KAIST-THP-90-702

KAIST-THP-90/702

Quantum correction of three-point couplings in Affine Toda Field Theories

H.S. Cho , J.D. Kim* , I.G. Koh**
Physics Department
Korea Advanced Institute of Science and Technology
P.O. Box 150, Cheongryang, Seoul, Korea.

ABSTRACT

The one loop quantum corrections of the three-point couplings in affine A_2, A_3, A_5 and D_4 Toda theories are calculated. We find that the ratios of 1 loop corrections to classical three-point couplings are not the same in different couplings in general affine Toda theory, while the ratios of 1 loop corrections to the classical masses are known to be the same.

* bitnet address : jdkim%sorak.kaist.ac.kr@relay.cs.net
* * bitnet address : koh%sorak.kaist.ac.kr@relay.cs.net

ROUTE TO
NAME
LOCATION

I. Introduction

The 2-dimensional Toda field theories based on semi-simple Lie algebra g have been studied extensively as examples of some rational conformal field theories[1]. The interesting feature of this description is that the affine generalizations of these theories represent certain integrable deformations of the conformal field theories.

It has been shown that certain conformal theories preserve their integrability away from critical point [2,3,4,5,6]. For example, when we perturbe the Ising model with $\phi_{1,2}$ operator, we still get the infinite number of conserved quantities. The integrals of motion appear when the spin values are the exponents of E_8 modulo its dual Coxeter number. Also if we perturbe the system with $\phi_{1,3}$ operator, the integrals of motion appear when the spin values are the exponents of A_1 modulo its dual Coxeter number. The Ising model away from criticality can be described by affine A_1 Toda field theory and affine E_8 Toda theory. Affine A_1 theory describes a thermal perturbation, whereas affine E_8 Toda field theory represents the Ising model in a non-zero magnetic field [2].

From the view point of conformal technology, we can construct minimal S-matrix with bootstrap, unitarity and crossing conditions, which then gives the mass spectrum determined by the conserved spin values. It happens to be identical to the eigenvector of incidence matrix correponding to the smallest eigenvalue.

The classical masses and three-point couplings for affine Toda field theories based on simply-laced Lie algebras are calculated [7]. The computation of classical masses and three-point couplings based on the non-simply-laced algebras are carried out with the aid of automorphism of Dynkin diagram [7]. The mass spectrum is identical to that of principal chiral model [8].

On the view point of bootstrap idea, the ratios of each particles' masses are utmost important. This aspect has been investigated at the one loop order by the H.W. Braden, et al.[7]. Because of the 2 dimensionality, the divergence appears only in tadpole diagrams. But these terms turn out to be completely decoupled from the finite part[9]. So the theories are finite, and these finite renormalizations give mass shift in simply-laced algebra cases. The one loop corrections of mass have a neat universal formula.

$$\frac{\delta m_i^2}{m_i^2} = \frac{\beta^2}{4h}\cot(\frac{\pi}{h}), \quad i = 1, \cdots, r. \tag{1}$$

,where β is scaling factor, h is the Coxeter number of the algebra and r is the rank of the

algebra [7, 8].

Around mid-seventies, much work on the perturbative analysis of sine-Gordon model (affine A_1 Toda theory with imaginary coupling β) were done[10]. In the meantime, appearance of the conformal algebra has stimulated a more extensive study of general affine Toda theories. General affine Toda field theories concern us here especially because of their multi-particle nature in the Lagrangian level(not soliton).

In this paper we would like to examine the quantum corrections of three-point couplings to one loop order. Especially we can expect that there might be some nice structure even in the three-point couplings because of the classical conservation of the higher spin currents[11] and group theoretic character. The aim of this paper is to show that the relations between classical three-point couplings and their one loop corrections in affine Toda field theories. It turns out that the neat feature analogous to Eq. (1) does not appear generally in the ratio of one loop coupling constant corrections to their classical values.

In section II, we will briefly review the affine Toda Lagrangian to be self-contained. We stress the usefulness of the complex root basis. Section III contains the results for the affine A_2 and A_3 Toda theories. These have a rather simple couplings due to the symmetry of Dynkin diagram. Since affine A_4 Toda theory has a complicated Lagrangian due to $e^{\frac{2\pi i}{5}}$, we skip to the affine A_5 case in section IV with the result of the affine D_4 Toda theory. Finally in section V, we conclude this paper.

II. Affine Toda Field Theories

We introduce the affine Toda field theories in arbitrary real root basis and rewrite them in the complex basis. The affine Toda field theories based on Lie algebra g with rank r represent the exponential interaction of the scalar fields ϕ^a corresponding to the components of simple roots α_i $(i = 1, \cdots, r)$ of g. The Lagrangian is given by

$$\mathcal{L}(\phi^a) = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{m^2}{\beta^2}\sum_{i=0}^{r} n_i e^{\beta\alpha_i\cdot\Phi} \tag{2}$$

,where

$$\alpha_0 = -\sum_{i=1}^{r} n_i\alpha_i, \quad \text{and} \quad n_0 = 1, \quad a = 1, \cdots, r.$$

In A_n algebra case, which we shall mostly consider from now on, $n_i = 1$. The field ϕ^a is a-th component of the field Φ.

The expansion of exponential potential term gives classical masses, three-point couplings, four-point couplings, and so on. If we expand the interaction term to quadratic β order, we find

$$\begin{aligned} V(\phi^a) &= \frac{m^2}{\beta^2}\sum_{i=0}^{r} n_i + \frac{m^2}{2}\sum_{i=0}^{r} n_i\alpha_i^a\alpha_i^b\phi^a\phi^b \\ &+ \frac{m^2\beta}{6}\sum_{i=0}^{r} n_i\alpha_i^a\alpha_i^b\alpha_i^c\phi^a\phi^b\phi^c + \frac{m^2\beta^2}{24}\sum_{i=0}^{r} n_i\alpha_i^a\alpha_i^b\alpha_i^c\alpha_i^d\phi^a\phi^b\phi^c\phi^d + \cdots \end{aligned} \tag{3}$$

The classical mass ratios are given by the eigenvalues of the square of mass matrix

$$(M^2)^{ab} = m^2\sum_{i=0}^{r} n_i\alpha_i^a\alpha_i^b. \tag{4}$$

The three-point couplings defined by

$$c^{abc} = m^2\beta\sum_{i=0}^{r} n_i\alpha_i^a\alpha_i^b\alpha_i^c \tag{5}$$

is symmetric in the three fields ϕ^a, ϕ^b, ϕ^c. And the four-point couplings are defined by

$$c^{abcd} = m^2\beta^2\sum_{i=0}^{r} n_i\alpha_i^a\alpha_i^b\alpha_i^c\alpha_i^d. \tag{6}$$

This can be continued to higher multiple-point couplings. But this real root basis needs to be diagonalized. Happily there exists a basis in which the square of mass matrix is automatically diagonal(see Eq. 15). A complex representation of the simple roots of A_n with affine root α_0 should satisfy the following two relations.

$$\alpha_i^* \cdot \alpha_j = \begin{cases} 0 & i \neq j, j \pm 1 \\ -1 & i = j \pm 1 \\ 2 & i = j \end{cases} \tag{7}$$

and

$$\sum_{i=0}^{n} \alpha_i = 0. \tag{8}$$

Let $\omega = e^{\frac{2\pi i}{n+1}}$ so that $\omega^{n+1} = 1$, and consider the vectors γ_i.

$$\gamma_i = (\omega^i,\ \omega^{2i}, \cdots,\ \omega^{ni}), \quad i = 0, 1, \cdots, n \tag{9}$$

Then their inner products are easily obtained.

$$\gamma_i^* \cdot \gamma_j = (n+1)\delta_{ij} - 1 \tag{10}$$

,where we have used the property $\sum_{i=0}^{n}\omega^i = 0$. If we normalize α_i such that

$$\alpha_i = \frac{1}{\sqrt{n+1}}(\gamma_{i+1} - \gamma_i)^*, \tag{11}$$

we now obtain the desired complex representation for A_n root space.

Because the Lagrangian should be real, we choose the complex basis for the scalar fields to satisfy the following condition.

$$(\phi^a)^* = \phi^{n+1-a}. \tag{12}$$

Then they give the nice property.

$$(\alpha_i^* \cdot \Phi)^* = \alpha_i^* \cdot \Phi. \tag{13}$$

The above property implies that the potential term in the Lagrangian is real when we take the Lagrangian to be

$$\mathcal{L}(\Phi) = \frac{1}{2}\partial\Phi \cdot \partial\Phi^* - \frac{m^2}{\beta^2}\sum_{i=0}^{n} e^{\beta\alpha_i^*\cdot\Phi}. \tag{14}$$

The square of mass matrix in this complex basis is

$$(M^2)^{ab} = \frac{m^2}{n+1}\sum_{i=0}^{n}(\gamma_{i+1} - \gamma_i)^a(\gamma_{i+1} - \gamma_i)^b = \begin{cases} 0 & a+b \neq n+1 \\ 4m^2\sin^2\frac{a\pi}{n+1} & a+b = n+1. \end{cases} \tag{15}$$

The scalar field ϕ^a and its conjugate field ϕ^{*a} have equal mass in this basis due to the property of sine function. The fusion rule of three-point couplings of mass eigenstates is the following.

$$\sum_{i=0}^{n}\alpha_i^a\alpha_i^b\alpha_i^c = \begin{cases} 0 & a+b+c \neq 0 \mod(n+1) \\ \frac{1}{\sqrt{n+1}}(\omega^a - 1)(\omega^b - 1)(\omega^c - 1) & a+b+c = 0 \mod(n+1). \end{cases} \tag{16}$$

And the fusion rule of four-point couplings of mass eigenstates is

$$\sum_{i=0}^{n}\alpha_i^a\alpha_i^b\alpha_i^c\alpha_i^d = \begin{cases} 0 & a+b+c+d \neq 0 \mod(n+1) \\ \frac{1}{n+1}(\omega^a - 1)(\omega^b - 1)(\omega^c - 1)(\omega^d - 1) & a+b+c+d = 0 \mod(n+1). \end{cases} \tag{17}$$

III. Affine A_2 and A_3 Toda Theory

In affine A_1 case, there is only one field corresponding to the single simple root. In affine A_2 case, there are two fields corresponding to the components of simple roots in the complex basis, one scalar field ϕ and its conjugate field ϕ^*, and one three-point coupling $(\phi\phi\phi)$ and its conjugate coupling $(\phi^*\phi^*\phi^*)$ exist.

Expansion of the potential term gives

$$V(\phi) = \frac{m^2}{\beta^2}3 + 3m^2\phi\phi^* + \beta m^2(\frac{i}{2}\phi\phi\phi - \frac{i}{2}\phi^*\phi^*\phi^*) + \frac{3}{4}\beta^2 m^2\phi\phi\phi^*\phi^* + \cdots \tag{18}$$

The three-point coupling at zero-th order is of β linear. The relation between vertex $I^{\alpha\beta\gamma}$ and three-point coupling $c^{\alpha\beta\gamma}$ is the following;

$$I^{\alpha\beta\gamma} = -ic^{\alpha\beta\gamma}, \quad I_1^{\alpha\beta\gamma} = -ic_1^{\alpha\beta\gamma} \tag{19}$$

,where subscript 1 denotes one loop correction. So the ratio of quantum corrections to classical coupling is

$$\frac{I_1^{\alpha\beta\gamma}}{I^{\alpha\beta\gamma}} = \frac{c_1^{\alpha\beta\gamma}}{c^{\alpha\beta\gamma}}. \tag{20}$$

We will bother to discuss I instead of c from now on. In general there are two types of diagrams which contribute to one loop correction(Figure 1). We shall call them type I and type II respectively. Type I represents the interaction of three three-point couplings, whereas type II does that of one three-point coupling and one four-point coupling. In affine A_2 case, the fusion rule (16),(17) shows only type II contributes in this complex basis (Table I). The notation in the tables refers to the labelings in Figure 1. If we change this basis to real one, type I is allowed but their sum is zero.

We used the ordinary Feynman rules for the scalar field. Value of each diagrams depends on external momenta. Here we set $p_i^2 = m_i^2$(on shell condition). Later in D_4 case, we will also consider the $p_i^2 = 0$ case. Calculations are rather straight forward and the results are the followings.

$$I^{\phi\phi\phi} = 3m^2\beta, \quad I^{\phi^*\phi^*\phi^*} = -3m^2\beta. \tag{21}$$

$$I_1^{\phi\phi\phi} = -\frac{\sqrt{3}}{4}m^2\beta^3, \quad I_1^{\phi^*\phi^*\phi^*} = \frac{\sqrt{3}}{4}m^2\beta^3. \tag{22}$$

$$\frac{I_1^{\phi\phi\phi}}{I^{\phi\phi\phi}} = \frac{I_1^{\phi^*\phi^*\phi^*}}{I^{\phi^*\phi^*\phi^*}} = -\frac{\sqrt{3}}{12}\beta^2. \tag{23}$$

The coupling and its conjugate coupling have the same magnitude with different sign and their one loop corrections also have the same magnitude with different sign, so the ratios of quantum corrections to classical couplings are equal.

Continuing to A_3 case, we get the following by expanding potential term.

$$\begin{aligned} V(\phi) &= \frac{m^2}{\beta^2}4 + m^2(2\phi_1\phi_1^* + 2\phi_2^2) + m^2\beta(i\phi_1^2\phi_2 - i\phi_1^{*2}\phi_2) \\ &+ m^2\beta^2(-\tfrac{1}{24}\phi_1^4 + \tfrac{1}{4}\phi_1^2\phi_1^{*2} + \phi_1\phi_1^*\phi_2^{*2} + \tfrac{1}{6}\phi_2^4 - \tfrac{1}{24}\phi_1^{*4}) + \cdots \end{aligned} \tag{24}$$

There are one heavy field ϕ_2(self conjugate), one light field ϕ_1 and its conjugate field ϕ_1^*. Between them there are only one three-point coupling $\phi_1^2\phi_2$ and its conjugate $\phi_1^{*2}\phi_2$.

$$I^{\phi_1^2\phi_2} = 2m^2\beta, \quad I^{\phi_1^{*2}\phi_2} = -2m^2\beta. \tag{25}$$

In this case both diagrams of the two types contribute to one loop correction. The values of each allowed diagrams is given in the table II. Their total sum is shown below.

$$I_1^{\phi_1^2\phi_2} = m^2\beta^3(\frac{1}{8\pi} - \frac{5}{16}), \quad I_1^{\phi_1^{*2}\phi_2} = -m^2\beta^3(\frac{1}{8\pi} - \frac{5}{16}). \tag{26}$$

This gives trivial result that the ratio of quantum correction to classical coupling is equal to that in conjugate coupling.

$$\frac{I_1^{\phi_1^2\phi_2}}{I^{\phi_1^2\phi_2}} = \frac{I_1^{\phi_1^{*2}\phi_2}}{I^{\phi_1^{*2}\phi_2}} = \beta^2(\frac{1}{16\pi} - \frac{5}{32}). \tag{27}$$

IV. Affine A_5 and D_4 Toda Theory

The theory that gives first nontrivial different couplings is A_4. But A_4 is not easy to treat numerically because $e^{\frac{2\pi i}{5}}$ is not a simple value. We now determine the ratios of three-point coupling to its quantum correction in affine A_5 case. Expansion of potential term gives

$$\begin{aligned} V(\phi) &= \frac{m^2}{\beta^2}6 + m^2(\phi_1\phi_1^* + 3\phi_2\phi_2^* + 2\phi_3^2) \\ &+ m^2\beta(\tfrac{\sqrt{2}}{4}i\phi_1^2\phi_2^* + \sqrt{2}i\phi_1\phi_2\phi_3 + \tfrac{\sqrt{2}}{4}i\phi_2^3 - \tfrac{\sqrt{2}}{4}i\phi_1^{*2}\phi_2 - \sqrt{2}i\phi_1^*\phi_2^*\phi_3 - \tfrac{\sqrt{2}}{4}i\phi_2^{*3}) \\ &+ m^2\beta^2(-\tfrac{1}{18}\phi_1^3\phi_3 - \tfrac{1}{8}\phi_1^2\phi_2^2 + \tfrac{1}{24}\phi_1^2\phi_1^{*2} + \tfrac{1}{2}\phi_1\phi_1^*\phi_2\phi_2^* + \tfrac{1}{3}\phi_1\phi_1^*\phi_3^2 + \tfrac{1}{2}\phi_1\phi_2^{*2}\phi_3 \\ &+ \tfrac{1}{2}\phi_1^*\phi_2^2\phi_3 + \tfrac{1}{3}\phi_2^2\phi_2^{*2} + \phi_2\phi_2^*\phi_3^2 + \tfrac{1}{9}\phi_3^4 - \tfrac{1}{18}\phi_3\phi_1^{*3} - \tfrac{1}{8}\phi_1^{*2}\phi_2^{*2}) + \cdots \end{aligned} \tag{28}$$

This shows that there exist three independent three-point couplings and their conjugate couplings.

$$\begin{aligned} &I^{\phi_1^2\phi_2^*} = \tfrac{\sqrt{2}}{2}m^2\beta, \quad I^{\phi_1\phi_2\phi_3} = \sqrt{2}m^2\beta, \quad I^{\phi_2^3} = \tfrac{3\sqrt{2}}{2}m^2\beta \\ &I^{\phi_1^{*2}\phi_2} = -\tfrac{\sqrt{2}}{2}m^2\beta, \quad I^{\phi_1^*\phi_2^*\phi_3} = -\sqrt{2}m^2\beta, \quad I^{\phi_2^{*3}} = -\tfrac{3\sqrt{2}}{2}m^2\beta. \end{aligned} \tag{29}$$

Symmetry between conjugate couplings allows one to consider only three independent couplings and to be blind to its conjugates. The diagrams which contribute to one loop correction and their values are given in table III. The results are

$$I_1^{\phi_1^2\phi_2^*} = (\frac{\sqrt{2}}{16\pi} - \frac{23}{432}\sqrt{6})m^2\beta^3, \; I_1^{\phi_1\phi_2\phi_3} = (\frac{2\sqrt{2}}{16\pi} - \frac{34}{432}\sqrt{6})m^2\beta^3, \; I_1^{\phi_2^3} = (\frac{3\sqrt{2}}{16\pi} - \frac{42}{432}\sqrt{6})m^2\beta^3 \tag{30}$$

$$\frac{I_1^{\phi_1^2\phi_2^*}}{I^{\phi_1^2\phi_2^*}} = \beta^2(\frac{1}{8\pi} - \frac{23}{216}\sqrt{3}), \quad \frac{I_1^{\phi_1\phi_2\phi_3}}{I^{\phi_1\phi_2\phi_3}} = \beta^2(\frac{1}{8\pi} - \frac{17}{216}\sqrt{3}), \quad \frac{I_1^{\phi_2^3}}{I^{\phi_2^3}} = \beta^2(\frac{1}{8\pi} - \frac{14}{216}\sqrt{3}). \tag{31}$$

It turns out that the ratios of quantum corrections to its classical couplings are not equal in different couplings. The universal formula for one loop correction is only confined to the coefficient of $\phi^a\phi^b$ term.

As a further check, we turn to the D_4 case. If we denote the four real scalar fields by $\phi^a(a = 1, \cdots, 4)$, we can formally associate ϕ_1 with vector representation, ϕ_2 with adjoint representation, ϕ_3 with spinor representation and ϕ_4 with conjugate spinor representation. Here we simply write down the potential term.

$$\begin{aligned} V(\phi) &= \frac{m^2}{\beta^2}6 + m^2(\phi_1^2 + 3\phi_2^2 + \phi_3^2 + \phi_4^2) + \tfrac{m^2\beta}{\sqrt{2}}(-\phi_1^2\phi_2 - \phi_3^2\phi_2 - \phi_4^2\phi_2 + \phi_2^3 - 2\phi_1\phi_3\phi_4) \\ &+ \tfrac{m^2\beta^2}{24}(\phi_1^4 + \phi_3^4 + \phi_4^4 + 9\phi_2^4 + 6\phi_1^2\phi_2^2 + 6\phi_1^2\phi_3^2 + 6\phi_1^2\phi_4^2 \\ &+ 6\phi_2^2\phi_3^2 + 6\phi_2^2\phi_4^2 + 6\phi_3^2\phi_4^2 + 24\phi_1\phi_2\phi_3\phi_4) + \cdots \end{aligned} \tag{32}$$

There are three independent three-point couplings.

$$I^{\phi_1^2\phi_2} = i\sqrt{2}m^2\beta, \quad I^{\phi_2\phi_3\phi_3} = -3i\sqrt{2}m^2\beta, \quad I^{\phi_1\phi_3\phi_4} = i\sqrt{2}m^2\beta. \tag{33}$$

In fact, there are additional $\phi_3^2\phi_2$, $\phi_4^2\phi_2$ couplings. But they are related by three-fold symmetry among ϕ_1, ϕ_3 and ϕ_4 fields to the $\phi_1^2\phi_2$ coupling. The diagrams which contribute to one loop correction and their values are given at table IV. J_1 in table IV's denotes the

vertex function at $p_i^2 = 0$. The results for I values are

$$I_1^{\phi_1^2\phi_2} = (\frac{\sqrt{2}}{8\pi} - \frac{35\sqrt{6}}{432})im^2\beta^3, \qquad \frac{I_1^{\phi_1^2\phi_2}}{I^{\phi_1^2\phi_2}} = \beta^2(\frac{1}{8\pi} - \frac{35}{432}\sqrt{3}),$$

$$I_1^{\phi_2\phi_2\phi_2} = (-\frac{3\sqrt{2}}{8\pi} - \frac{45\sqrt{6}}{432})im^2\beta^3, \qquad \frac{I_1^{\phi_2\phi_2\phi_2}}{I^{\phi_2\phi_2\phi_2}} = \beta^2(\frac{1}{8\pi} + \frac{15}{432}\sqrt{3}), \tag{34}$$

$$I_1^{\phi_1\phi_3\phi_4} = (\frac{\sqrt{2}}{8\pi} - \frac{45\sqrt{6}}{432})im^2\beta^3, \qquad \frac{I_1^{\phi_1\phi_3\phi_4}}{I^{\phi_1\phi_3\phi_4}} = \beta^2(\frac{1}{8\pi} - \frac{45}{432}\sqrt{3}).$$

A few observations can be made. See the first lines of type I case.

$$\begin{aligned} I_1 &: \ \frac{7}{72\sqrt{6}}, \qquad -\frac{27}{72\sqrt{6}}, \qquad \frac{9}{72\sqrt{6}}. \\ J_1 &: \ \frac{1}{\sqrt{2}\pi}(\frac{1}{8} - \frac{1}{16}log3), \quad -\frac{1}{\sqrt{2}\pi}(\frac{3}{8}), \quad \frac{1}{\sqrt{2}\pi}(\frac{1}{8}). \end{aligned} \tag{35}$$

These diagrams are the cases which consist of only $\phi_1^2\phi_2$, ϕ_2^3, and $\phi_1\phi_3\phi_4$ coupling at each three intermediate vertex of type I respectively. The ϕ_2^3 and $\phi_1\phi_3\phi_4$ couplings show the nice classical $-3:1$ ratio. But $\phi_1^2\phi_2$ coupling is slightly out of order. It is originated from the mass difference between ϕ_1 field and ϕ_2 field. On the other hand, we can expect this difference adds up to zero, summed over the contributing diagrams. But they fail, as shown in the last lines in each tables. So here we can see the qualititive origin of irregular ratios due to their multi-particle nature.

V. Discussion

We studied the ratios of one loop quantum corrections to classical three-point couplings in affine A_2, A_3, A_5 and D_4 Toda field theories. We have found the ratios of different couplings are not the same in general except the case where the different three-point couplings are related by symmetry of Dynkin diagram. Such an universal formula as that of finite mass corrections does not appear in three-point couplings. The β dependence of exact S matrix in affine Toda theories(conjectured by H.W. Braden, et al.[7]) may be consistent to this irregular correction, but it is not yet clarified.

Acknowledgements

One of the authors(JDK) is grateful to Dr. C. Ahn and Dr. W.T. Kim for useful discussions. This research is supported in part by KOSEF and Basic Science Research Institute, Kyung Hee University.

References

[1] T. J. Hollowood and P. Mansfield, Phys. Lett. B226 (1989) 73.

[2] A. B. Zamolodchikov, Int. J. Mod. Phys. A 3 (1988) 73.

[3] A. B. Zamolodchikov, Integrable Field Theory from Conformal Field Theory, Proceedings of the Taniguchi Symposium, Kyoto(1988)

[4] P. Christe, G. Mussardo, Integrable systems away from criticality: the Toda Field Theory and S matrix of the Tricritical Ising Model Preprint UCSBTH 89-19 to appear in Nucl. Phys. B

[5] P. Christe, G. Mussardo, Elastic S-matrices in (1 + 1) dimensions and Toda Field Theories, UCSBTH -89-44 to appear in Nucl. Phys. B

[6] T. Eguchi, S. K. Yang, Phys. Lett. B 224 (1989) 373.

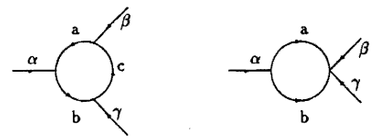
[7] H. W. Braden, E. Corrigan, P. E. Dorey and R. Sasaki, Affine Toda Field Theory and exact S-matrices, Preprint RRK 89-29, UDCPT 89/53
H. W. Braden, E. Corrigan, P. E. Dorey and R. Sasaki, Phys. Lett. B227 (1989) 411.

[8] E. Ogievetsky and P. Wiegmann, Phys. Lett. B168 (1986) 360.

[9] M. Koca and G. Mussardo, Mass Formulae in Toda Field Theories, Preprint CERN-TH 5659/90, UCSBTH 90-12.

[10] H. J. de Vega, Nucl. Phys. B115 (1976) 411.
P. Weiz, Nucl. Phys. B122 (1977) 1.
J. Verwaest, Nucl. Phys. B123 (1977) 100.
L. F. Abbott, Nucl. Phys. B139 (1978) 159.

[11] A. V. Mikhailov, M. A. Olshanetsky and A. M. Perelomov, Commun. Math. Phys. 79 (1981) 473.
D. Olive and N. Turok, Nucl. Phys. B257 (1985) 277.



Type I. Type II.

Figure 1. one loop diagrams of three-point coupling.

	α	β	γ	a	b	c	$I_1(m^2\beta^3)$
Type II	ϕ	ϕ	ϕ	ϕ^*	ϕ^*		$-\frac{\sqrt{3}}{1}$

Table I. one loop correction of affine A_2(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(m^2\beta^3)$
Type I	ϕ_1	ϕ_1	ϕ_2	ϕ_2	ϕ_1^*	ϕ_1	$\frac{1}{8\pi}+\frac{1}{16}$
Type II	ϕ_1	ϕ_1	ϕ_2	ϕ_1^*	ϕ_2		$-\frac{1}{1}$
	ϕ_2	ϕ_1	ϕ_1	ϕ_1	ϕ_1		$-\frac{1}{16}$
	ϕ_2	ϕ_1	ϕ_1	ϕ_1^*	ϕ_1^*		$-\frac{1}{16}$
total							$\frac{1}{8\pi}-\frac{3}{16}$

Table II. one loop corrections of affine A_3(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(m^2\beta^3)$
Type I	ϕ_1	ϕ_1	ϕ_2^*	ϕ_2^*	ϕ_1^*	ϕ_3	$\frac{\sqrt{2}}{24\pi}+\frac{1}{216}\sqrt{6}$
	ϕ_1	ϕ_1	ϕ_2^*	ϕ_3	ϕ_2^*	ϕ_2	$\frac{\sqrt{2}}{48\pi}+\frac{1}{216}\sqrt{6}$
Type II	ϕ_1	ϕ_1	ϕ_2^*	ϕ_2^*	ϕ_3		$-\frac{1}{36}\sqrt{6}$
	ϕ_1	ϕ_1	ϕ_2^*	ϕ_1^*	ϕ_2		$-\frac{1}{72}\sqrt{6}$
	ϕ_2^*	ϕ_1	ϕ_1	ϕ_1^*	ϕ_1^*		$-\frac{1}{216}\sqrt{6}$
	ϕ_2^*	ϕ_1	ϕ_1	ϕ_3	ϕ_1		$-\frac{1}{108}\sqrt{6}$
	ϕ_2^*	ϕ_1	ϕ_1	ϕ_2	ϕ_2		$-\frac{1}{144}\sqrt{6}$
total							$\frac{\sqrt{2}}{16\pi}-\frac{23}{432}\sqrt{6}$

Table III-a. one loop corrections of $I^{\phi_1\phi_1\phi_2^*}$ in affine A_5(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(m^2\beta^3)$
Type I	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_1^*	$\frac{\sqrt{2}}{24\pi}+\frac{1}{54}\sqrt{6}$
	ϕ_1	ϕ_2	ϕ_3	ϕ_2^*	ϕ_1^*	ϕ_2	$\frac{\sqrt{2}}{24\pi}+\frac{1}{108}\sqrt{6}$
	ϕ_1	ϕ_2	ϕ_3	ϕ_3	ϕ_2^*	ϕ_1	$\frac{\sqrt{2}}{24\pi}+\frac{1}{54}\sqrt{6}$
Type II	ϕ_1	ϕ_2	ϕ_3	ϕ_1^*	ϕ_2		$-\frac{1}{72}\sqrt{6}$
	ϕ_1	ϕ_2	ϕ_3	ϕ_2^*	ϕ_3		$-\frac{1}{36}\sqrt{6}$
	ϕ_2	ϕ_1	ϕ_3	ϕ_1	ϕ_1		$-\frac{1}{108}\sqrt{6}$
	ϕ_2	ϕ_1	ϕ_3	ϕ_1^*	ϕ_3		$-\frac{1}{54}\sqrt{6}$
	ϕ_2	ϕ_1	ϕ_3	ϕ_2^*	ϕ_2^*		$-\frac{1}{72}\sqrt{6}$
	ϕ_3	ϕ_1	ϕ_2	ϕ_1^*	ϕ_2^*		$-\frac{1}{48}\sqrt{6}$
	ϕ_3	ϕ_1	ϕ_2	ϕ_2	ϕ_1		$-\frac{1}{48}\sqrt{6}$
total							$\frac{\sqrt{2}}{8\pi}-\frac{34}{432}\sqrt{6}$

Table III-b. one loop corrections of $I^{\phi_1\phi_2\phi_3}$ in affine A_5(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(m^2\beta^3)$
Type I	ϕ_2	ϕ_2	ϕ_2	ϕ_1^*	ϕ_1	ϕ_3	$\frac{3\sqrt{2}}{16\pi}+\frac{1}{12}\sqrt{6}$
Type II	ϕ_2	ϕ_2	ϕ_2	ϕ_1	ϕ_1		$-\frac{1}{24}\sqrt{6}$
	ϕ_2	ϕ_2	ϕ_2	ϕ_1^*	ϕ_3		$-\frac{1}{12}\sqrt{6}$
	ϕ_2	ϕ_2	ϕ_2	ϕ_2^*	ϕ_2^*		$-\frac{1}{18}\sqrt{6}$
total							$\frac{3\sqrt{2}}{16\pi}-\frac{42}{432}\sqrt{6}$

Table III-c. one loop corrections of $I^{\phi_2^3}$ in affine A_5(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(im^2\beta^3)$	$J_1(im^2\beta^3)$
Type I	ϕ_1	ϕ_1	ϕ_2	ϕ_2	ϕ_1	ϕ_1	$\frac{7}{72\sqrt{6}}$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8}-\frac{1}{16}log3)$
	ϕ_1	ϕ_1	ϕ_2	ϕ_1	ϕ_2	ϕ_2	$\frac{3}{\sqrt{2}}(-\frac{1}{27\sqrt{3}}+\frac{1}{36\pi})$	$(-\frac{3}{\sqrt{2\pi}})(-\frac{1}{24}+\frac{1}{16}log3)$
	ϕ_1	ϕ_1	ϕ_2	ϕ_4	ϕ_3	ϕ_3	$\frac{1}{36\sqrt{2}}(\frac{9}{\sqrt{3}}+\frac{3}{\pi})$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8})$
	ϕ_1	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_4	$\frac{1}{36\sqrt{2}}(\frac{9}{\sqrt{3}}+\frac{3}{\pi})$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8})$
Type II	ϕ_1	ϕ_1	ϕ_2	ϕ_1	ϕ_2		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{\sqrt{2\pi}})(\frac{1}{4}log3)$
	ϕ_1	ϕ_1	ϕ_2	ϕ_3	ϕ_4		$-\frac{\sqrt{6}}{18}$	$(-\frac{1}{\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_2	ϕ_1	ϕ_1	ϕ_1	ϕ_1		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{4\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_2	ϕ_1	ϕ_1	ϕ_3	ϕ_3		$\frac{\sqrt{6}}{72}$	$(\frac{3}{4\sqrt{2\pi}})(\frac{1}{6})$
	ϕ_2	ϕ_1	ϕ_1	ϕ_4	ϕ_4		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{4\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_2	ϕ_1	ϕ_1	ϕ_2	ϕ_2		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{4\sqrt{2\pi}})(\frac{1}{2})$
total							$\frac{1}{4\sqrt{2\pi}}-\frac{35}{72\sqrt{6}}$	$(\frac{1}{\sqrt{2\pi}})(-\frac{1}{4}+log3\times(-\frac{1}{2}))$

Table IV-a. one loop corrections of $I^{\phi_1\phi_1\phi_2}$ in affine D_4(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(im^2\beta^3)$	$J_1(im^2\beta^3)$
Type I	ϕ_2	ϕ_2	ϕ_2	ϕ_2	ϕ_2	ϕ_2	$-\frac{27}{72\sqrt{6}}$	$(-\frac{27}{\sqrt{2\pi}})(\frac{1}{72})$
	ϕ_2	ϕ_2	ϕ_2	ϕ_1	ϕ_1	ϕ_1	$-\frac{1}{\sqrt{2}}(\frac{1}{3\sqrt{3}}+\frac{1}{4\pi})$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8})$
	ϕ_2	ϕ_2	ϕ_2	ϕ_3	ϕ_3	ϕ_3	$-\frac{1}{\sqrt{2}}(\frac{1}{3\sqrt{3}}+\frac{1}{4\pi})$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8})$
	ϕ_2	ϕ_2	ϕ_2	ϕ_4	ϕ_4	ϕ_4	$-\frac{1}{\sqrt{2}}(\frac{1}{3\sqrt{3}}+\frac{1}{4\pi})$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8})$
Type II	ϕ_2	ϕ_2	ϕ_2	ϕ_1	ϕ_1		$-\frac{\sqrt{6}}{12}$	$(-\frac{3}{2\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_2	ϕ_2	ϕ_2	ϕ_2	ϕ_2		$\frac{3\sqrt{6}}{8}$	$(\frac{81}{2\sqrt{2\pi}})(\frac{1}{6})$
	ϕ_2	ϕ_2	ϕ_2	ϕ_3	ϕ_3		$-\frac{\sqrt{6}}{12}$	$(-\frac{3}{2\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_2	ϕ_2	ϕ_2	ϕ_4	ϕ_4		$-\frac{\sqrt{6}}{12}$	$(-\frac{3}{2\sqrt{2\pi}})(\frac{1}{2})$
total							$-\frac{3}{4\sqrt{2\pi}}-\frac{43}{72\sqrt{6}}$	$(\frac{1}{\sqrt{2\pi}})(\frac{9}{2}+log3\times(0))$

Table IV-b. one loop corrections of $I^{\phi_2\phi_2\phi_2}$ in affine D_4(notation refers to Fig. 1).

	α	β	γ	a	b	c	$I_1(im^2\beta^3)$	$J_1(im^2\beta^3)$
Type I	ϕ_1	ϕ_3	ϕ_4	ϕ_4	ϕ_3	ϕ_1	$\frac{9}{72\sqrt{6}}$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8})$
	ϕ_1	ϕ_3	ϕ_4	ϕ_1	ϕ_2	ϕ_4	$\frac{1}{12\sqrt{2\pi}}$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8}-\frac{1}{16}log3)$
	ϕ_1	ϕ_3	ϕ_4	ϕ_2	ϕ_1	ϕ_3	$\frac{1}{12\sqrt{2\pi}}$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8}-\frac{1}{16}log3)$
	ϕ_1	ϕ_3	ϕ_4	ϕ_3	ϕ_4	ϕ_2	$\frac{1}{12\sqrt{2\pi}}$	$(\frac{1}{\sqrt{2\pi}})(\frac{1}{8}-\frac{1}{16}log3)$
Type II	ϕ_1	ϕ_3	ϕ_4	ϕ_1	ϕ_2		$-\frac{\sqrt{6}}{72}$	$(-\frac{1}{2\sqrt{2\pi}})(\frac{1}{4}log3)$
	ϕ_1	ϕ_3	ϕ_4	ϕ_3	ϕ_4		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{2\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_3	ϕ_1	ϕ_4	ϕ_1	ϕ_4		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{2\sqrt{2\pi}})(\frac{1}{2})$
	ϕ_3	ϕ_1	ϕ_4	ϕ_2	ϕ_3		$-\frac{\sqrt{6}}{72}$	$(-\frac{1}{2\sqrt{2\pi}})(\frac{1}{4}log3)$
	ϕ_4	ϕ_1	ϕ_3	ϕ_2	ϕ_4		$-\frac{\sqrt{6}}{72}$	$(-\frac{1}{2\sqrt{2\pi}})(\frac{1}{4}log3)$
	ϕ_4	ϕ_1	ϕ_3	ϕ_1	ϕ_3		$-\frac{\sqrt{6}}{36}$	$(-\frac{1}{2\sqrt{2\pi}})(\frac{1}{2})$
total							$\frac{1}{4\sqrt{2\pi}}-\frac{15}{72\sqrt{6}}$	$(\frac{1}{\sqrt{2\pi}})(-\frac{1}{4}+log3\times(-\frac{9}{16}))$

Table IV-c. one loop corrections of $I^{\phi_1\phi_3\phi_4}$ in affine D_4(notation refers to Fig. 1).