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# Is it possible to determine $\alpha_s$ using the semi leptonic $\tau$ decay data ?

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Abstract: It is shown that there are considerable uncertainties in the determiniation of the QCD running coupling constant $\alpha_s$ using the finite energy sum rule and the perturbative QCD method for the semi leptonic $\tau$ decay. The usual claim of a good agreement with the value of $\alpha_s$ determined from the LEP data could be accidental due to the special value of the physical $\tau$ mass. A new and more reliable sum rule is derived when asymptopia is above the $\tau$ mass scale; it can also give informations on the axial vector spectral function beyond the $\tau$ mass.

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There have been interests in using the semi leptonic $\tau$ decay data, under the assumptionn of the premature validity of the asymptotic freedom or equivalently the validity of the perturbative Quantum Chromodynamics ( QCD ) calculation at low energy, in order to determine the running coupling constant $\alpha_s$ at the $\tau$ mass scale [1-4]. When this coupling constant is runned to the Z mass scale, it was found that one obtains a result which is consitent with the value of $\alpha_s$ determined from the hadronic Z decay width and jet physics [ 1-4 ].

The purpose of this note is to point out that there are many uncertainties in the determination of $\alpha_s$ from the $\tau$ decay. The apparent good agreement with that determined from the Z decay could be accidental due to the special value of the $\tau$ mass. Because the average energy available to the hadronic system is less than 1.5 Gev, in contrast with the much higher energy scale of the Z mass, it is doubtful that the perturbative QCD and the short distance expansion analysis can be applied at this energy scale. This is so because the $\tau$ semi -leptonic decay is dominated by resonances rather the point like structure observed at the much higher energy Z mass scale. The uncertainties in recent theoretical works can be seen by examining the experimental data on the hadrons produced in the $e^+e^-$ collisions.

The compilation by the 1990 PDG [ 6 ] of the ratio of $R = \sigma ( e^+e^- \longrightarrow \text{hadrons} ) / \sigma ( e^+e^- \longrightarrow \mu^+ \mu^-)$ shows little evidence for R to have a point like behavior, i.e. R tends to a constant limit below 3 Gev. The experimental data are however rough and conflicting; new measurements will be valuable. Although there are some evidences for the point like structures above 5 Gev c.o.m. $e^+e^-$ energy, but R increases with energy instead of decreasing to a constant value as predicted by the asymptotic freedom. Above 11 Gev the experimental data appear to be consistent with the theoretical prediction of the QCD as analysed by recent theoretical works [7-9]. From this discussion, it would even be impossible to determine $\alpha_s$ using the $e^+e^-$ hadronic data in the 5-10 Gev region ( when the hadronic contribution from the $\tau$ decay is subtracted out ). We can stop this article at this point, let us however be constructive and entertain the possibility that the experimental data in this energy region is either wrong or this peculiar behavior is not shared by the lighter quarks. In fact the more recent values of R published in 1981 by Bacci et al [10], which was not used by the PDG group, showed the point like structure above 1.8 Gev. We have not, however, examined the reliability of this analysis. Because of the conflicting results in the 2 Gev region, we *strongly* urge the experimentalists to remeasure the ratio R in this region, including the $6\pi$ cross section in order to settle the important question of starting at what energy the perturbative QCD can

be trusted. The well-known proof of the asymptotic freedom [11] was based on the study of the correlation function $\Pi(s)$ in the deep Euclidean region. One must however look at the experimental data to get a hint of the asymptopia.

Although theoretical works based on the QCD sum rules seem to suggest that the perturbative QCD in combination with the short distance expansion can be used at a very low energy scale, between 1 and 2 Gev [12], but this was done with the hope that the low energy hadronic properties can be calculated with an accuracy of 10 to 20 % but not of a few percent which is needed in order to determine $\alpha_s$ accurately from the $\tau$ decay data. The apparent good agreement between the value of $\alpha_s$, calculated by recent theoretical works [1-4] using the semi leptonic $\tau$ decay data and extrapolated to the Z mass scale, with that determined from the observed Z hadronic width and jet physics could be accidental.

To show this is the case, we first calculate the ratio $R_\tau^V$ of the decay rate of $\tau \rightarrow (1^-$ even G-Parity states $)+\nu$ to the electronic mode, using the $e^+e^- \rightarrow$ G= even parity pion data and the conservation of the vector current ( CVC ) hypothesis. We then equate this result with that obtained from the perturbative QCD in order to determine the strong QCD running coupling constant $\alpha_s(M)$ at the $\tau$ mass M. We then change M from its observed value to lower and larger values. It will be shown that for M varies from 1.6 to 2.1 Gev, the values of $\alpha_s$ deduced from the data change rapidly from 0.35 to 0.19, where the last figure is obtained with M=2.1 Gev, in contrast with the slower variation of the order of 10% expected from the perturbative QCD calculation.

This result is totally expected because, as discussed above, at the $\tau$ mass scale, we are near to the resonance region i.e. non perturbative, and not the asymptotic region. The idea of changing the $\tau$ mass as a mean to test the reliability of the use of the perturbative QCD result for $\tau$ decay was previously suggested [13] but was criticized by the reference [4].

Because of the CVC, the vector correlation tensor can be written as $\Pi^{\mu\nu}(q) = (-g^{\mu\nu}q^2 + q^\mu q^\nu)\,\Pi(q^2)$. The ratio $R_\tau^V$ of the vector hadronic decay the electronic state decay is given by:

$$R_\tau^V = 12\pi \int_{4m^2}^{M^2} \frac{ds}{M^2}\left(1 - \frac{s}{M^2}\right)^2 \left(1 + 2\frac{s}{M^2}\right) \mathrm{Im}\Pi(s + i\varepsilon) \qquad (1)$$

where M and m are, respectively, the $\tau$ lepton and pion masses $\mathrm{Im}\Pi(s)$, is given by unitarity:

$$\mathrm{Im}\Pi(s+i\varepsilon) = (1/6\pi)\,\sigma[e^+e^- \to (I=1)\text{ hadrons}] / \sigma[e^+e^- \to \mu^+\mu^-]\cos^2\theta_c \qquad (2)$$

where $\theta_c$ is the Cabibbo angle and the isospin rotation is used.

Consider the function $G(s) = (1/M^2)(1 - s/M^2)^2(1 + 2s/M^2)\,\Pi(s)$ as an analytic function in the s plane except a cut on the real s positive axis. By the Cauchy integral , Eq (1) can be written as an integral around the circle in the complex s-plane with a radius $|s| = M^2$ which runs in the counter-clockwise direction:

$$R_\tau^V = 6\pi i \int_{|s|=M^2} \frac{ds}{M^2}\left(1 - \frac{s}{M^2}\right)^2\left(1 + 2\frac{s}{M^2}\right)\Pi(s) \qquad (3)$$

This equation is rigourous as a mathematical theorem which means that when $\Pi(s)$ is analytically continued to the real axis, it will have exactly the discontinuity as given by $2i\,\mathrm{Im}\Pi(s)$ which must therefore describes correctly the experimental data. It is usually assumed in recent theoretical studies that $\Pi(s)$ used in Eq (3) is given by the perturbation theory. This is a very *strong assumption at the $\tau$ mass scale*, because we are near to the resonance region. It is highly desirable that this assumption must be *verified* by experiments. We cannot of course check this assumption for complex s but can analytically continue the perturbative amplitude onto the real positive s axis and compare it with the experimental data. It is usually argued that the weight factor $(1-s/M^2)^2$ would minimise the contribution of $\Pi(s)$ around $s= M^2$. This would be the case if the experimental data for $s > M^2$ agree with the perturbation series to a good precision as will be shown below.

The usual assumption in recent theoretical studies is that the function $\Pi(s)$ on the contour is given by the perturbative QCD calculation [ 1-4, 9 ]:

$$\Pi_p(s) = A - (N_c\cos^2\theta_c / 12\pi^2)\,[\log(-s/\Lambda^2) + (12/(33 - 2n_f)\log\log(-s/\Lambda^2)] + \ldots \qquad (4)$$

where $N_c$ is the QCD color number, A is a constant which drops out in the calculation. For simplicity and clarity, we write down only the perturbative QCD result including only the $\alpha_s$ correction, but in the following calculation higher order $\alpha_s$ corrections are also taken into account. In order to compare our work with the previous publications, we also assume that the contribution of the higher dimension operators in the operator product expansion are small [1-4]. The recent resolution of the so-called one prong problem [14] by the ALEPH group[15] and others implies the

approximate equality of the vector and axial spectral function contribution to the $\tau$ decay as suggested earlier [16] but cannot be used to estimate the contribution of the dimension 6 operator [17].

The analytic continuation of the perturbative function $\Pi(s)$ on the real axis is simply a smooth function varying slowly with s; under no circumstances, it can approximate the behaviour of the resonances. What one can hope is that at $|s| = M^2$, the true function $\Pi(s)$, can be approximated by that given by Eq(4). We shall therefore have some approximate average equality which must depend sensitively on the value of the $\tau$ mass because of the dominance of the resonances in $\tau$ decay. We now show this is indeed the problem which prevents us from using the perturbative calculation in combination of the experimental data on $R_\tau^V$ as a method of determining $\alpha_s$.

We calculate $R_\tau^V$ as given by the perturbative QCD and compare it with that given by the $e^+e^-$ data using CVC for 4 values of $\tau$ masses M = 1.6, 1.784, 1.9 and 2.1 Gev. The perturbative result is straightforwardly given by [1-4]:

$$R_\tau^V = ( N_c \cos^2\theta_c / 2 ) [ 1 + \alpha_s( M^2) / \pi + (\alpha_s(M^2)/\pi)^2 ( F_3 - 19\beta_1/24 ) ] \qquad (5)$$

where $\alpha_s( M^2 )$ is the QCD coupling constant evaluated at the $\tau$ mass M, $F_3 = 1.9857 - 0.1153\, n_f$ with $n_f$ denoting of the number of quark flavors and $\beta_1 = ( 2n_f - 33 ) / 6$. For simplicity we give only the expression for $R_\tau^V$ up to two loop corrections, but the $\alpha_s^3$ order correction is known and can be taken into account in a straightforward manner. Next we determine $\alpha_s$ using Eq.(5) and equate it with the value of $R_\tau^V$ determined from experiments.

It is somewhat more involved to calculate $R_\tau^V$ using the experimental data on $e^+e^-$ and Eq (1) because of the conflicting measurements. We use $e^+e^- \to \pi^+\pi^-$ as given by Barkov et. al. [18] and the $e^+e^- \to 4\pi$ data a below 1.2 Gev as given by the Novosibirsk group [19,20] and the more recent accurate results of DM2 above 1.35 Gev [21]. The $e^+e^-$ cross section data presented by the DM2 group for $\pi^+\pi^-\pi^0\pi^0$ are significantly lower than those given by the previous publications.

The $e^+ e^-$ experimental data for the cross sections of the G = + parity pion states between 1.0 to 2.1 Gev are not consistent with each other [20]. For example around 1.4 Gev, the ND data [20] yield $\sigma(e^+e^- \to \pi^+ \pi^-\pi^0\pi^0 )$ cross sections which are larger than the OLYA [20] measurements by 40% and a factor of 2 larger than the more recent accurate DM2 results. We should also like to point out that the the combination $\sigma_\tau = \sigma( e^+e^- \to \pi^+ \pi^-\pi^0\pi^0 ) + 1/2\, \sigma( e^+e^- \to$

$\pi^+\pi^-\pi^+\pi^-$) cross sections of the DM2 [21] agree well with the Argus [22] results for $\tau$ decay which agree only with the ND and OLYA experimental data below 1.2 Gev but not at higher energy. ( There is a possibility that the Argus result could be wrong by 20% due to the difficulty of detecting the $\pi^0$ in the decay mode $\tau \rightarrow \pi^+ \pi^-\pi^-\pi^0\nu$ but the error of this magnitude will not influence the following calculation).

In the following, we first use the following set of experimental data : The $\sigma( e^+e^- \rightarrow \pi^+ \pi^-\pi^+\pi^- )$ present no problem because all the existing data including the new results from the DM2 group agree with each other. From 1 Gev to 1.2 Gev we use the ND and OLYA data for $\sigma( e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0 )$ because they agree with the experimental determination from the Argus data [22]. From 1.35 Gev up to 2 Gev we use the recent accurate data from the DM2 group [21] which are consistent with the previous measurements at Orsay and also with the less accurate Argus data. We extrapolate the experimental data between 1.2 to 1.35 Gev. Because there is no accurate measurements for the $\sigma ( e^+e^- \rightarrow 6\pi )$, and while waiting for the new DM2 result for this process, we can suppose that the $\sigma ( e^+e^- \rightarrow 6\pi )$ is negligible at 1.5 Gev and below, and rise to about 20 nb at 1.8 Gev. These cross sections would yield $B(\tau \rightarrow (6\pi)^-\nu ) = 0.12\%$ in agreement with the world average value of $0.11 \pm 0.027\%$. Above this energy, the even G Parity cross sections are supposed to have the asymptotic value predicted by the parton model, $R^V = 3/2$ as roughly given by the $\gamma\gamma 2$ experiment [10]. (The low energy measurements of the $6\pi$ cross section of the $\gamma\gamma 2$ group would yield $B(\tau \rightarrow (6\pi)^-\nu ) = 0.34\% \pm 0.06\%$ [19] which is much larger than the world average value.). The calculated $R_\tau^V$, using this procedure, is referred in the table 1 as $R_\tau^V(1)$. Admittedly this way of using the experimental data is not conventional, we simply want to illustrate our main point, and leave the job of fitting procedures to the experts.

To illustrate that our conclusion is not affected by this method of handling the experimental data, $R_\tau^V$ is also calculated using the ND data from the threshold up to 1.35 Gev for both $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$ cross sections and thereafter the DM2 data. The calculated value for $R_\tau^V$ in this way is referred in the table 1 as $R_\tau^V(2)$.

In the table 1, the calculated value of $\alpha_s$ as a function of the $\tau$ mass is given for both values of $R_\tau^V$. If one takes only into account of the statistical errors, the error of $R_\tau^V$ is about 1%, but this could be misleading due to the larger systematic errors. As M increases from 1.6 to 2.1 Gev, $\alpha_s$ decreases by almost a factor of 2, independently of the method of handling the

experimental data. This is a much larger variation from the expected perturbative QCD result which would give a decrease of the value of $\alpha_s$ of only 10% in this mass range. The reason for a fast decrease in $\alpha_s$ is due to the $1/M^2$ behavior of the ratio $\tau \to \rho\nu$ to the electronic mode; this fast decrease as a function of M is not sufficiently compensated by the contribution from the opening of the even number multi- pions channels to keep $R_\tau^V$ almost constant in order to support the perturbative calculation.

A factor 2 of uncertainty in $\alpha_s(M)$ is not so bad, because at the Z mass scale, it will become only 25%: for $\alpha_s(M)$=0.35 and 0.19 we have respectively, $\alpha_s(M_Z) = 0.12$ and 0.095 [22]. The rapid variation of $\alpha_s(M)$ as a function of M can however be serious, because it may reflect the presence of a large non perturbative effect at the $\tau$ mass scale which has not been taken into account.

We now want to turn the problem around and extend the contour integral to a higher masss scale where the perturbative calculation is expected to be more reliable. Let us use the contour integral along the real axis from the threshold to an energy squared $nM^2$ then into a larger circle in the complex s plane with the radius $|s| = nM^2$ with n>1. Eq (3) can now be rewritten as a sum of the contribution of the discontinuity on the real axis from $M^2$ to $nM^2$ and that from the circle of the radius $n|M^2|$ in the counterclockwise direction :

$$R_\tau^V = 12\pi \int_{M^2}^{nM^2} \frac{ds}{M^2}\left(1 - \frac{s}{M^2}\right)^2\left(1+2\frac{s}{M^2}\right) \mathrm{Im}\Pi(s+i\varepsilon) + 6\pi i \int_{|s|=nM^2} \frac{ds}{M^2}\left(1 - \frac{s}{M^2}\right)^2\left(1+2\frac{s}{M^2}\right) \Pi(s) \quad (6)$$

where $\mathrm{Im}\Pi$ from the first integral *must* be taken from the experimental data. It is simple to see that if the data on the real axis from $M^2$ to $nM^2$ was exactly the same as that given by the imaginary part of the perturbative QCD, there would be no difference between the results of Eq (6) and that given by Eq (3), using the perturbative QCD result for complex s. Using the perturbative results on the contour $|s| = nM^2$ we can rewrite Eq (6) as:

$$12\pi \int_{M^2}^{nM^2} \frac{ds}{M^2}\left(1 - \frac{s}{M^2}\right)^2\left(1 + 2\frac{s}{M^2}\right) \mathrm{Im}\Pi(s+i\varepsilon) = N_c\cos^2\theta_c \left\{ \left(n-n^3+\frac{n^4}{2}\right)\left[1+a+a^2\left(F_3 - \frac{1}{2}\beta_1 \log n\right)\right] + \frac{1}{2}\beta_1 a^2\left[-n+\frac{n^3}{3} - \frac{n^4}{8} + \frac{n}{2}(2 - 2n^2 + n^3)\log n\right] \right\} - R_\tau^V \quad (7)$$

where $a = \alpha_s(nM^2)/\pi$; Eq (7) gives informations on the average behaviour of the vector spectral functions between $M^2$ and $nM^2$ as a function of the experimental value of $R_\tau^V$. For n = 1 we

recover Eq (5). We must choose n in such a way that at $nM^2$ the asymptotic freedom is approximately valid. For a feel, let us take n = 2 corresponding to an upper limit of the integration of about (2.5 Gev $)^2$, the first term on the RHS of Eq (7) is larger than the last term by a factor of 4 which should be easily tested once the accurate $e^+e^-$ into the even G-Parity state will become available. Furthermore for n sufficiently large, the contribution from the resonance region as represented by the experimental value $R_\tau^V$ becomes small with respect to the remaining terms on the RHS. The validity of Eq (7 ) then depends mostly on how well the observed even G- Parity $e^+e^-$ cross sections can be fitted with the perturbative results. Eq (7) represents however an improvement over the " eye ball " fit to the experiental data, but the essential point is that one must be in the asymptotic region; this can be checked by looking directly at the experimental data.

The above discussion also makes the role of the weight factor $(1-s/M^2)^2$ not as important as it is usually thought. Indeed, if we write $n = 1+\varepsilon$ i.e. with $\varepsilon << 1$, i.e. we change slightly the $\tau$ mass, then the first term on the RHS is of the O $(\varepsilon^2)$. However this is no longer true when $\varepsilon$ is not small, then the weight factor makes the high energy contribution away from $s = M^2$ important. This problem is associated with the well known fact that it is difficult to make a good approximation for an analytic function for a wide range of energy.

We can generalise Eq (7) to include the axial spectral function contribution; the Im$\Pi$ on the LHS should be interpreted to include both the vector and the axial contributions; on the RHS the coefficient $N_c$ is multiplied by a factor of 2 and $R_\tau^V$ is replaced by the total hadronic to electronic ratio. Because the vector part contribution can be evaluated using $e^+e^-$ data, the remaining unknown axial contribution from $M^2$ to $nM^2$ can then be expressed as a sum rule. It is also simple to generalise Eqs (6, 7) to include the strangeness changing decay ; in this case we have to add to the RHS of Eqs (6,7) the contribution of similar terms proportional to $\sin^2\theta_c$ which amounts to omitting the $\cos^2\theta_c$ on the RHS and reinterprete these equations in an obvious way.

In conclusion, we raise in this letter the question of the applicability of the perturbative QCD at the $\tau$ mass scale. Our analysis casts doubt about this possibility, but it is by no means definite, pending an improvement of the experimental data, especially the ratio R above 1.7 Gev. Further studies of the non perturbative effects will be considered elsewhere.

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**Table 1**

| $M_\tau$( Gev ) | $R_\tau^V(1)$ | $R_\tau^V(2)$ | $\alpha_s$ calc.by $R_\tau^V(1)$ | $\alpha_s$ calc.by $R_\tau^V(2)$ |
|---|---|---|---|---|
| 1.6 | 1.72 | 1.74 | 0.35 | 0.37 |
| 1.784 | 1.64 | 1.66 | 0.26 | 0.29 |
| 1.9 | 1.60 | 1.63 | 0.22 | 0.25 |
| 2.1 | 1.58 | 1.61 | 0.19 | 0.23 |

Table Caption

$R_\tau^V(1)=R_\tau^V$ calculated using ND and OLYA data up to 1.2 Gev and DM2 data as described in text.

$R_\tau^V(2)=R_\tau^V$ calculated using ND and OLYA data up to 1.35 Gev and DM2 data as described in text

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