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Rolling Vacuum in Brans-Dicke and Supergravity Theories

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Abstract

The role played by the Brans-Dicke dilaton in resolving the cosmological constant problem is investigated both in the Brans-Dicke theory and in the supergravity theories. The supergravity theory is understood to be a natural generalization of the Brans-Dicke theory. It is found that the idea of the rolling vacuum is consistent with the constancy of the gauge coupling in the 4-dimensional Calabi-Yau compactification of the 10-dimensional supergravity theory. The appearance of the various mass scales in nature is attributed to the possibility of expanding the physical quantities in terms of the ratio of the size of the internal space with the Planck length. The crucial test of the theory will be to find the time dependence of the mass scales.

1 Introduction

Brans-Dicke theory[1] has been investigated so far as a variant of Einstein gravity without positive experimental evidence [2]. Recently, however, La and Steinhardt [3] suggested that the Brans-Dicke theory is capable of solving the graceful exit problem [4] which existed in the original inflation model of Guth [5].

On the other had it has been known that there exists a scalar field which is similar to the Brans-Dicke dilaton both in 4-dimensional [6] and 10-dimensional supergravity theories[7]. Its physical meaning, however, has not been fully investigated so far. The purpose of this work is to clarify the role played by the Brans-Dicke scalar in resolving the cosmological constant problem [8].

The basic idea is the following. The Brans-Dicke scalar is characterized by the fact that it couples to the trace of the energy momentum tensor:

$$\Box\phi = T^{\mu}_{\mu}(\phi, \lambda) \tag{1}$$

This equation shows that ϕ couples to the vacuum energy i.e. the cosmological constant when it is not vanishing. If ϕ takes some non-vanishing constant value in the vacuum state, it causes the well known difficulty of non-vanishing cosmological constant. The vanishing ϕ is achieved only when the vacuum energy is zero as is seen from (1). Completely different situation emerges when the vacuum corresponds to the solution which is "rolling down the hill". The cosmological constant term will be multiplied by the exponential of the dilaton field in the Einstein frame and it will vanish when the dilaton field tends to $-\infty$ as the time passes. This is the situation we investigate in this work.

The strong argument [9] against the vacuum which is "rolling down the hill" comes from the observation that it might result in an unacceptable time-dependence of such fundamental constant as the electric charge [10]. The purpose of this work, therefore, is to show that this observation is false. It is shown in particular that in the Calabi-Yau compactification [11] of the 10-dimensional supergravity we have the time independent gauge coupling at least or most likely only in the case of 6-dimensional internal space. We find that the scalar field which originates in the 11-dimensional supergravity conspires with the scalar field which corresponds to the size of the internal space to make the electric charge time-independent.

Brans-Dicke theory is not a conformably invariant theory. We must therefore select a specific conformal frame in which the comparison is made of the experimentally observed physical quantity with the theory. We find that we can consistently interpret the finite cosmological constant only in the so called Einstein frame where the gravitational constant is time-independent rather than in the Jordan frame where the cosmological constant is time- dependent. As a matter of fact we always write the Lagrangian of the 4 dimensional [6] or of the 10-dimensional [7] supergravity theory in the Einstein frame assuming implicitly that the gravitational constant is time-independent.

In section 2 the Brans-Dicke theory with the cosmological constant is investigated. Rolling solution for the vacuum in the Jordan frame is compared with that in the Einstein frame. It is shown that the cosmological constant can be absorbed in the boundary condition in the Einstein frame and so there is no way to set any bound on its value. Whereas it follows the usual argument in the Jordan frame. Section 3 is devoted to the study of its implication to cosmology. It is pointed out that the finite temperature universe expands faster than the zero temperature universe, which is nothing but the vacuum.

We also point out that part of the observed cosmological red shift is compensated by the time change of the mass of the elementary particles. In section 4 we discuss the space-time compactification of superstring theory. We show that in the case of 6-dimensional internal space (4-dimensional space-time) there exists a vacuum solution in which the size of the internal space is slowly expanding and the gauge coupling is independent of time. We find that the breaking of the supersymmetry must occur at 10^{11} GeV if the grand unification scale is assumed to be 10^{15} GeV which is the inverse size of the internal space. We also find that so to speak the two bridge poles in the hidden sector at 10^{11} GeV and at 10^{7} GeV suspend the bridge which interconnect the grand unified world with the low energy world. We make some concluding remarks in section V.

2 Brans-Dicke Theory with Cosmological Constant

Brans-Dicke theory [1] in the Jordan conformal frame is characterized by the following action

$$S = \int \sqrt{(-\tilde{g})} \left[-\tilde{\phi}\tilde{R} - \lambda - \frac{\omega}{\tilde{\phi}} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \right] + S_m \left(\tilde{\psi}, \tilde{g}_{\mu\nu} \right). \tag{2}$$

Here the notation $\tilde{A}$ is to indicate that the variable A is in the Jordan frame. Cosmological constant λ is absent in the conventional Brans-Dicke theory, while it plays the fundamental role in this work. $\tilde{\psi}$ stands for an arbitrary matter field. Note that $S_m(\tilde{\psi}, \tilde{g}_{\mu\nu})$ does not depend on the Brans-Dicke dilation field $\tilde{\phi}$. The Brans-Dicke action is not invariant under the conformal transformation. This implies that we obtain a different theory if we perform a conformal transformation to equation (2). Einstein frame is characterized by the fact that the gravitational constant is field-independent in it. It can be obtained from the Jordan frame by the following conformal transformation.

$$(2\kappa^2 \tilde{\phi}) \tilde{g}_{\mu\nu} = g_{\mu\nu}, \tag{3}$$

with

$$\tilde{\phi} = \frac{1}{2\kappa^2} e^{-2\beta\sigma}, \tag{4}$$

where

$$\beta = \frac{1}{2}(\omega + \frac{3}{2})^{-\frac{1}{2}} \tag{5}$$

and

$$\kappa^2 = m_p^{-2}, \tag{6}$$

m_p being the Planck mass.

The action becomes

$$S = \int \sqrt{(-g)} \left[\frac{-R}{2\kappa^2} - \lambda e^{4\beta\sigma} - \frac{g^{\mu\nu}}{2\kappa^2} \partial_\mu \sigma \partial_\nu \sigma \right] + S_m(\psi, e^{2\beta\sigma} g_{\mu\nu}). \tag{7}$$

In contrast to the Jordan frame the Einstein frame gives the matter action which depends on the Brans-Dicke scalar through $e^{2\beta\sigma} g_{\mu\nu}$. This is shown to

result in the time-dependence of particle masses. Furthermore, the cosmological constant has an ordinary stringent bound in the Jordan frame but it evades the bound in the Einstein frame as is shown below. It is shown that it is absorbed in the initial condition of the vacuum solution which is rolling down the hill.

In the case when no matter field has the vacuum expectation value we obtain the following set of vacuum equations in the Einstein frame.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \lambda\kappa^2 e^{4\beta\sigma}g_{\mu\nu} + \frac{1}{2}g_{\mu\nu}g^{\rho\kappa}\partial_\rho\sigma\partial_\kappa\sigma - \partial_\mu\sigma\partial_\nu\sigma, \tag{8}$$

and

$$g^{\mu\nu}D_\mu\partial_\nu\sigma = 4\beta\kappa^2\lambda e^{4\beta\sigma}, \tag{9}$$

where D_μ is the conventional covariant derivative. Assuming that the vacuum space is maximally symmetric;

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin\theta^2d\varphi^2\right), \tag{10}$$

we obtain the following solution [3] for equation (8) and (9):

$$a = a_0 + 2\beta\alpha t, \tag{11}$$

with

$$\sigma = \sigma_0 - \frac{1}{2\beta}log\left(1 + \frac{2\beta\alpha}{a_0}t\right), \tag{12}$$

where

$$\alpha^2 = \frac{2\omega+3}{\omega-\frac{1}{2}}k, \tag{13}$$

and β is given previously. a_0 and σ_0 are related by,

$$a_0 = \frac{\alpha}{\sqrt{\kappa^2\lambda}}e^{-2\beta\sigma_0}. \tag{14}$$

Corresponding solution in the Jordan frame is

$$\tilde{a} = \beta\alpha t + a_0e^{\beta\sigma_0}, \tag{15}$$

with

$$\tilde{\phi} = \frac{\lambda}{2\alpha^2}\left(\beta\alpha t + a_0e^{\beta\sigma_0}\right)^2. \tag{16}$$

If we take the existence of finite cosmological constant seriously it is impossible to have a matter dominating universe except for a period of very early universe where quantum effect may be important. Throughout the almost entire period of the universe the expansion rate will be given by the cosmological constant. The discussion of the case of finite temperature will be given in the next section. At the present stage of the universe where the temperature is very low the universe will be close to the vacuum given either by equation (11) or by equation (15). The theory in the Jordan frame gives the following expansion rate,

$$H = \frac{\dot{\tilde{a}}}{\tilde{a}} = \frac{\beta\alpha}{\beta\alpha t + a_0 e^{\beta\sigma_0}} = \beta\sqrt{\frac{\lambda}{2\tilde{\phi}}} \tag{17}$$

We have, therefore,

$$\lambda = \frac{2H^2\tilde{\phi}}{\beta^2} \sim H^2 m_p^2, \tag{18}$$

where m_p stands for the Planck mass. Here we used the fact that the gravitational constant is expressed in terms of the field ϕ in the action given by equation (2). H is the Hubble constant at the present time. Thus the λ is restricted by the usual limit and it is as small as $(10^{-3}\ \mathrm{ev})^4$. In the Einstein frame we have

$$H = \frac{\dot{a}}{a} = \frac{1}{t + \frac{1}{2\beta}\sqrt{\frac{m_p^2}{\lambda}}e^{-2\beta\sigma_0}} \tag{19}$$

λ is completely arbitrary in this case and can be absorbed in the initial value σ_0. This is the key observation for the later discussions and because of this we regard the theory in the Einstein frame to be the right theory rather than that in the Jordan frame.

The σ-dependence of the matter action $S_m(\psi, e^{2\beta\sigma}g_{\mu\nu})$ in equation (3) gives the time dependence of the physical variables. The matter action for the scalar, the gauge vector and the Dirac spinor in the Jordan frame takes the form;

$$\begin{aligned} S_m &= \int\sqrt{(-\tilde{g})}\Big[-\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - V(\tilde{\phi}) - \frac{1}{4}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\kappa}\tilde{F}_{\mu\rho}\tilde{F}_{\nu\kappa} \\ &\quad - i\bar{\tilde{\psi}}\gamma^\alpha D_\alpha\tilde{\psi} - g\bar{\tilde{\psi}}\tilde{\psi}\tilde{\phi}\Big], \end{aligned} \tag{20}$$

where

$$D_\alpha = \tilde{e}^\mu_\alpha(\partial_\mu - ie\tilde{A}_\mu) + \frac{1}{2}\sigma^{\beta\gamma}\tilde{e}^\nu_\beta\tilde{e}^\mu_\alpha(\tilde{e}_{\gamma\nu})_{;\mu}, \tag{21}$$

with $\tilde{e}^\mu_\alpha$ being the vierbein and $;\mu$ stands for the gravitational covariant differentiation. Under the conformal transformation

$$\tilde{g}_{\mu\nu} = e^{2\beta\sigma}g_{\mu\nu}, \tag{22}$$

we get

$$\sqrt{(-\tilde{g})} = e^{4\beta\sigma}\sqrt{(-g)} \tag{23}$$

Equation (20) shows that the gauge coupling remains constant even in the Einstein frame because of equation (23) which is valid only in the 4-dimensional space time. Changing the variable from $\tilde{\phi}$ and $\tilde{\psi}$ to $\phi = e^{\beta\sigma}\tilde{\phi}$ and $\psi = e^{\frac{3}{2}\beta\sigma}\tilde{\psi}$ respectively we obtain the matter action in the Einstein frame;

$$\begin{aligned} S_m &= \int\sqrt{(-g)}\Big[-\frac{g^{\mu\nu}}{2}\big\{\partial_\mu\phi\partial_\nu\phi - \beta\partial_\mu\sigma\phi\partial_\nu\phi - \beta\partial_\nu\sigma\phi\partial_\mu\phi + \beta^2\partial_\mu\sigma\partial_\nu\sigma\phi^2\big\} \\ &\quad - \Big(\frac{1}{2}\mu^2e^{2\beta\sigma}\phi^2 + \xi\phi^4\Big) - \frac{1}{4}g^{\mu\rho}g^{\nu\kappa}F_{\mu\nu}F_{\rho\kappa} - \bar{\psi}\left(i\gamma^\alpha D_\alpha - g\phi\right)\psi\Big], \end{aligned} \tag{24}$$

where we have taken the explicit form for $V(\phi)$:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \xi\phi^4. \tag{25}$$

The vacuum solution in this case has to take into account the non-vanishing ϕ. The exact solution in this case for a σ is given by equation (12). The only change is that we have to replace λ by $\lambda + \frac{1}{2}\mu^2\phi_0^2 + \xi\phi_0^4$ in equation (14) with ϕ_0 being the solution of

$$\mu^2\phi_0 + 4\xi\phi_0^3 = 0. \tag{26}$$

ϕ is given by:

$$\phi = \phi_0\sqrt{\frac{a_0}{a}} = \frac{\phi_0}{\sqrt{e^{-2\beta\sigma_0} + 2\beta\alpha t/a_0}}. \tag{27}$$

This solution implies that ϕ may start with the non-zero value ϕ_0 but it eventually tends to zero as the time passes. The rate of the transition is directly related to the expansion rate of the vacuum which is approximately that of our universe itself.

3 Cosmology

Our discussion in section 2 has been on the vacuum solution which is rolling down the hill. Our universe may be approximated by the vacuum solution at a very low temperature as it is now. This is the consequence of the finite cosmological constant which is much larger than the matter density. We have seen in the previous section that it does not cause any problem for the theory in the Einstein frame. Let us discuss briefly the case of finite temperature β^{-1}. We have,

$$-\ddot{\sigma} = \kappa^2 \frac{\partial F}{\partial \sigma}, \tag{28}$$

and

$$\dot{a}^2 + k = \frac{\kappa^2}{3} a^2 \rho, \tag{29}$$

with

$$6\ddot{a} = -\kappa^2(\rho + 3p)a. \tag{30}$$

Here F is given in the free gas approximation [12],

$$F = V_o(\sigma) + \frac{1}{(2\pi)^3 \beta} \int d^3k log\left(1 - exp(-\beta\sqrt{M^2 + k^2})\right) \tag{31}$$

where $V_o(\sigma)$ is the effective potential at zero temperature and M^2 is given by,

$$M^2 = \frac{\partial^2 V_0(\sigma)}{\partial \sigma^2} = (4\beta_0)^2 \lambda e^{4\beta_0 \sigma}, \tag{32}$$

where β in section 2 has been replaced by β_0 not to be confused with the temperature. ρ and p are related to F by,

$$\begin{aligned} \rho &= F + \beta \frac{\partial F}{\partial \beta} \\ and \quad p &= -F. \end{aligned} \tag{33}$$

We see from (31) that the effective potential is a monotonic function of σ. This implies that the universe will correspond to the solution which is rolling down the hill as in the case of the vacuum. The cosmological constant does not stay constant and it seems that the exponential expansion does not occur

even at the finite temperature. Nevertheless, it is very important as will be discussed later that the expansion rate of the universe is much faster than that of the vacuum. We expect that this is realized qualitatively because of the higher density at higher temperature. We note that equations (28) through (33) guarantee that the universe expands like $\sqrt{t}$ at the initial stage. Quantitative discussion throughout the entire period will be given elsewhere. More complete picture can be obtained by studying the equation;

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}\int \Pi_x d\sigma(x) e^{-\int d^4x\sqrt{g}\left[\frac{g^{\mu\nu}}{2}\partial_\mu\sigma\partial_\nu\sigma+\lambda e^{4\beta\sigma}\right]}, \tag{34}$$

where the boundary condition is $\sigma(\beta) = \sigma(0)$. Equation (34) is the Euclidean gravity with the temperature β^{-1}. By transforming it back to the Minkoski metric we will obtain the solution which describes the universe for the entire period.

Assuming that the present universe is approximately described by the rolling vacuum solution given in section 2 we study the effect of time dependent masses when we compare the theory with the experiment. The expansion rate at the present time t_0 is given by,

$$H = \frac{\dot{a}}{a} = \frac{1}{a_0/2\beta\alpha + t_0}. \tag{35}$$

The ratio of the frequency $\nu(t)$ of the light emitted at t to the frequency $\nu_1(t_0)$ emitted now from the same phenomenon is given by

$$\frac{\nu(t)}{\nu_1(t_0)} = \frac{m(t)}{m(t_0)} = \sqrt{\frac{a(t_0)}{a(t)}} \tag{36}$$

$\nu(t)$ will be cosmologically red shifted to $\nu(t_0)$ by,

$$\frac{\nu(t)}{\nu(t_0)} = \frac{a(t_0)}{a(t)} \tag{37}$$

We get, therefore,

$$Z \equiv \frac{\lambda(\nu(t_0)) - \lambda(\nu_1(t_0))}{\lambda(\nu_1(t_0))} = \sqrt{\frac{a(t_0)}{a(t)}} - 1. \tag{38}$$

This leads to the following formula for the relation between the luminosity distance d_L and Z;

$$d_L = (1+Z)^{\frac{3}{2}} a(t_0) sin\left(\frac{2}{a(t_0)H(t_0)} log(1+z)\right), \tag{39}$$

for the case of the closed universe [13]. Here sin should be replaced by either 1 or sinh in the case of marginal or open universe respectively. In deriving equation (39) the effect of the time dependence of the mass has to be taken into account in relating the apparent luminosity with the brightness of the universe. The detailed comparison of equation (39) with the observation remains to be done. We can check that it corresponds roughly to the case of the deceleration parameter q_0 being equal to -1 in the standard model.

4 Calabi-Yau compactification

Supergravity theory can be regarded as the natural extension of the Brans-Dicke gravity theory. Before going into the detailed discussion of the 10-dimensional supergravity we make a brief preliminary study of the 4-dimensional supergravity. The full action has been obtained by Cremmer et al.[6]. They start with the action in the Jordan frame just as in the simple Brans-Dicke theory and make an appropriate conformal transformation to arrive at the action in the Einstein frame. The relevant part of the action which is needed to study the vacuum structure reads;

$$\begin{aligned}(-g)^{-\frac{1}{2}}\mathcal{L} &= -\frac{1}{2}R + e^{-G}\left(3 + G'_k G''^{-1k} G'^{\ell}\right) \\ &\quad - \frac{1}{2}\tilde{g}^2 \left(Ref^{-1}_{\alpha\beta}\right)\left(G'^i T_i^{\alpha j} z_j\right)\left(G'^k T_k^{\beta \ell} z_\ell\right) \\ &\quad + G''^i_j \partial_\mu z_i \partial^\mu z^{*j},\end{aligned} \tag{40}$$

where G is given by,

$$G(z,z^*) = J(z,z^*) - log\frac{1}{4} \mid g(z,z^*) \mid^2, \tag{41}$$

with g being a superpotential. Here we have

$$G'^i = \frac{\partial G}{\partial z_i}. \tag{42}$$

$G_i''^{-1j}$ stands for the inverse matrix to

$$G_i''^j = \frac{\partial^2 G}{\partial z^{*i} \partial z_j}. \tag{43}$$

$T_i^{\alpha j}$ is the representation matrix of the generators T^α of a certain Lee Algebra. $\tilde{g}$ is the coupling constant. Minimal theory is defined [6] to be the theory with $f_{\alpha\beta} = \delta_{\alpha\beta}$ and $G_j''^i = -\frac{1}{2}\delta_j^i$. Although the discussion given below does not need these assumptions, we restrict ourselves only to this case for simplicity except for the Brans-Dicke dilaton. The Brans-Dicke dilaton is one of the chiral multiplet (z_i, X_{Li}, h_i) which we denote by (ϕ, ξ_L, h_ϕ). The superpotential g has a special dependence on ϕ in order that the vacuum should exhibit the time dependence as has been discussed in section 2. We assume the following specific form for g;

$$g = e^{-(\phi+\phi^2)}/4, \tag{44}$$

where ϕ is the complex dilaton field,

$$\phi = \phi_R + i\phi_I. \tag{45}$$

With this specific form for g we can rewrite equation (40) in the following form;

$$\begin{aligned}(-g)^{-\frac{1}{2}}\mathcal{L} = & -\frac{1}{2}R + e^{-\frac{\phi_R}{2}+\phi_I^2-G}\left[3 - 2\left(\phi_I^2 + \frac{1}{16}\right) - 2G''^i G_i'\right] - \frac{1}{2}D^\alpha D^\alpha \\ & - \frac{1}{2}\partial_\mu\phi_R\partial^\mu\phi_R - \frac{1}{2}\partial_\mu\phi_I\partial^\mu\phi_I - \frac{1}{2}\partial^\mu z_i\partial^\mu z^{*i}\end{aligned} \tag{46}$$

Here G has the dependence on the z's except for ϕ. We obtain the vacuum equations from equation (46) assuming equation (10) as in the case of the Brans-Dicke theory. The following ansatz is made to solve these equations based on the experience in section 2;

$$a(t) = \alpha + \beta t, \tag{47}$$

and

$$\varphi_R = \gamma log(\delta + \epsilon t). \tag{48}$$

The vacuum equations then reduce to the equations among the constants;

$$\begin{aligned} \gamma &= 4, \\ 2\beta\gamma\epsilon + \frac{1}{2}\rho^{-1}e^{-G}\left(\frac{23}{8} - 2G'^{i}G'_{i}\right) &= 0, \\ \beta^2 + k + \rho^{-2}\left\{e^{-G}\left(\frac{23}{8} - 2G^{i}G'_{i}\right) + 8\epsilon^2\right\} &= 0, \end{aligned} \tag{49}$$

with $\rho = \alpha/\delta = \beta/\epsilon$, and

$$\begin{aligned} &\ddot{z}^{*i} + \frac{3\beta}{\alpha + \beta t}\dot{z}^{*i} - 2\frac{e^{-G}}{(\delta + \epsilon t)^2}\left\{G'^{i}\left(2G'^{j}G'_{j} - \frac{15}{8}\right) - 2G''^{ji}G'_{j}\right\} \\ &+ \frac{\partial\left(D^{\alpha}D^{\alpha}\right)}{\partial z_i} = 0 \end{aligned} \tag{50}$$

We also have,

$$\varphi_I = 0, \tag{51}$$

Equation (50) together with equation (28) determines z_i ($z_i = 0$ is a trivial solution) and other equations determine the values of β and ϵ. We note that not all z_i can be zero since otherwise either β or ϵ will become negative because of the second equation of (49),

$$\frac{23}{8} - 2G'^{i}G'_{i} < 0. \tag{52}$$

The rest of the discussion parallels the conventional one in many ways except that we can associate the origin of the mass scales (for example the scale of the hidden sector) with the time variation of the dilaton field. For example, the supersymmetry breaking scale is given by,

$$\mu^2 = M_p^2 e^{-\frac{G}{2}} G'_i(\delta + \epsilon t)^{-1}, \tag{53}$$

for certain i. We then have

$$M_{\frac{3}{2}} = \frac{\mu^2}{M_p} G'_i \tag{54}$$

where $M_{3/2}$ is the gravitino mass. This is key equation to justify the supersymmetry breaking in the hidden sector [14]. We obtain,

$$M_{\frac{3}{2}} = M e^{-\frac{G}{2}}(\delta + \epsilon t)^{-1}. \tag{55}$$

Suppose that $(\delta + \epsilon t)^{-1}$ is as small as $10^{-16}\mu$ becomes of the order of 10^{11} GeV and $M_{3/2}$ becomes 10^3 GeV. This point will become much clearer when we discuss the compactification of the 10-dimensional supergravity.

Equation (50) determines the time dependence of z_i's. We assume z_i's in the hidden sector has no time dependence. Then they satisfy

$$G^{ii}\left(2G'^{j}G'_{j} - \frac{15}{8}\right) - 2G''^{ji}G'_{j} = 0. \tag{56}$$

Here i stands for the hidden sector and j runs over all the z_j's but because of small $(\delta + \epsilon t)^{-1}$ only the hidden sector components are relevant. If we assume that the supersymmetry is broken only in the hidden sector we have

$$\ddot{z}^{*i} + \frac{3\dot{a}}{a}\dot{z}^{*i} + 4\frac{e^{-G}}{\rho^2 a^2}G''^{0i}G'_0 = 0, \tag{57}$$

for the other z_i's. Here a is the size of the rolling vacuum and the index zero stands for the hidden sector. The contribution to G is dominated by the hidden sector component. The last term of equation (57) vanishes when z_i does not couple to the hidden sector. Assume that G contains the following term;

$$G = \sum_i c_i z_0 z^{*i} z_i. \tag{58}$$

We then get

$$\ddot{z}^{*i} + \frac{3\dot{a}}{a}\dot{z}^{*i} + \frac{\xi_i}{a^2}z^{*i} = 0 \tag{59}$$

with

$$\xi_i = \frac{4c_i e^{-G}}{\rho^2}G'_0. \tag{60}$$

This can be easily shown to have the following solution;

$$z^i = (\alpha + \beta t)^{\nu}\bar{z}^i, \tag{61}$$

where $\bar{z}^i$ are constants and ν is given by

$$\nu = -1 \pm \sqrt{1 - 4\xi_i}. \tag{62}$$

We can obtain any mass scale by adjusting the value of ξ_i. It is not unnatural, for example, to have a violation of $SU(2) \times U(1)$ at the tree level in this scheme.

Although the 4-dimensional supergravity theory is in itself an interesting theory, it is too ambiguous to confront with the experimental observation. We need such ad hov assumptions as the minimality etc. to obtain a concrete result. We, therefore, turn to the discussion of the 10-dimensional (superstring) supergravity [7] which leads to a rather remarkable consequence.

The bosonic part of the 10-dimensional supergravity action with the super Yang-Mills field is,

$$(-g)^{-\frac{1}{2}}\mathcal{L} = -\frac{1}{2\kappa^2}R - \frac{3}{4}\phi^{-\frac{3}{2}}H^2_{MNP} - \frac{9}{16\kappa^2}(\partial_M\phi/\phi)^2 - \frac{1}{4}\phi^{-\frac{3}{4}}F^\alpha_{MN}F^{MN\alpha}, \quad (63)$$

where R is the 10-dimensional scalar curvature and,

$$H_{MNP} = \partial_{[M}F_{NP]} - \frac{\kappa}{\sqrt{2}}\omega_3, \quad (64)$$

with

$$\omega_3 = A^\alpha_{[M}F^\alpha_{NP]} - \frac{1}{3}gf_{\alpha\beta\gamma}A^\alpha_{[M}A^\beta_N A^\gamma_{P]} - \omega_{3L}, \quad (65)$$

with ω_{3L} being the gravitational Chern-Simon form. The symbol $[MNP]$ means the total anti-symmetrization as usual. Our task is to find the vacuum solution to the equations derived from action (63) in which the dilaton field ϕ is time-dependent. At first sight it seems impossible because of the explicit appearance of $\phi^{-\frac{3}{4}}$ multiplying the F^2 term which then will give the time dependence of the gauge couplings [9]. Our goal is to show that it does not happen in the compactified 4-dimensional theory. The compactified vacuum solution has so far been discussed under the assumption of supersymmetry. This led inevitably to the vanishing of the cosmological constant or the Minkovsky space-time. On the other hand the finite cosmological constant plays the fundamental role in our approach. We, therefore, must violate the exact supersymmetry at some energy scale depending on the magnitude of the cosmological constant. It will be shown later that the energy scale is determined to be of the order of 10^{11} GeV if we assume the grand unification scale is 10^{15} GeV. As was shown by Candelas et al. [11], there are 2 conditions to be satisfied if the supersymmetry is to be preserved in the process of compactification. One is the condition of Ricci flatness and Kähler property for the compactified space and the other is the Kähler-Yang-Mills equation for the gauge field. We can certainly give up all these conditions to break the supersymmetry. But we choose to give up only the Kähler-Yang-Mills

equation to make the discussion simpler. This means that we break the supersymmetry in the super-Yang-Mills sector rather than in the supergravity sector. We denote the internal Calabi-Yau components by a, b, c, $\cdots$ and the external components by $\mu, \nu, \rho, \cdots$. The dimension of the Calabi-Yau space is left open;

$$D_e + D_i = 10. \tag{66}$$

with D_e and D_i being the external and the internal space dimensions respectively. The internal space has the metric

$$f(x) g_{a\bar{b}}(z, \bar{z}), \tag{67}$$

with

$$g_{a\bar{b}}(z, \bar{z}) = \frac{\partial^2 \Phi}{\partial z^a \partial \bar{z}^{\bar{b}}}. \tag{68}$$

The square of the size of the Calabi-Yau space $f(x)$ can depend on the external coordinate x especially on time t. The scalar curvature R can then be calculated with the result;

$$\begin{aligned} R &= \frac{D_i}{2} f^{-1} D_\mu \left(g^{\mu\nu} \partial_\nu f\right) + \frac{D_i}{2} g^{\mu\nu} D_\mu \left(f^{-1} \partial_\nu f\right) \\ &\quad + \frac{5}{4} D_i g^{\mu\nu} f^{-2} \partial_\mu f \partial_\nu f + R^{(D_e)} \end{aligned} \tag{69}$$

where $R^{(D_e)}$ is the De-dimensional curvature. Ricci flatness of the metric $g_{a\bar{b}}$ has been used to derive equation (69). This leads to the following formula for the gravitational part of the action given in equation (63) (we take $2\kappa^2 = 1$);

$$\begin{aligned} S_G &= -\int \sqrt{-g^{(10)}} R^{(10)} d^{10}x \\ &= \int \sqrt{-g^{(D_e)}} d^{D_e}x S^{D_i} \left(-R^{(D_e)} + (2D_i^2 - 7D_i) \frac{g^{\mu\nu} \partial_\mu S \partial_\nu S}{S^2}\right), \end{aligned} \tag{70}$$

where the dimensionality has been explicitly denoted and $f = S^2$. Equation (70) shows that the size of the internal space $S(x)$ plays the role of Brans-Dicke dilaton in addition to the original ϕ field. We have also calculated the left hand side of equation (70) in the case of maximally symmetric internal

space just for comparison with the result;

$$\begin{aligned} S_G &= -\int \sqrt{-g^{(10)}} R^{(10)} d^{10}X \\ &= \int \sqrt{-g^{(D_e)}} d^{D_e}x S^{D_i} \Big\{ -R^{(D_e)} \\ &\quad + D_i(D_i-1)\left(\frac{g^{\mu\nu}\partial_\mu S \partial_\nu S}{S^2} + \frac{k}{S^2}\right)\Big\}, \end{aligned} \tag{71}$$

We see from (70) and (71) that the Calabi-Yau space is in a way much simpler than the maximally symmetric space in the sense that the former has no potential term in the action. We also found that the computation which leads to equation (70) is less tedious than that leading to equation (67). We need another tedious step to perform the conformal transformation in the space-time of dimension D_e. In general we get,

$$R = \Omega^2 \tilde{R} - 2(D_e - 1)\Omega\tilde{\Box}\Omega + D_e(D_e-1)\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega, \tag{72}$$

where the transformation is defined by,

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}. \tag{73}$$

The Einstein frame corresponds to

$$\Omega^2 = S^A, \tag{74}$$

with

$$A = \frac{2D_i}{D_e - 2}. \tag{75}$$

Here the $\tilde{g}_{\mu\nu}$ is the metric in the Einstein frame. Throughout this section the quantity A in the Einstein frame is written as $\tilde{A}$ rather than that in the Jordan frame. The gravitational part of the action (70) becomes,

$$S_G = \int \sqrt{-\tilde{g}}\left[-\tilde{R} - K\frac{\tilde{g}^{\mu\nu}\partial_\mu S \partial_\nu S}{S^2}\right], \tag{76}$$

where K is given by;

$$K = \frac{(D_e-2)(D_e-1)}{4}A^2 - D_i(2D_i - 7). \tag{77}$$

The rest of the action (63) can also be computed with the result;

$$S_m = \int \sqrt{-\tilde{g}} \left[-\frac{9}{8} \frac{\tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}{\phi^2} + \mathcal{L}_F - \frac{1}{4} \phi^{-\frac{3}{4}} S^A \tilde{g}^{\mu\rho} \tilde{g}^{\nu\kappa} F^\alpha_{\mu\nu} F^\alpha_{\rho\kappa} \right], \tag{78}$$

where $\mathcal{L}_F$ is given by,

$$\begin{aligned} \mathcal{L}_F &= \int \Big[-\frac{1}{2} \phi^{-\frac{3}{4}} S^{-A+4} g_{a\bar{a}} g_{b\bar{b}} \left(F^{\alpha ab} F^{\alpha \bar{a}\bar{b}} + F^{\alpha a\bar{a}} F^{\alpha b\bar{b}} \right) \\ &\quad + \phi^{-\frac{3}{4}} S^{-2A+2} g_{a\bar{a}} \partial_0 A^{\alpha a} \partial_0 A^{\alpha \bar{a}} \Big]. \end{aligned} \tag{79}$$

Here the gauge field with the internal indices a or $\bar{a}$ are allowed to depend on time t (∂_0 is the time derivative). To obtain equation (79) we neglected the H_{MNP} components. This can be justified later within the framework of the expansion scheme in terms of S^{-1} [15]. The determinant of the internal metric has been set equal to 1 owing to the Ricci flatness condition. Equation (79) is the result of the extra R^2 term added to equation (63) coming from $(\alpha')^2$ correction from the string theory. We also made use of the formula [15],

$$\int F^\alpha_{ij} F^{\alpha ij} = \int \left[2 g^{a\bar{a}} g^{b\bar{b}} F^\alpha_{ab} F^\alpha_{\bar{a}\bar{b}} + 2 g^{a\bar{a}} g^{b\bar{b}} F_{a\bar{a}} F_{b\bar{b}} - 2 k tr F \wedge F \right] \tag{80}$$

and the last term has been cancelled with its counter part coming from R^2 term. As a matter of fact this is the only place that our theory deviates from the minimal 10-dimensional supergravity. $\mathcal{L}_F$ provides the equation of motion for F which replaces the Kähler-Yang-Mills of Condelas et al [11]. Not all the Calabi-Yau metric are the automatic solution to the equation of motion. We get the following equations for F and the Kähler potential Φ;

$$\frac{\partial^2}{\partial z^a \partial \bar{z}^{\bar{a}}} \left[\partial_0 A^{\alpha a} \partial_0 A^{\alpha \bar{a}} - S^{A+2} \frac{\partial^2 \Phi}{\partial z^b \partial \bar{z}^{\bar{b}}} \left(F^{\alpha ab} F^{\alpha \bar{a}\bar{b}} + F^{\alpha a\bar{a}} F^{\alpha b\bar{b}} \right) \right] = 0, \tag{81}$$

and

$$\begin{aligned} & g_{a\bar{b}} \partial_0 \left(\sqrt{-\tilde{g}} \phi^{-\frac{3}{4}} S^{-2A+2} \partial_0 A^{\alpha \bar{b}} \right) + \sqrt{-\tilde{g}} \phi^{-\frac{3}{4}} A^{-A+4} \Big\{ \partial^b \left(g_{a\bar{a}} g_{b\bar{b}} F^{\alpha \bar{a}\bar{b}} \right) \\ & + g f^{\alpha\beta\gamma} g_{a\bar{a}} g_{b\bar{b}} A^{\gamma b} F^{\beta \bar{a}\bar{b}} + \partial^{\bar{a}} \left(g_{a\bar{a}} g_{b\bar{b}} F^{\alpha b\bar{b}} \right) + g f^{\beta\alpha\gamma} g_{a\bar{a}} g_{b\bar{b}} A^{\gamma \bar{a}} F^{\beta b\bar{b}} \Big\} \\ & = 0. \end{aligned} \tag{82}$$

We first discuss the case when $F^{\alpha a\bar{b}}$, $F^{\alpha ab}$ and $F^{\alpha\bar{a}\bar{b}}$ are time independent and the second term of equation (79) can be neglected. We will discuss more general case later. Combining equation (76), (78) and (79) we obtain;

$$\begin{aligned} S &= S_G + S_m \\ &= \int \sqrt{-\tilde{g}}\Big(-\tilde{R} - K\frac{\tilde{g}^{\mu\nu}\partial_\mu S\partial_\nu S}{S^2} - \frac{9}{8}\frac{\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{\phi^2} - \lambda\phi^{-\frac{3}{4}}S^{-A+4} \\ &\quad - \frac{1}{4}\phi^{-\frac{3}{4}}S^A\tilde{g}^{\mu\rho}\tilde{g}^{\nu\kappa}F^\alpha_{\mu\nu}F^\alpha_{\rho\kappa}\Big)d^{D_e}X, \end{aligned} \tag{83}$$

with

$$\lambda = \frac{1}{2}\int g_{a\bar{a}}g_{b\bar{b}}\left(F^{\alpha ab}F^{\alpha\bar{a}\bar{b}} + F^{\alpha a\bar{a}}F^{\alpha b\bar{b}}\right) \tag{84}$$

The last term in equation (83) has been written explicitly to clarify the issue although $F^\alpha_{\mu\nu}$ will have no vacuum expectation value. We see that the factor $\phi^{-\frac{3}{4}}S^A$ multiplying the F^2 term rather than mere $\phi^{-\frac{3}{4}}$ in the 10-dimensional case. Our problem is to see whether this factor can be time independent when the vacuum is rolling down the hill. The action given in equation (83) can be rewritten in the form:

$$\int \sqrt{-\tilde{g}}\left[-\tilde{R} - (\partial_\mu\varphi_1\partial^\mu\varphi_1 + \partial_\mu\varphi_2\partial^\mu\varphi_2) - \lambda e^{-N\varphi_1}\right], \tag{85}$$

with

$$\varphi_1 = \left(\frac{1}{\sqrt{2}}\varphi + \frac{A-4}{\sqrt{\kappa}}\sigma\right)/N, \tag{86}$$

and

$$\varphi_2 = \left(\frac{A-4}{\sqrt{K}}\varphi - \frac{1}{\sqrt{2}}\sigma\right)/N, \tag{87}$$

where φ and σ are defined through;

$$S = e^{\frac{\sigma}{\sqrt{K}}}, \tag{88}$$

and

$$\phi = e^{\frac{2\sqrt{2}}{3}\varphi}. \tag{89}$$

Here

$$N = \sqrt{\frac{1}{2} + \frac{(A-4)^2}{K}} \tag{90}$$

K being given in equation (77). The condition of the constancy of the gauge coupling is reduced to,

$$\varphi_2 = \varphi_0, \tag{91}$$

where φ_0 is defined through,

$$\phi^{-\frac{3}{4}} S^A = e^{\varphi_0} = exp\left(-\frac{1}{\sqrt{2}}\varphi + \frac{A}{\sqrt{K}}\sigma\right). \tag{92}$$

The equation (91) leads to

$$\frac{A(A-4)}{K} = \frac{1}{2}, \tag{93}$$

Combining this equation with equations (75) and (77) we obtain,

$$D_e = 4. \tag{94}$$

It is remarkable that we have the time independent gauge coupling only in the 4-dimensional space-time. It is not clear at present how this result is related to the argument given in section 2 where we also found the constant gauge coupling only in 4 dimensional space-time but without compactification process. We can explicitly solve the vacuum equation with the ansatz given in equation (10) and,

$$a = \alpha + \beta t, \varphi_1 = \gamma log(\delta + \epsilon t). \tag{95}$$

The result is;

$$\varphi_2 = const \equiv \varphi_{20}, \tag{96}$$

$$\gamma^2 = 6, \tag{97}$$

$$\frac{\alpha}{\delta} = \sqrt{\frac{6k}{\lambda}}, \tag{98}$$

$$\epsilon^2 = \frac{1}{12}\lambda, \tag{99}$$

and

$$\beta = \epsilon\sqrt{6k/\lambda} = \sqrt{\frac{k}{2}}. \tag{100}$$

We also have

$$S = \left[\sqrt{\frac{\lambda}{6k}}(\alpha + \beta t)\right]^{\frac{1}{4}} e^{-\frac{1}{4\sqrt{2}}\varphi_{20}}. \tag{101}$$

This shows that the internal space is expanding as,

$$S \propto a^{\frac{1}{4}}. \tag{102}$$

We now investigate the more general case in which an ad hoc assumption that $F^{\alpha ab}$ etc. are time independent is not made. We will keep, nevertheless, the favorable property of the solution we just found. We first note that the fermion part of the action which belongs to the same multiplet as the gauge bosons has the following form;

$$\begin{aligned} \mathcal{L}_\chi &= \int \sqrt{-\tilde{g}}\Big[-\frac{S^{-\frac{A}{2}}}{2}\bar{\chi}^\alpha \chi^\alpha \\ &\quad - \frac{1}{2}S^{-A-1}\bar{\chi}^\alpha e^{a\bar{a}}\Gamma_{\bar{a}}\left(\frac{1}{2}\omega_a^{\tilde{c}\tilde{b}}\Sigma_{\tilde{c}\tilde{b}}\delta^{\alpha\beta} + gS^2 f^{\alpha\beta\gamma} g_{aa'} A^{a'}\right)\chi^\beta\Big] \end{aligned} \tag{103}$$

where the indices a, a' etc. cover both a and $\bar{a}$ and $\tilde{a}$, $\tilde{b}$ etc. correspond to the tangential space to it. This equation shows that $A^{\alpha a}$ contribute,

$$m_\chi = S^{-2} A^{\alpha a} \tag{104}$$

to the fermion mass. Assuming that the time dependence of $A^{\alpha a}$ comes only through its dependence on S we expand $A^{\alpha a}$ in the following way [15];

$$A^{\alpha a} = S^2 A^{\alpha a}_{(2)} + S A^{\alpha a}_{(1)} + A^{\alpha a}_{(0)} + S^{-1} A^{\alpha a}_{-1} + \cdots. \tag{105}$$

The highest mass term comes from $m_\chi = A^{\alpha a}_2$ which is of the order of Planck mass. This is why we do not have much higher order than S^2 in equation (105). We know from equation (101) that there is no way to determine the magnitude of S although it is slowly expanding with time. What does the experimental observation suggest on this matter? If we assume the existence of mass scale at the level of 10^{15} GeV, we can associate this mass scale with the second term in equation (105). This means that we have,

$$S^{-1} \cong 10^{15} GeV. \tag{106}$$

Equation (105) then gives the mass hierarchy separated by 10^4 GeV;

$$m_{(n)} = A^{\alpha a}_{(n)}(m_p S)^{n-2} m_p \quad (n = 2, 1, 0, -1, \cdots). \tag{107}$$

We have the expansion formula for $F^{\alpha ab}$ etc. corresponding to equation (51);

$$F^{\alpha ab} = S^4 F^{\alpha ab}_{(4)} + S^3 F^{\alpha ab}_{(3)} + S^2 F^{\alpha ab}_{(2)} + S F^{\alpha ab}_{(1)} + \cdots, \tag{108}$$

where $F^{\alpha ab}_{(m)}$ is given by,

$$F^{\alpha ab}_{(m)} = g^{aa'} \partial_{a'} A^{\alpha b}_{(m+2)} - g^{bb'} \partial_{b'} A^{\alpha a}_{(m+2)} + g f^{\alpha\beta\gamma} \sum_{n \leq 2} A^{\beta a}_{m-n} A^{\gamma b}_{n}. \tag{109}$$

Corresponding formula for $F^{\alpha \bar{a}\bar{b}}$ and $F^{\alpha a\bar{b}}$ is understood. The solution obtained assuming that $F^{\alpha ab}$, $F^{\alpha \bar{a}\bar{b}}$ and $F^{\alpha a\bar{b}}$ are constant in time remains so if $F_{(4)}$, $F_{(3)}$, $F_{(2)}$ and $F_{(1)}$ satisfy the Kähler-Yang-Mill's equation as is seen from equation (79). We will see later that this condition is to be replaced by the condition of vanishing Chern-Simon form up to S^1 order. Note that the second term in equation (79) is S^{-10} times smaller than the first term in this case. These conditions guarantee that the supersymmetry is unbroken down to the energy 10^{11} GeV which is the typical energy obtained in the hidden sector hypothesis [14]. We obtain this result from the condition of time independence of the gauge coupling. There is the following problem to this value of S. We know that there is the observed cosmic red shift in the various atomic lines. The mass scale of the electron will correspond either to $m_{-2} = A^{\alpha a}_{(-2)}(m_p s)^{-4} m_p = A^{\alpha a}_{(-2)} \times 1TeV$ or more likely to a much lower scale. This means that the red shift is completely cancelled or it will turn into the blue shift. This will be remedied if the expansion rate of our universe was much larger than in the past when the temperature was much higher. The quantitative discussion on this point will be given elsewhere. An alternative choice is to make S as large as $10^{-11} GeV^{-1}$. Then we have,

$$m_0 = A^{\alpha a}_0 (m_p S)^{-2} m_p = A^{\alpha a}_{(0)} \times 1TeV. \tag{110}$$

This is the typical low energy scale where the supersymmetry is broken and the red shift is half way compensated as in section 2. This is because $S^{-2} \cong ((a)^{\frac{1}{4}})^{-2}$ as is shown in equation (102). The problem in this case is to

accommodate the grand unification scale of 10^{11} GeV with the other experiments such as the lack of proton decay or the estimation of the Weinberg angle based on the grand unification. This solution, therefore, is not our choice.

Since the non-vanishing character of $dH = F^2 - R^2$ is crucial to our discussion, the contribution of the anti-symmetric tensor H needs more clarification. The Kähler-Yang Mill's equations for $F_{(4)}, F_{(3)}, F_{(2)}$ and $F_{(1)}$ guarantee that dH has no positive exponent in S. Suppose that this property is also valid for H itself i.e. the Chern-Simon term has the maximum exponent zero as well as $\partial_M F_{[NP]}$ term in expression (64) for H (gravitational part is trivial), then we can easily prove that H can be neglected in equation (63); The contribution of H to the cosmological constant reads,

$$\mathcal{L}_H = \int \sqrt{-\tilde{g}} \left(-\frac{3}{4} S^{-A} \phi^{-\frac{3}{2}} S^6 g_{aa'} g_{bb'} g_{cc'} H^{abc} H^{a'b'c'} \right), \tag{111}$$

which is negligibly small compared to L_F when H is constant and $\phi^{-\frac{3}{2}} \sim S^{-12}$. It should also be pointed out that the equation of the vanishing Chern-Simon form up to the S^1 order completely replaces the Kähler-Yangs-Mills equation up to the S^1 order. In summary we get

$$\omega_3 = A^{\alpha[a} F^{\alpha bc]} - \frac{1}{3} g f^{\alpha\beta\gamma} A^{\alpha[a} A^{\beta b} A^{\gamma c]} = 0(S^\circ), \tag{112}$$

and

$$\frac{\partial^2}{\partial z^a \partial \bar{z}^{\bar{a}}} \left[\frac{\partial^2 \Phi}{\partial z^b \partial \bar{z}^{\bar{b}}} \sum_m \left(F^{\alpha ab}_{(m)} F^{\alpha \bar{a}\bar{b}}_{(-m)} + F^{\alpha a\bar{a}}_{(m)} F^{\alpha b\bar{b}}_{(-m)} \right) \right] = 0, \tag{113}$$

and

$$\partial^b \left(g_{a\bar{a}} g_{b\bar{b}} F^{\alpha\bar{a}\bar{b}}_{(0)} \right) + g f^{\alpha\beta\gamma} g_{a\bar{a}} g_{b\bar{b}} \sum_m A^{\gamma b}_m F^{\bar{a}\bar{b}}_{(-m)} + \partial^{\bar{a}} \left(g_{a\bar{a}} g_{b\bar{b}} F^{\alpha b\bar{b}}_{(0)} \right)$$
$$+ g f^{\alpha\beta\gamma} g_{a\bar{a}} g_{b\bar{b}} \sum_m A^{\gamma\bar{a}}_{(m)} F^{\beta b\bar{b}}_{(-m)} = 0. \tag{114}$$

Equations (113) and (114) are obtained by expanding equations (81) and (82) respectively. $[a\alpha bc]$ or $[a\beta b\gamma c]$ in equation (112) stands for the anti-symmetrization of a, b and c. The electromagnetic charge is known be time independent to the accuracy of 10^{-12} [10]. This implies that $F_{(-1)}$ and $F_{(-2)}$

should not contribute to the cosmological term (equation (84)). The simplest way is to assume Kähler-Yang-Mill's equation for these F's.

In the case of heterotic string theory [16] where the symmetry is $E_8 \times E_8$ we have the following scenario for the symmetry breaking.

$$A^{(\alpha,\alpha')} = S^2 A_{(2)}^{(\alpha,\alpha')} + S A_{(1)}^{(\alpha,\alpha')} + \cdots, \tag{115}$$

where α and α' denote the observed and hidden E_8 respectively. Correspondingly we have

$$F^{(\alpha,\alpha')} = S^4 F_{(4)}^{(\alpha,\alpha')} + S^3 F_{(3)}^{(\alpha,\alpha')} + \cdots, \tag{116}$$

where F is given by equation (109). In the expansion (115) $A_{(2)}^{(\alpha,\alpha')}$ is singlet in the hidden E_8 and only the observed E_8 will be broken down to much smaller group such as E_6. $A_{(1)}^{(\alpha,\alpha')}$ is also a singlet in the hidden E_8 and serves to break the grand unified group to $SU(3) \times SU(2) \times U(1)$. $A_{(0)}^{(\alpha,\alpha')}$ and $A_{(-1)}^{(\alpha,\alpha')}$ are singlet in the observed E_8 and the hidden E_8 will be completely broken by these two terms. $A_{(-2)}^{(\alpha,\alpha')}$ correspond to the $SU(2) \times U(1)$ breaking. These quantities must satisfy equation (112), (113) and (114) together with the Kähler-Yang-Mill's equation for $F_{(-1)}$ and $F_{(-2)}$.

5 Conclusion

We have discussed the supergravity theory regarding it as the generalization of the Brans-Dicke theory. Brans-Dicke theory has the remarkable property that the cosmological constant term is multiplied by the exponential of the dilaton field σ and will be damped down when $\sigma \to -\infty$ as the time passes. It is precisely what happens when the equation of motion is solved. The theory is defined in the so called Einstein frame where the gravitational constant is independent of time. In return the particle masses depend on time. This provides us the crucial test for the discussion given in this work. In the simplest version of Brans-Dicke theory the dependence is such that it partly cancels the cosmological red shift. The 4-dimensional gravity theory gives us a solution in which the time dependence of the masses cancels the red shift completely if the expansion rate of the universe is the same as that of the vacuum. It is, therefore, very important that the universe expanded faster than the vacuum in the past. The 10-dimensional supergravity

is more complicated. If we assume the conventional grand unification scale of 10^{15} GeV, we are lead to the same situation as in the case of 4-dimensional supergravity.

The solution we have obtained describes the vacuum which is 'rolling down the hill'. The crucial observation of this work is that we have the constancy of the gauge coupling in and perhaps only in the space-time dimension of 4. It is rather trivial in the simplest Brans-Dicke theory. But it seems rather remarkable in the compactified theory of superstring. One might suspect that these two situations are related. In any way the fact that the observed space-time dimension of 4 seems to be closely related to the constancy of the gauge coupling constant. It is assumed in this work that the supersymmetry is broken in the gauge boson sector by giving up the Kähler-Yang-Mill's equation. Supergravity sector remains to keep the supersymmetry thus leading to the Calabi-Yau compactification. We find that the size of the Calabi-Yau space is slowly expanding and gives the origin of the grand unification scale m_G. The supersymmetry is then violated at the scale of m_G^2/m_p which turns out to be 10^{11} GeV for $m_G = 10^{15}$ GeV. We have a new scale of 10^7 GeV in the hierarchy of the masses which presumably occurs in the hidden sector. We have the equations to determine both the Calabi-Yau metric and the gauge field in the vacuum (equation (81) and equation (82) or equation (113) and equation (114)). It is not obvious at this time how restrictive these equations are. The cosmology takes a new form in the Brans-Dicke theory with the cosmological constant. Large cosmological constant dominates the matter density throughout almost the entire period. The damping mechanism by the Brans-Dicke dilaton seems to persist at the finite temperature. The exponential expansion of the universe, therefore, seems to be hard to realize in this approach although the expansion rate can be much larger than that of the rolling vacuum itself.

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