = 0.09$$

(35)

with $\chi^2_{min} = 0$, since we take the fit (34) as the model independent parametrization of the $\nu_e - e$ data [48]. The result is also shown in Fig. 3.

Atomic Parity Violation

As for the APV experiments, we used the result of the analysis [50] on the parity violating transitions in the cesium atom ($A/Z = 135/55$);

$$Q_{W}(135, 55) = -71.04 \pm 1.81$$

(36)

where we sum the experimental and theoretical errors by quadrature.

Our simple formula (4) reproduces the $u$- and $d$-quark contributions of ref. [51], but not the photonic correction to the axial vector $Zee$ vertex nor the $Z\gamma$ box corrections that are sensitive to the nucleon structure. We adopt the results of ref. [51] for these corrections, and find

$$g_2(0) = -0.6138 \cdot g_2^e(0) + 0.5681 \pm 0.0083.$$  

(37)

The result is shown in Fig. 3.

e-$D$ scattering

Finally, for the SLAC $eD$ polarization asymmetry experiment [52], we make a model-independent fit to the original data by using the two parameters, $2C_{1u} - C_{1d}$ and $2C_{2u} - C_{2d}$ of ref. [53], by taking into account uncertainties due to the sea-quark contributions and finite $N_s = x_s/\sqrt{N}$ [54], and those due to higher twist contributions [55, 56]. The former uncertainties are found to be very small, confirming the results of ref. [54], while the latter are found to be model dependent [57]. We adopt the estimates [56] based on the MIT-Bag model, which find rather small corrections, as in the neutrino scattering off nucleon targets [58]. Further study on the higher twist effects may be needed to achieve precision measurements of the electroweak parameters in these reactions. After allowing for uncertainties in the Bag model parameters of ref. [56], we find

$$2C_{1u} - C_{1d} = -0.94 \pm 0.26$$
$$2C_{2u} - C_{2d} = 0.80 \pm 1.23$$

(38)

with $\chi^2_{min} = 9.95$ for 11 data points. Because of the strong correlation, only a linear combination of the two coupling factors is measured well.

The electroweak corrections in the SM are found in ref. [59]. Our formula (4) leads to all relevant correction factors except for the external photonic corrections. We use the explicit form of ref. [51] for these external photonic correction factors, and checked the insensitivity of our fit to the uncertainty in the $Z\gamma$ box corrections. The
QED coupling $\alpha^2(t)$ and the vertex functions $\Gamma_1(t)$ and $\Gamma_2(t)$ in our amplitudes ($4$) are evaluated at $(-t) = 1.5$ GeV$^2$. We find

$$\alpha^2(0) = 0.3305 \pm 0.0067 \pm 0.0054,$$

$$\chi^2_{min} = 1.78 \cdot \alpha^2(0) + 1.44.$$  \hfill (39b)

where again we take the fit (38) as the model-independent parametrization of the data. The above parametrization is valid only in the vicinity of the SM predictions, $\alpha^2(0) \sim 0.55$, as shown in Fig. 5.

The results of our two parameter fit to all the neutral current data are summarized in Fig. 5 by 1-$\sigma$ allowed regions in the $(\alpha^2(0), \alpha^2(m^2))$ plane. They are consistent with each other and, after combining the above four sectors, we find

$$\alpha^2(0) = 0.5460 \pm 0.0035 \quad \alpha^2(m^2) = 0.2351 \pm 0.0045,$$

$$\rho_{\text{err}} = 0.53.$$  \hfill (40a)

The fit is excellent as the effective degrees of freedom of the fit is 8 2 = 6. The combined fit above (40a) is shown by the thick 1-$\sigma$ contour in Fig. 5.

### 3.1.3 Charged Current

The $W$ mass data have been updated this summer by the CDF and D0 collaborations [60]. We obtain

$$m_w = 80.25 \pm 0.24 \text{GeV},$$

by combining the two most recent measurements [60] after adding all the quoted errors by quadrature.

The electroweak parameter $\varphi_\mu(0)$ is then obtained from the $\mu$ life-time via the identity (9). By using the SM estimate $\varphi_\mu = 0.0055$ and the perturbative approximation $\varphi_\mu = \varphi_\mu(0)$, we find

$$\varphi_\mu(0) = 0.4226 \pm 0.0025.$$  \hfill (42)

No other experiment in the charged current sector is accurate enough to add useful information in our electroweak analysis. Precision measurements of the $W$ width [61] and its leptonic branching fraction may determine $\varphi_\mu(0)$ in the future.

All the electroweak precision data have now been represented by the charge form factor values of eqs.(31,40,42). We find that all results are consistent with the assumptions of the SU(2)$_L \times$ U(1)$_Y$ universality and the SM dominance of the vertex and box corrections. In the following, we perform the fit to the data in three steps by systematically strengthening the model assumptions.

### 3.2 Testing the running of the charge form factors

Only two of the four form factors, $\varphi^2(q^2)$ and $\varphi^2(q^2)$, have been measured sufficiently accurately at two energy scales, $q^2 = 0$ and $m^2_z$. From eqs.(31,40), we find

$$\frac{4\pi}{\varphi^2(m^2_z)} - \frac{4\pi}{\varphi^2(0)} = -0.36 \pm 1.2(a_p - 0.12) \pm 0.16,$$

$$\frac{\alpha^2(m^2_z)}{\alpha^2(0)} - a = -2.45 \pm 1.1(a_p - 0.12) \pm 0.63,$$

$$\rho_{\text{err}} = 0.48.$$  \hfill (43)

The $\varphi^2(0)$ and $\varphi^2(m^2_z)$ are represented by the thick 1-$\sigma$ contours in Fig. 5.
The SM predictions for these quantities are, respectively

$$\frac{4\pi}{\gamma^2(m_t^2)} \gamma^2(m_0^2) = -0.2998 + 0.0013 - 0.0021,$$

$$\frac{\alpha^4}{\alpha^3(m_t^2)} \frac{\alpha^2}{\alpha^2} = -3.0760 + 0.0058 - 0.0015,$$

in the range $100 < m_t(GeV) < 250$ and $100 < m_H(GeV) < 1000$. Both results are consistent at the $1\sigma$ level with the assumption that the running of these form factors is governed by the SM particles only (see Fig. 6). Since the running of the form factors is affected only by particles of mass in the vicinity of $m_z$, we conclude that there is no indication of new particles of mass $m < m_z$.

The errors in (43) are determined by those of the low energy experiments. Further improvements in the low energy precision experiments are needed to detect a signal of relatively light new particles.

### 3.3 Testing the 3 parameter universality

By using the SM running of the form factors (44), we can combine the $Z$ parameter fit (31) and the low energy $NC$ fit (40). This is schematically shown in Fig. 6, where the combined low energy $NC$ fit of Fig. 5 is reproduced in the $(\gamma^2(m_t^2), \gamma^2(m_0^2))$ plane. The uncertainty in the running of the parameters within the SM is visualized by the thickness of the contour which spans the range $m_t = 100 - 200$ GeV, $m_H = 100 - 1000$ GeV in eq.(44). The low energy parameters are consistent with the $Z$ parameters, which are also shown as the 'LEP+SLC' contour. All the neutral current data are now combined to give

$$\gamma^2(m_0^2) = 0.5547 - 0.033(\alpha_s - 0.12) \pm 0.0015,$$

$$\gamma^2(m_t^2) = 0.2312 + 0.008(\alpha_s - 0.12) \pm 0.0007,$$

$$\delta = -0.0064 - 0.43(\alpha_s - 0.12) \pm 0.0034,$$

$$\chi^2_{min} = 5.50 \pm (\alpha_s - 0.13, 0.12) \pm 0.0125^2.$$

The above fit is almost independent of $(m_t, m_H)$ values assumed in the running of the charge form factors. The $\chi^2_{min}$ value of 7.3 for $\alpha_s = 0.12$ is excellent for the effective degrees of freedom of the fit, 18 - 3 = 15.

There is one notable point at this stage which becomes apparent by comparing the global fit of Fig. 6 with the individual fit to low energy $NC$ data in Fig. 5. Both the data on $\nu_e + e$ and $\nu_\mu + e$ experiments are perfectly consistent with the global fit, whereas the APV result and the $e$ asymmetry fit are just 1-$\sigma$ away. Further studies of polarization asymmetries in the $e^-\mu$ sector, as well as quantitative studies of the neutral current processes at TRISTAN energies might be potentially rewarding.
Both the $S$ and $U$ parameters are consistent with zero at the 1-$\sigma$ level. Note also that the $S$ parameter is particularly sensitive to the hadronic uncertainty $\delta_{\alpha}$ of $1/(\alpha(m_1^2))$, whose mean value can change by a quarter of its error for $\delta_{\alpha} = \pm 0.10$ [10].

### 3.4 Testing the Minimal Standard Model

In the minimal SM, the parameters $\beta_\alpha^2(m_1^2), \beta_\beta^2(0), \beta_\gamma^2(0)$ and $\delta(m_1^2)$ are uniquely determined by the two mass parameters $m_t$ and $m_H$. Insertion of the SM ($m_t, m_H$) dependences into our global fits (31), (40) and (42) gives the constraints on $m_t$ and $m_H$.

In Fig. 8, we show the result of our global SM fit to all the electroweak data in the $(m_t, m_H)$ plane for three representative $\alpha_\alpha$ values. One can clearly see the strong correlation between the preferred values of $m_t$ and $m_H$, which is found independently of the assumed $\alpha_\alpha$ value. On the other hand, the preferred range of $m_H$ depends rather sensitively on $\alpha_\alpha$. For $\alpha_\alpha(m_z) < 0.125$, smaller $m_H$ is preferred, whereas for $\alpha_\alpha(m_z) > 0.130$, larger $m_H$ is slightly favored. The $m_H$ dependence of the fit is very mild and no strict bound on $m_H$ can be given without imposing a constraint on $\alpha_\alpha(m_z)$.

We find the following parametrization for our global SM fit to all the electroweak data in terms of $m_t, m_H, \alpha_\alpha(m_z)$ and $\delta_{\alpha}$:

$$
\chi^2_{SM}(m_t, m_H, \alpha_\alpha, \delta_{\alpha}) = \frac{(m_t - m_{t,\text{pred}})^2}{\Delta m_t} + \chi^2_{\text{data}}(m_H, \alpha_\alpha, \delta_{\alpha})
$$

where

$$
\Delta m_t = 147 + 12.7 \ln \frac{m_H}{100} + 0.9 \ln \frac{m_H}{100} - 3 \left( \frac{\alpha_\alpha - 0.12}{0.11} - 5 \left( \frac{\delta_{\alpha}}{0.10} \right) \right)
$$

$$
\chi^2_{\text{data}}(m_H, \alpha_\alpha, \delta_{\alpha}) = 16 - 0.28 \ln \frac{m_H}{100} (0.044 - 0.006 \ln \frac{m_H}{100}) (m_t - 150)
$$
and
\[
\chi^2_{\text{m}}(m_H, \alpha, \lambda_n) = 7.0 + \left( \frac{\lambda_n - 0.30}{0.45} \right)^2 + \left( \frac{\alpha - 0.1179 + 0.005b}{0.006} \right)^2
- \left( \frac{\alpha}{0.014} + \frac{b}{0.57} - 8.66 \right) \ln \frac{m_H}{100} + \left( \frac{\alpha}{0.078 - 1.9} \right) \ln^2 \frac{m_H}{100} + \left( \frac{\lambda_n}{0.10} \right)^2.
\]

Here, \(m_H\) and \(m_H\) are measured in GeV units. This parametrization reproduces the correct \(\chi^2\) within a few % accuracy in the range \(50 < m_H(\text{GeV}) < 300\), \(60 < m_H(\text{GeV}) < 1000\) and \(0.11 < \alpha(m_H) < 0.13\). The best-fit value of \(m_H\) for a given set of \(m_H, \alpha\) and \(\lambda_n\) is immediately obtained from eq.(48) with its approximate error of (49). Likewise, the dependence of \(\chi^2\) on \(m_H, \alpha\), and \(\lambda_n\) is obtained from the above parametrization for a given set of the remaining parameters. It is also easy to find the results that are independent of \(\alpha\) or \(\lambda_n\), or those after imposing external constraints on them, since the \(\chi^2\)-function above is of a quadratic form in \(\alpha\) and \(\lambda_n\) which can be readily integrated out. The parametrization also gives accurately the aforementioned \(\alpha\) dependence of the preferred \(m_H\) range, which confirms the trend as observed in refs. [62,63]. As an example, we show in Fig. 9 the minimal of the total \(\chi^2\) of the SM fit to all the electroweak data as functions of \(m_H\) and \(\alpha\) or \(m_H\) and \(\lambda_n\). The dashed lines are obtained by using the parametrization (50). Also shown in Fig. 9 is the absolute \(\chi^2\) minimum which is obtained by allowing both \(m_H\) and \(\alpha\) to be freely fitted by the electroweak data. It is approximated as
\[
\chi^2_{\text{min}} = 7.0 + 0.12\ln(m_H/14)^2 \quad \text{for free } \alpha(m_H).
\]
In the region \(60 \text{ GeV} < m_H < \infty\), this leads to a formal constraint on \(m_H, m_H < 3.1 \text{ TeV (90\% C.L.)}\). The upper bound is, however, clearly outside the region of validity of our perturbative framework. If we allow an arbitrary \(m_H\) values [62], \(0 < m_H(\text{GeV}) < \infty\), this bound becomes \(m_H < 570 \text{ GeV (90\% C.L.)}\). Severe upper bound can be obtained by restricting to smaller \(\alpha\) values \((\alpha, \leq 0.115)\), as can be seen from Fig. 9.

Finally, by noting that the effective number of the data we used in our analysis is 18, we conclude, from Fig. 9 and the parametrization (47), that an excellent agreement of the data with the SM predictions is observed in the unshaded ranges of \(m_H\) and \(m_H\) in Fig. 8 for arbitrary values of \(\alpha\) and \(\lambda_n\) in the possible ranges; \(0.11 \leq \alpha, \leq 0.13\) and \(-0.1 \leq \lambda_n \leq 0.1\). In other words, we find no signal of new physics beyond the SM in the present precision experiments.

4 Electroweak physics at TRISTAN

In this section, the electroweak physics at TRISTAN is described in our formalism. The charge form factors which would be determined at the TRISTAN energy are \(\bar{g}_Z(q^2)\) and \(\bar{g}_A(q^2)\) with \(\sqrt{s} = 58 \text{ GeV}\). It is notable that the measurement of the QED effective coupling \(\bar{g}_Z(q^2)\) at TRISTAN [64] has a clear advantage over that in LEP/SLC, since the latter receive huge backgrounds from the Z-exchange.

Though most of the formula given in the \(Z\) parameter analysis are common for the TRISTAN analysis, it is instructive to repeat them and to express the observables in terms of the scalar amplitudes (4) by neglecting the \(\cos \theta\)-dependence of the box corrections.

The cross sections for \(e^+ e^- \rightarrow f \bar{f}\) are given by
\[
\sigma_f = \sigma(e^+ e^- \rightarrow f \bar{f}) = \frac{8}{48\pi} \left( \left| M_{0L}^f + M_{2L}^f \right|^2 + \left| M_{0R}^f + M_{2R}^f \right|^2 \right) \left| C_{Zf} \right|^2
+ \left( \left| M_{1L}^f - M_{1R}^f \right|^2 + \left| M_{1R}^f - M_{1L}^f \right|^2 \right) \left| C_{Af} \right|^2 \left( 1 + 3 \left( \frac{\alpha}{\alpha^2} \right) \right).\]

(52)

where the factors \(C_{Zf}\) and \(C_{Af}\) which contains the external QCD corrections for the vector part [29] and for the axial vector part [30] together with the finite mass corrections of the final state fermions [31] are given in eq.(18). Neglecting the lepton masses, we have
\[
\sigma_f = \frac{8}{48\pi} \left( \left| M_{0L}^f + M_{2L}^f \right|^2 + \left| M_{0R}^f + M_{2R}^f \right|^2 \right) \left( 1 + 3 \frac{\alpha^2}{\alpha} \right).\]

(53)

(53)

Using the above cross sections, the ratios \(R_L\) and \(R_R\) at TRISTAN energies are defined by
\[
R_L = \sigma_L / \sigma_{\mu}, \quad (54a)
R_R = \sigma_R / \sigma_{\mu}, \quad (54b)
\]
with
\[
\sigma_{\mu} = \frac{4 \alpha^2}{3 \pi}.\]

(55)

The Forward-Backward asymmetries for \(f = \ell, c\) and \(b\) are given by
\[
A_{fR} = \frac{3}{2} \frac{2 \beta \left( \left| M_{0L}^f + M_{2L}^f \right|^2 - \left| M_{0R}^f + M_{2R}^f \right|^2 \right) + \beta^2 \left| M_{1L}^f - M_{1R}^f \right|^2 + \left| M_{1R}^f - M_{1L}^f \right|^2}{2 \beta^2 \left( \left| M_{0L}^f + M_{2L}^f \right|^2 + \left| M_{0R}^f + M_{2R}^f \right|^2 \right) + \beta^2 \left| M_{1L}^f - M_{1R}^f \right|^2 + \left| M_{1R}^f - M_{1L}^f \right|^2}.\]

(56)
Table 3 Vertex form factors in the SM at $\sqrt{s} = 58$ GeV

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\nu_L$</th>
<th>(\ell_L^*)</th>
<th>$\ell_R^*$</th>
<th>$u_L$</th>
<th>$u_R$</th>
<th>$\ell_L$</th>
<th>(\ell_R^*)</th>
<th>$d_L$</th>
<th>$d_R^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00107 + 0.000198 + 0.00345 - 0.00261 &amp; \ldots &amp; \ldots &amp; \ldots &amp; \ldots &amp; \ldots &amp; \ldots &amp; \ldots &amp; \ldots</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 4 Box form factors in the SM at $s = -2t = (58 \text{ GeV})^2$

<table>
<thead>
<tr>
<th>$f$</th>
<th>$sB(e_L, f_L)$</th>
<th>$sB(e_R, f_L)$</th>
</tr>
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<tbody>
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</table>

with

$$\beta_L = \sqrt{1 - \frac{4m_t^2}{s}}. \quad (57)$$

Observable asymmetries for charm and bottom jets should in general have QCD corrections:

$$A_{FB}^{\text{neutral}} = A_F + \beta_{\text{i.e.}}^2 (1 + k_A \beta_{\text{H}}) \quad (58)$$

where the factor $k_A$ depends on details of the jet axis defining algorithm (58).

The magnitudes of the non-universal corrections (vertex and box) in the SM are given in Tables 3 and 4. As seen from Table 4, the box corrections are numerically very small also in this energy region as compared to the propagator and vertex corrections.

We give in Fig. 10 the SM predictions for these parameters as functions of $\sqrt{s}$. It should be noted here that the asymmetries are very large because of the large interference between the $\gamma$ and the $Z$-exchange amplitudes. Hence they give qualitatively different information from the $Z$-pole asymmetries which determine only the $\delta_1(m_3^2)$ parameter: the asymmetries at TRISTAN are sensitive to $\delta_2(s)$ as well.

In principle, the TRISTAN experiments as well as those at PEP/PETRA colliders can measure the three charge form factors $\delta_1(s)$, $\delta_2^0(s)$, $\delta_3(s)$ by assuming the SM.
dominance to the vertex and box corrections. If we assume the SM running of the \( g_2^l(x) \) and \( e^l(x) \) form factors between the TRISTAN and LEP/SLC energies, then these experiments measure \( e^l(x) \). On the other hand, if we assume the SM running for \( e^l(x) \), then they measure two parameters \( g_2^l(x) \) and \( s^l(x) \). This latter measurement has similar physical consequences to those of the low energy neutral current measurements, especially to those of the APV and e+e- experiments that both measure the neutral currents in the e-\( q \) sector. The fits (37) and (39) give the combined information

\[
\begin{align*}
g_2^l(0) &= 0.5524 \pm 0.0166 \\
s^l(0) &= 0.2274 \pm 0.0088 \\
\end{align*}
\]

on the neutral current couplings as measured in the e-\( q \) sector. The e+e- experiments off the Z-peak can give additional information in this sector, as well as the new information in the e-\( l \) (purely charged lepton) sector. The SM prediction for these observables are shown by solid lines in Fig. 10. The regions sandwiched by the dashed lines in these figures show the 1-st allowed predictions that are obtained by using the above constraint (59) on the neutral current form factors, and the SM running of all the three form factors. These regions somewhat deviate from the SM predictions because the present low energy experiments in the e-\( q \) sector show a 1-st deviation from the SM predictions with the LEP/SLC inputs: see Figs. 5 and 6. If the e+e- experiments find accurate constraints on these observables that are comparable to these predictions of the low energy data, we will effectively have new information on the electroweak physics.

5 Summary

We introduce four charge form factors \( e^l(q^2) \), \( h^l(q^2) \), \( g_2^l(q^2) \) and \( g_2^b(q^2) \) associated with the four gauge boson propagators, and one vertex form factor \( h_b(q^2) \) associated with the Zb\( \bar{b} \) vertex in the analysis of the electroweak data at the quantum level.

By assuming negligible new physics contributions to vertex and box corrections, except for the Zb\( \bar{b} \) vertex, we can determine these charge form factors accurately from precision experiments at the one-loop level. Our approach allows us to test the electroweak theory at several qualitatively different levels. We find that the data show excellent agreement with the SM at all stages of these tests.

We clearly need further improvements in the precision experiments in order to identify a signal of new physics beyond the SM. We find that the two polarization asymmetries at high energies, \( F_L \) and \( A_{L\tau} \), are most effective in this regard since they constrain the parameter \( s^l(m_Z^2) \) directly without suffering from the QCD uncertainty. At low energies, two polarization experiments in the e-\( q \) sector, the polarized eD scattering and the APV measurements, may have the potential of identifying physics beyond the \( SU(2)_L \times U(1)_Y \) universality.

It is also shown that the electroweak physics at TRISTAN can be studied naturally in our framework. It will be an important and exciting work in future to compare and combine the TRISTAN data with the measurements at other energies.

We should note, however, that a better measurement of the QCD coupling strength \( \alpha_s(m_Z^2) \) and that of the hadronic vacuum polarization effect \( \delta_b = \delta([G但仍]_b)\) are needed in order for us to look beyond the SM through the electroweak radiative effects.

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