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INHOMOGENEITY CAN REMOVE THE BIG-BANG SINGULARITY

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Abstract

It is clear that spherical symmetry cannot sustain inhomogeneous non-static fluid models. For general generic conditions, incorporation of inhomogeneity is desirable that can be done by adopting cylindrical (instead of spherical) symmetry, which also admits singularity free solutions of Einstein's equation with acceptable physical properties. The whole set of such models has been identified. The family of singularity free cosmological models can also be thought of arising out of natural inhomogenisation of the open Friedman-Robertson-Walker model. The big-bang singularity then seems to be an artifact of spherical symmetry and not a general property of the theory. For homogeneous cosmological models spherical seems to be the proper symmetry whereas cylindrical seems to be the case for inhomogeneous and singularity free models.

Finally at whatever scale the universe is supposed to be inhomogeneous, it must at best be cylindrically symmetric and can as well be singularity free.

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It is generally believed that occurrence of the big-bang singularity is an essential feature of the relativistic cosmology. This belief is based on the fact that the universe is expanding and hence it should have had a singular (physical volume tending to zero) big-bang beginning in the finite past. The Friedman-Robertson-Walker (FRW) model not only imbibes this crucial feature but is also successful enough to describe the present state of the universe to be termed as the standard model. The FRW model represents a radially symmetric, homogeneous and isotropic universe filled with perfect fluid. One may ask, whether occurrence of the singularity is due to the special features of radial symmetry, homogeneity and isotropy of the model or is it a generic property of the theory, general relativity (GR) ? Then followed the powerful singularity theorems [1] which proved in all generality that occurrence of singularity is inescapable in GR under reasonable physical assumptions. This went on to strengthen the belief that the big-bang singularity is unavoidable in relativistic cosmology so long as we adhere to GR.

The general strategy adopted by cosmologists is very pragmatic, not to fight with the hard facts but rather learn to live with them. That is the big-bang is there, now the only pertinent question is how to "manage" it ? For this one generally invokes quantum effects and new fields or modifies GR. A lot of effort has been (and is being) invested in this project giving rise to the field of quantum cosmology dealing with the early (closer to the bang) universe at the scale of the Planck length. It should however be remembered that the only observational imprint of the big-bang we have is the cosmic microwave background radiation which will only require a dense state of the universe, not necessarily going down to the Planck length.

In 1990 Senovilla [2] found a non-singular solution of Einstein's equation representing a cosmological model filled with incoherent radiation, $\rho = 3p$. The model was examined and found free of singularity of any kind [3] and yet satisfied all physically reasonable conditions. This was a dramatic and important discovery. How did this happen when the theorems [1] prohibit it ? Existence of compact trapped surfaces (i.e. gravity becomes so strong that even photons cannot leave a surface) is one of the assumptions of the theorems. How strong should gravity become must entirely be governed by the field equations. We should not in general make additional prescriptions for field. This concept was invented for understanding gravitational collapse leading to formation of black hole. It played a very useful role there. But there seems to be no physical imperative for it in the cosmological context. Non-occurrence of compact trapped surfaces will not imply any loss of physical reasonableness of cosmological models. Obviously owing to cylindrical symmetry of the Senovilla solution [2], it does not obey this assumption rendering the theorems ineffective.

Inspired by the existence of singularity free models in GR, I wish to reexamine the question: Is the big-bang singularity in the FRW model due to its special properties of radial symmetry, homogeneity and isotropy ? In this essay I shall attempt to show that it is so. It turns out that the spherical symmetry (of the FRW model) cannot allow non-static inhomogeneous fluid models. For general generic conditions, it is desirable that inhomogeneity of fluid distribution is permissible. For that to happen we will have to go over to less restrictive symmetry (from 3-radial symmetry to 2-radial symmetry) of cylindrical symmetry. In cylindrical symmetry with separability

of metric, there exist a family of singularity free inhomogeneous models [4] and further it is shown to form the complete set of such solutions [5].

According to the Raychaudhuri equation [6], for avoidance of singularity it is necessary to have acceleration and/or rotation non-zero . We shall take fluid to be irrotational and hence the spacetime will be vorticity free. That means it will be necessary to have non-zero acceleration for non-singular spacetime. From the definations of shear, expansion and acceleration it can be easily seen that acceleration can be non-zero for a separable (in space and time) metric only if shear is non-zero [7]. Thus singularity free (separable) spacetimes will have necessarily to be both inhomogeneous and anisotropic. Since spherical symmetry cannot sustain inhomogeneity, we will have to instead employ cylindrical symmetry for constructing non-singular cosmological models in GR. This is how cylindrical symmetry gets associated with singularity free models.

In cosmology, we are concerned with overall smoothed out behaviour of the universe as a whole. That is micro level effects and phenomena, which may be extremely complex and involved, are not taken into consideration in cosmological models. It is therefore perhaps not too unreasonable to assume separability of the spacetime metric into space and time variables. The overriding consideration for this however is mathematical simplicity that makes the problem tractable. This is why it is a common property shared by all cosmological models.

We thus begin with an orthogonal (i.e. vorticity free) metric

$$ds^2 = Ddt^2 - A(dx^1)^2 - B(dx^2)^2 - C(dx^3)^2 \tag{1}$$

where A, B, C, D are separable functions of space and time variables (i.e. $A = A_1(t)A_2(x^i), i = 1,2,3$) . We further take that spatial dependence only comes through the radial distance, i.e. $A = A_1(t)A_2(r)$ where $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$.

Now let the metric (1) represent a universe filled with perfect fluid, which would require the following conditions: $T_{ao} = 0, T_{ab} = 0$ for $a \neq b; a, b = 1,2,3$ and $T_1^1 = T_2^2 = T_3^3$. These conditions after considerable manipulations (see Tolman [8] for expressions of $T_{\alpha\beta}$) lead to the only three following models (a) FRW with vanishing shear and acceleration and non-zero expansion, (b) Bianchi I with zero acceleration and non-zero shear and expansion and (c) static models.

It is interesting to note that radial space dependance dictates that the model can, ruling out the static case, either be FRW or Bianchi I. That means this does not allow a non-static inhomogeneous model at all. For bringing in inhomogeneity into fluid distribution, we have to seek spatial dependence which is less restrictive. The obvious choice would be cylindrical radial distance instead of spherical.

We are thus led to cylindrical symmetry in a natural way for inclusion of inhomogeneity in the system. As stated earlier that a complete set of singularity free inhomogeneous cylindrical models has been identified [4,5]. Consider the metric (1) with cylindrial symmetry, now the fluid conditions lead to the following cases [9]: (i) a complete set of singularity free inhomogeneous models [5], (ii) inhomogeneous models with singularities, not necessarily of big-bang type [4], (iii) Bianchi I models, and (iv) static models.

The important point to note is that it can be shown that the first three cases can be

obtained by inhomogenising the FRW metric for $k = -1, 1, 0$ (in the case of $k = 1$, only a subset of (ii) results) respectively. Let us demonstrate this process explicitly for the case $k = -1$, exhibiting the fact that inhomogenisation of FRW model leads to the unique singularity free family of models [10].

We begin with the FRW metric with $k = -1$

$$ds^2 = dt^2 - T^2(t)\left(\frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) \tag{2}$$

which can be written in the cylindrical coordinates as

$$dz^2 = dt^2 - T^2(t)\left(\frac{d\bar{r}^2}{1+\bar{r}^2} + (1+\bar{r}^2)dz^2 + \bar{r}^2 d\phi^2\right)$$

by the transformation

$$r = (sinh^2 z + \bar{r}^2 cosh^2 z)^{1/2}, tan\theta = \frac{\bar{r}}{sinhz\sqrt{1+\bar{r}^2}} \cdot \tag{3}$$

Further writing $m\bar{r} = sinh(m\hat{r})$ and then dropping caps to get

$$ds^2 = dt^2 - T^2(t)(dr^2 + cosh^2(mr)dz^2 + m^{-2}sinh^2(mr)d\phi^2).$$

Now inhomogenise this in the natural way by using the functions $T(t)$ and $cosh(mr)$ to write

$$\begin{aligned} ds^2 &= T^{2\alpha}cosh^{2a}(mr)(dt^2 - dr^2) - T^{2\beta}cosh^{2b}(mr)dz^2 \\ &- m^{-2}sinh^2(mr)T^{2\gamma}cosh^{2c}(mr)d\phi^2. \end{aligned} \tag{4}$$

where we have used the coordinate freedom to write $g_{tt} = |g_{rr}|$. The fluid conditions will now determine

$$T = cosh(nt), \alpha = \gamma \tag{5}$$

and relations between the parameters $\alpha, \beta, a, b, c, n$ and m. The metric (4) with (5) is the unique singularity free family. That is inhomogenisation of the FRW metric leads in a natural way to the singularity free family of cosmological models [10]. This metric has been shown to be geodesically complete without reference to Einstein's equation with some constraints on the range of the parameters [11].

What we have attempted to establish in the above is that for inclusion of inhomogeneous fluid distributions in cosmological models, we are naturally led to cylindrical symmetry which unlike spherical symmetry admits physically acceptable singularity free models. Though the present day universe is homogeneous and isotropic, it is of course well known that these special features could not be sustained at all times. For formation of large scale structures in the universe as well as to have general generic initial conditions, the consideration of inhomogeneity and anisotropy of the universe becomes essential and desirable. For incorporation of inhomogeneity, as mentioned earlier cylindrical (or lesser) symmetry is required. We are thus led to the inference that the scale at which we wish to allow inhomogeneity in the fluid distribution, the universe must at best be cylindrically symmetric and it need not be singular as there exist a family of non-singular solutions.

Now the prime question is : how to make the early cylindrical, inhomogeneous and anisotropic universe compatible with the present spherical, homogeneous and isotropic universe ? This is an important question whose affirmative answer will have profound effect on our cosmological perception and thinking in general and the early universe cosmology in particular.

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