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TWO-PARTICLE CORRELATION FUCTION AND GRAVITATIONAL GALAXY CLUSTERING

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Abstract

A theory for a system clustering under gravitational interaction is developed for $b(n,T)$, ratio of gravitational correlation energy to twice the kinetic energy of peculiar velocities, in terms of a partial differential equation using thermodynamic technique. Various solutions of the differential equation relates $b(n,T)$ with density n and temperature T of the gravitating system. The physical validity of various solutions of $b(n,T)$ on the basis of certain boundary conditions and probability density distribution function is discussed. Results indicate that for single component systems the density distribution function of galaxy clustering depends on the specific combination nT^{-3}. The theory also provides a new insight into gravitational clustering.

Subject headings: Cosmology — galaxies: Clustering — hydrodynamics — method: Analytical.

Introduction

The gravitational clustering of galaxies has played an important role in the evolution of the observed Universe. The presently observed clustering of galaxies suggests that their motions have been dominated by mutual gravitational dynamics. The non-linear clustering phenomenon is determined by physical process involving a lengthy and complex sequence of events. One approach for understanding such galaxy clustering in the Universe deals with the evaluation of n-particle correlation functions between galaxies. This can be done by solving system of Liouville's equations or BBGKY - hierarchy equations and have been discussed by many workers, e.g. Saslaw (1972), Inagaki (1976), Fall and Saslaw (1976) and Peebles (1980), etc. But BBGKY - hierarchy equations are too complicated to handle for higher order correlation functions. However, the lowest order, two-point correlation function, is useful for discussing the phenomenon of galaxy clustering which contains information on all the higher n-particle correlations in the full BBGKY - hierarchy (Peebles 1980, Zhan 1989, Zhan & Dyer 1989 and Hamilton 1993). An alternative simple and more effective statistical approach to two-point correlation functions for non-linear galaxy clustering has been developed first by Saslaw and Hamilton (1984) with the help of gravitational thermodynamic results. The physical validity of applying thermodynamic theory to the gravitational clustering of galaxies has been discussed on the basis of computer N-body simulations by Zhan (1989), Saslaw et al (1990), Itoh et al (1993), Saslaw & Sheth (1993). On the basis of the pairwise gravitational interaction of point masses, statistical homogeneity and quasi-equilibrium evolution, an analytical expression for the probability distribution function $f(N)$ of finding N galaxies in a randomly positioned volume V in terms of b, is given by (Saslaw & Hamilton 1984):

$$f(N) = \frac{\overline{N}(1-b)}{N!}[\overline{N}(1-b)+Nb]^{N-1}e^{-[\overline{N}(1-b)+Nb]}, \qquad (1)$$

where $\overline{N} = \overline{n}V$, $\overline{n}$ is the average number density of the particles and b is the ratio of gravitational correlation energy, W_c, to the kinetic energy, K, of the peculiar

motions:

$$b = -\frac{W_c}{2K}. \tag{2}$$

The theory can be extended to obtain a non-linear velocity distribution function for quasi-equilibrium gravitational clustering in an expanding Universe (Saslaw et al 1990). The value of b measures the influence of the two-particle correlation function for clustering in the non-linear regime. On the basis of certain physical arguments, Saslaw and Hamilton (1984) assumed a simple mathematical expression for b with physical limits $b = 0$ (for the perfect gas) and $b = 1$ (for the hierarchical system). The extra-ordinary agreement of $f(N)$ with the computer N-body simultations and observed galaxy distribution leads to the expectation that it might be based on more fundamental considerations (Saslaw et al 1990). The importance of two-particle correlation function (or b) on time is revealed by its simple relation with the expansion scale R of Universe (Saslaw 1992).

$$R = R_*[b(1-b)^{-7}]^{1/8}, \tag{3}$$

where R_* is constant given by initial state. In a recent paper Saslaw and Sheth (1993) studied details of the non-linear and time evolution of gravitational galaxy clustering.

The value of b depends on the two-point correlation function which in turn is a function of density n and temperature T of the system in a grand canonical resemble. Thus it is valuable to understand the functional form of $b(n,T)$ and to derive its expression in terms of n and T from first principles to help in understanding the non-linear nature of the problem. It is the aim of the present paper to develop a theory for $b(n,T)$ in terms of n and T of the system by using gravitational thermodynamic relations. Firstly a partial differential equation is developed from the energy equation of thermodynamics in combination with the equation of state, taking gravitational interactions between particles into consideration. The solutions of the differential equation give the required analytical dependence of b on n and T. We are interested in a solution which satisfies boundary conditions of the system for clustering under gravitational interactions. The theory here is based on the

quasi-equilibrium hypothesis of thermodynamics. The N-body computer simulation results (Itoh et al 1993) explicitly verify the basic assumption that thermodynamic theory applies to gravitational clustering of galaxies.

This paper is organised as follows: In Section 2, the applicability of quasi-Equilibrium hpothesis to galaxy clusters is discussed and then a partial differential equation is developed by using the energy equation and the equation of state of thermodynamics taking gravitational interaction between galaxies into consideration. The differential equation when solved gives a number of explicit solutions in terms of n and T. In Section 3, the physical validity of various solutions is established on the basis of certain boundary conditions and the probability density distribution function. Finally, in Section 4 the results are discussed.

2. Gravitational Thermodynamics

2.1 *The Quasi-Equilibrium approximation :*

The applicability of quasi-equilibrium thermodynamics to galaxy clustering was hypothesized Previously (Saslaw and Hamilton 1984, Saslaw 1985), and computer N-bogy experiments strongly confirm the applicability of this approximation (Itoh et al 1988, 1993). It is further supported by theoretical arguments (Saslaw 1992;, Zhan 1989; Zhan and Dyer 1989, Saslaw and Fang 1996). Since local clustering in the expanding Universe is generally faster than the expansion time scale, quasi-equilibrium evolution seems to be a good approximation. The evolution can proceed approximately adiabatically through a sequence of equilibrium states. This has been tested by N-body simulation results (Itoh et al 1988), which show that relaxed evolution occurs through equilibrium states and applies to a wide range of conditions. The fact that the gravitational force does not saturate in gravitating systems means there is no inherent equilibrium. At first sight, quasi-equilibrium evolution may seem to be unpromising. However application of quasi-equilibrium approximation to the galaxy clustering has the following bases :

1. On relaxed scales, generally where the two point correlation function $\xi(r) \geq 1$, the system has lost its memory of initial conditions. If these initial conditions were Poisson (or nearly so with no long range coherance) they would contain

minimal initial information. So evolution would take place from a relaxed state with no initial memory.

2. The long range part of the gravitational field (mean field) is removed by the expansion of the universe (Peebles 1980, Saslaw & Fang 1996), so the thermodynamics only involves the fluctuating short range component of the gravitational force. This expansion effectively subtracts off the mean field, leaving only the fluctuations in the force which create density fluctuations.

3. Extremely condensed states (two point masses with $r_{ij} \longrightarrow 0$ which give an energy state $-\infty$) may occur, but they are very improbable for galaxy clustering since almost all clusters virialize, at least approximately. Therefore, these states do not contribute significantly to the thermodynamics.

4. The time scale to develop local bound finite sub-systems is $\sim \frac{1}{\sqrt{G\rho}}$ which is less than the global dynamical relaxation time $\sim \frac{1}{\sqrt{G\rho_0}}$.

5. It is possible to use the usual macroscopic thermodynamic variables U,S,V,N to describe the macroscopic state of these infinite statistically homogeneous gravitating systems. This is because their fluctuations over a grand canonical ensemble are sufficiently small that average values of these variables are well defined.

6. There is a time-dependence to the thermodynamics, represented by macroscopic variables, including $b(t)$, which changes over the Hubble time scale or longer (Saslaw 1992). This time dependence is slow enough that when the locally relaxed regions respond to it, they retain their description in terms of thermodynamic variables (eg. fluctuations remain relatively small). This is the usual condition for ordinary non-equilibrium thermodynamics to apply (Kreuzer 1986).

7. Some standard thermodynamic approaches (eg. Callen 1985) take the somewhat circular but consistent view that if the properties of a system agree with thermodynamic theory (as the N-body simulations and observations of the galaxy distribution functions show) then it is effectively in equilibrium or

for gravity in quasi-equilibrium.

2.2 *Energy equation and two-particle correlation function*

The fundamental energy equation expresses the internal energy of the system as a function of the various thermodynamic variables defining the system. Consider an infinite system of N single component particles (galaxies) distributed homogenously in a volume V having internal energy U and pressure P.

In general P, V and T are related by $P = P(V,T)$ and $U(V,T)$ for fixed N and by the relation:

$$dU = \left(\frac{\partial U}{\partial V}\right) dV + \left(\frac{\partial U}{\partial T}\right) dT \tag{4}$$

Combining the first and second laws of thermodynamics with the above equation and apply the condition that entropy is a perfect differential leads to the energy equation (Landau and Lifshitz 1981) :

$$\left(\frac{\partial U}{\partial V}\right)_{T,N} = T\left(\frac{\partial P}{\partial T}\right)_{V,N} - P \tag{5}$$

The energy equation in combination with the equations for state for a gravitating system determines the properties of the system and can be used to derive a partial differential equation for $b(n,T)$ in terms of density and temperature of the system. We assume galaxies to be point masses which are interacting pairwise gravitationally. The internal energy U and pressure P are related to the two-particle correlation function (Saslaw 1985, Sheth & Saslaw 1996) by:

$$U = \frac{3}{2} NT(1 - 2b) \tag{6}$$

and

$$P = \frac{NT}{V}(1 - b), \tag{7}$$

where b measures the influence of gravitational correlation energy W_c and is related to the two-particle correlation function $\xi(n,T,r)$ by

$$b(n,T) \equiv -\frac{W_c}{2K} = \frac{2\pi G m^2 \bar{n}}{3T} \int_V \frac{dV}{4\pi r} \tag{8}$$

here $\bar{n} = (N/V)$ is the average number density. The kinetic energy K of peculiar motion is related to temperature T (in energy units with Boltzmann's constant $k = 1$) by:

$$K = \frac{3}{2}NT = \frac{1}{2}\sum_{i=1}^{N} mV_i^2 \tag{9}$$

The value of $b(n,T)$ depends on the form of two-particle correlation function $\xi(n,T,r)$ and it was shown that $\xi \sim r^{-2}$ over a finite range maximises the gravitational entropy of clustering (Saslaw 1980). Thus specifying the correct form of b as a function of n and T is essential in understanding the thermodynamics. Saslaw and Hamilton (1984) assumed a mathematical form of $b(n,T)$ on the basis of three arguments: (i) scale invariance of $b(n,T)$, (ii) the entropy as a total differential, and (iii) mathematical simplicity

Without using such approximations here, our main concern is to develop a differential equation for b in terms of n and T from basic principles. This can be done by using thermodynamic energy equation in the equation of state. The combination of equations (6) and (7) in (5) leads to a first order partial differential equation:

$$3n\left(\frac{\partial b}{\partial n}\right)_T + T\left(\frac{\partial b}{\partial T}\right)_n = 0. \tag{10}$$

The solutions of this partial differential equation can relate $b(n,T)$ to the density and temperature for the gravitational galaxy clustering. The partial differential equation can be solved using standard method (Sneddon 1985), and admits

$$b(n,T) = f(nT^{-3}) \tag{11}$$

as its general solution. The choice of f gives different particular solutions for $b(n,T)$ in terms of nT^{-3}. We are interested in a solution which satisfies the boundary conditions of the system for clustering under gravitational interactions. One solution of the differential equation (10) is

$$b(n,T) = \frac{(\beta nT^{-3})^{\alpha}}{1 + (\beta nT^{-3})^{\alpha}} \tag{12}$$

where β is a positive constant.

Here α is an integer, $\alpha = 1, 2, 3.....$, and for $\alpha = 1$, the equation (12) leads exactly to the assumed expression of $b(n, T)$ by Saslaw and Hamilton (1984). Fig. 1 shows how b varies with nT^{-3} for different values of α.

Next we derive various thermodynamic quantities such as the entropy, chemical potential, specific heat and fluctuations for some solutions of $b(n, T)$. We confine our results for the solutions of $b(n, T)$ given by equation (12). The entropy, specific heat at constant volume C_v and chemical potential μ can be obtained from the thermodynamic relations (Landau and Lifshtiz 1981)

$$d\left(\frac{S}{N}\right) = \frac{1}{T}d\left(\frac{U}{N}\right) + \frac{P}{T}d\left(\frac{V}{N}\right), \tag{13}$$

$$C_v = \frac{1}{N}\left(\frac{\partial U}{\partial T}\right)_{T,N} \tag{14}$$

and

$$(\mu/T) = \frac{U}{NT} - \frac{S}{N} + \frac{\psi}{N}, \tag{15}$$

where the grand canonical potential ψ is

$$\psi = \frac{PV}{T} = \overline{N}(1 - b) \tag{16}$$

The use of equations (6), (7) and (12) in (13) and (14) gives S and C_v for the system governed by the two-particle correlation function related to b by :

$$\left(\frac{S}{N}\right) = ln(n^{-1}T^{3/2}) - 3b - \frac{1}{\alpha}ln(1 - b) + s_o, \tag{17}$$

$$C_v = \frac{3}{2}[1 + 2(3\alpha - 1)b - 6\alpha b^2]. \tag{18}$$

The critical value of b at which the specific heat becomes negative is given by

$$b_c = \frac{(3\alpha - 1) + \sqrt{9\alpha^2 + 1}}{6\alpha}. \tag{19}$$

For $\alpha = 1, b_c = 0.8604$(Saslaw & Sheth 1993), and for $\alpha = 2, b_c = 0.9236$ etc.

The chemical potential (μ/T) can be calculated by the use of equations (6), (16) and (17) in (15)

$$(\mu/T) = ln(nT^{-3/2}) - b + \frac{1}{\alpha} ln(1-b) \tag{20}$$

Next, we calculate mean square number and energy fluctuations in a given volume governed by grand canonical ensemble from the relations (Callen 1985)

$$< \triangle N^2 >=< N^2 > - < N >^2 = \left. \frac{\partial^2 \psi}{\partial^2 (\mu/T)} \right|_{T,V}, \tag{21}$$

$$< \triangle U^2 >=< U^2 > - < U >^2 = - \left. \frac{\partial^2 \psi}{\partial^2 (I/T)} \right|_{T,V}, \tag{22}$$

which give number fluctuations and energy fluctuations for the values of $b(n,T)$ given by equation (12)

$$< \triangle N^2 >= \frac{N}{(1-b)(1-\alpha b)} \tag{23}$$

and

$$< \triangle U^2 >= \frac{3NT^2[5 - 2(9+\alpha)b + 2(8+9\alpha)b^2 - 2\alpha(5+3\alpha)b^3]}{4(1-b)(1-\alpha b)} \tag{24}$$

Saslaw et al 1990 modified the earlier (Saslaw and Hamilton 1984) assumed expression for $b(n,T)$ using the more general form :

$$n(1-b) = \frac{\Phi(b)}{\beta T^{-3}}, \tag{25}$$

where $\Phi(b)$ can be choosen as an arbitrary function of b which satisfies two conditions. First, $\Phi(b) = 0$ for $b = 0$, and secondly the Poisson limit $b\Phi(b) - \Phi'(b) = 0$. The condition of the 'Poisson limit' is determined on the basis of the standard two-point correlation function (Landau and Lifshitz 1981)

$$\frac{1}{V^2} \int \int \xi_{12} dV_1 dV_2 = \frac{1}{\overline{N}} \left[\left(\frac{\triangle N}{\overline{N}} \right)^2 - 1 \right] \tag{26}$$

The term on the right-hand side is due to the mean square number fluctuation and is given by equation (23).

3. Physical Validity of Various Solutions

The partial differential equation (10) has number of solutions for $b(n,T)$ in terms of n and T. We can investigate the physical validity of these solutions on the basis of:

(a) Boundary conditions to be fulfilled by the two-point correlation function for gravitational galaxy clustering.

(b) Probability density distribution functions.

3.1 *Boundary Conditions*

Two-point correlation function of gravitational galaxy clustering can be assigned certain boundary conditions. A physically valid solution of $b(n,T)$ should satisfy the following five boundary conditions which are summarised in Table 1 for different solutions.

(1) The gravitational clustering of galaxies in a homogeneous Universe requires $b(n,T)$ to be scale invariant. One can express this boundary condition on the hierarchical scale as $n \longrightarrow \lambda^{-3}n$ and $T \longrightarrow \lambda^{-1}T$, so $b(\lambda^{-3}n, \lambda^{-1}T) = b(n,T)$, where λ is a scalar multiplier. All the solutions comply with this scale invariance condition (Table 1). The scaling of temperature indicates that clusters at a higher level of the hierarchy have lower random velocities relative to each other than smaller clusters have relative to each other (Saslaw and Hamilton 1984).

(2) When the two-particle correlation function increases or $b \longrightarrow 1$, the galaxies become more and more clustered. This is because of virial equilibrium, which suggests that at low temperatures and high densities more strongly bound clusters are formed. This boundary condition requires that $b \longrightarrow 1$, when $nT^{-3} \longrightarrow \infty$, and is satisfied by all solutions except I in Table I.

(3) As $b \longrightarrow 0$, the entropy of the ensemble must tend to a 'Maxwellian gas' distribution i.e. $S \sim ln\left(\frac{NT^{3/2}}{V}\right) + S_o$. This boundary condition is satisfied only for $b(n,T)$ given by III for all values of α.

Two more boundary conditions which $b(n,T)$ must satisfy are based on the general description of b (Saslaw and et al 1990). These conditions are based on equations (25) and (26).

(4) If $b = 0$, then $\Phi(b) = 0$, which implies that the ensemble represents a perfect gas. This condition is a modified form of boundary condition 3. One can see that this boundary condition is satisfied only by III for any value of α.

(5) The last boundary condition demands that for the Poisson limit $b\Phi' - \Phi = 0$. It is interesting to note that no solution of $b(n,T)$ except $\alpha = 1$ satisfies this condition. It corresponds to the assumed expression for $b(n,T)$ of Saslaw and Hamilton (1984).

3.2 *Galaxy distribution function*

It is found that the first four boundary conditions are satisfied by the solution of the partial differential equation given by equation (11) for any value of α. The last boundary condition 5 is satisfied only for $\alpha = 1$, which corresponds to the Poisson limit. For values $\alpha > 1$, the mean square number fluctuation expression $< \triangle N^2 >$ is negative which is an unphysical result. This would give a Poisson limit equation (26) for $\alpha > 1$ which would require that the integral of the two point correlation function be negative instead of the Poisson value of zero. Now we would like to investigate, what the Poisson limit means for $\alpha < 1$ for gravitational galaxy clustering on the basis of density distribution function. Saslaw and Hamilton (1984) derived an elegant and simple form for the probability distribution, which is given in equation (1). The probability distribution function for finding N-galaxies in a volume V of arbitrary shape is given by

$$f(N) = e^{-\psi} \frac{e^{\mu/TN} (e^{\psi})_{\circ}^{(N)}}{N!}, \tag{27}$$

where ψ and (μ/T) are given by equations (16) and (20) respectively. $\left(e^{\psi}\right)_{\circ}^{(N)}$ is given by

$$\left(e^{\psi}\right)_{\circ}^{(N)} = \left\{ \left(\frac{d}{de^{\mu/T}} \right)^{N} exp[\psi(e^{\mu/T})] \right\}_{e^{\mu/T}=0} \tag{28}$$

We can calculate $(e^{\psi})_{\circ}^{(N)}$ for a few values of N from the relations:

$$\left(e^{\psi}\right)_{\circ}^{(1)} = \left\{ e^{\psi} e^{-\mu/T} \frac{d\psi}{d(\mu/T)} \right\}_{e^{\mu/T}=0} \tag{29}$$

$$\left(e^{\psi}\right)_{\circ}^{(2)} = \left\{e^{\psi}e^{-2\mu/T}\left[\left(\frac{d\psi}{d(\mu/T)}\right)^{2} - \left(\frac{d\psi}{d(\mu/T)}\right) + \left(\frac{d^2\psi}{d(\mu/T)^2}\right)\right\}_{e^{\mu/T}=0} \tag{30}$$

and

$$\left(e^{\psi}\right)_{\circ}^{(3)} = \left\{e^{\psi}e^{-3\mu/t}\left[\left(\frac{d\psi}{d(\mu/T)}\right)^{3} - 3\left(\frac{d\psi}{d(\mu/T)}\right)^{2} + 3\left(\frac{d\psi}{d(\mu/T)}\right)\left(\frac{d^2\psi}{d(\mu/T)^2}\right) + 2\left(\frac{d\psi}{d(\mu/T)}\right) - 3\left(\frac{d^2\psi}{d(\mu/T)^2}\right) + 3\frac{d^3\psi}{d(\mu/T)^3}\right]\right\}_{e^{\mu/T}=0}. \tag{31}$$

By use of equations (12), (16) and (20), we have following relations:

$$\psi = \frac{Vb^{1/\alpha}}{\beta'(1-b)^{-1+1/\alpha}}, \tag{32}$$

$$e^{-\mu/T} = \frac{\beta' T^{3/2} e^{b}}{b^{1/\alpha}}, \tag{33}$$

$$\frac{d\psi}{d(\mu/T)} = \frac{Vb^{1/\alpha}}{\beta'(1-b)^{1/\alpha}}, \tag{34}$$

$$\frac{d^2\psi}{d(\mu/T)^2} = \frac{Vb^{1/\alpha}}{\beta'(1-\alpha b)(1-b)^{1+1/\alpha}}, \tag{35}$$

and

$$\frac{d^3\psi}{d(\mu/T)^3} = \frac{Vb^{1/\alpha}(1+\alpha^2 b - 2\alpha^2 b^2)}{\beta'(1-\alpha b)^3(1-b)^{2+1/\alpha}}, \tag{36}$$

where

$$\beta' = \beta^{1/\alpha} T^{-3} \tag{37}$$

with the use of equations (32) to (36) in (29) to (31), we have

$$\left(e^{\psi}\right)_{\circ}^{(1)} = \frac{V}{\beta'}\beta' T^{3/2}, \qquad \textit{for all values of } \alpha \tag{38}$$

$$\left(e^{\psi}\right)_{\circ}^{(2)} = \left[e^{\frac{Vb^{1/\alpha}}{\beta'(1-b)^{-1+1/\alpha}}}\beta'^2 T^3 e^{2b}\left\{\frac{V^2}{\beta'^2(1-b)^{2/\alpha}} + \frac{V}{\beta'}\frac{(1+\alpha)b^{1-1/\alpha} - \alpha b^{2-1/\alpha}}{(1-\alpha b)(1-b)^{1+1/\alpha}}\right\}\right]_{b=0}. \tag{39}$$

For $b = 0$, we have

$$\left(e^{\psi}\right)_{\circ}^{(2)} = \frac{V}{\beta'} T^3 \beta'^2 \left(\frac{V}{\beta'} + 2\right) \qquad for\ \alpha = 1 \tag{40}$$

$$= \infty \qquad for\ \alpha < 1 \tag{41}$$

Similarly, we have

$$\begin{aligned}\left(e^{\psi}\right)_{\circ}^{(3)} = \Bigg[& \frac{V b^{1/\alpha}}{e^{\beta'(1-b)^{-1+1/\alpha}}} \beta'^3 T^{9/2} e^{3b} \Bigg\{ \frac{V^3}{\beta'^3 (1-b)^{3/\alpha}} \\ & + \frac{V^2}{\beta'^2} \frac{3(\alpha+1)b^{1-1/\alpha} - 3\alpha b^{2-1/\alpha}}{(1-\alpha b)(1-b)^{1+2/\alpha}} + \frac{V}{\beta'} [\![(\alpha^2 - 1) b^{1-2/\alpha} \\ & + (\alpha^2 + 6\alpha + 2) b^{2-2/\alpha} - \alpha(2\alpha^2 + 9\alpha + 6) b^{3-2/\alpha} \\ & + 2\alpha^2 (2\alpha^2 + 3) b^{4-2/\alpha} - 5\alpha^3 b^{5-2/\alpha}]\!] \Big/ (1-\alpha b)^3 (1-b)^{2+1/\alpha} \Bigg\} \Bigg]_{b=0}\end{aligned} \tag{42}$$

giving limiting values at $b = 0$, we have

$$\begin{aligned}\left(e^{\psi}\right)_{\circ}^{(3)} &= \frac{V}{\beta'} T^{9/2} \beta'^3 \left(\frac{V}{\beta'} + 3\right)^2 \qquad for\ \alpha < 1 \\ &= \infty \qquad for \alpha < 1.\end{aligned} \tag{43}$$

For $\alpha = 1$, one can calculate the general term of $(e^{\psi})_{\circ}^{(N)}$ and arrive at the same results for the $f(N)$ distribution function given by equation (1). But it is interesting to observe that for $\alpha < 1$, $\left(e^{\psi}\right)_{\circ}^{(2)}$ onward diverges . Thus the distribution function does not converge for values other than $\alpha = 1$. The function should be differentiable at all points and possess derivatives for all orders. For $\alpha < 1$, the derivatives at $b = 0$ become infinite at higher orders and if $\alpha = 1$, the situation is saved. Thus it can be concluded that the partial differential equation has only one solution which satisfies all the boundary conditions and gives a distribution function which converges for $b = 0$.

4. Discussion

On the basis of quasi-equilibrium approach it is possible to use thermodynamics to derive a simple partial differential equation which must be satisfied by the distribution function of galaxy clustering. The solution of this partial

differential equation determine $b(n,T)$ with density and temperature having Specific combination of nT^{-3}. Several solutions do not satisfy all the physical boundary conditions for $b(n,T)$ but all the solutions satisfy the scale invariane of $b(n,T)$. Most of these boundary conditions are satisfied by the partial differential equation. The important unsatisfied condition is that in the Poisson limit the System should behave like an ordinary 'Maxwellian gas'. Also the mean square number fluctuation $< \Delta N^2 >$ becomes negative (unphysical) for other solutions except for $b(n,T)$ given by

$$b(n,T) = \frac{\beta n T^{-3}}{1 + \beta n T^{-3}} \tag{44}$$

This is further confirmed by the density contribution $f(N)$ which diverges for values other than $b(n,T)$ given above. The divergence is shown from the derivation of $f(N)$ distribution function. The higher order coefficients in the distribution function become infinity for any solution other thatn $b(n,T)$ given above. Thus we can assign the Poisson limit ($b \longrightarrow 0$) as an essential boundary condition for the distribution function to be satisfied for gravitational clustering. A wide range of class for solution of $b = f(nT^{-3})$ in terms of functions like trignometrical, hyperbolic and polynomials etc. do not satisfy all the physical boundary conditions and analyticity of distribution function.

The main results found here are summarised as follows :

(1) The galaxy distribution function for gravitational galaxy clustering depends on the density n and temperature T. It depends on the specific combination of nT^{-3}. This combination gives a scale invariants form of $b(n,T)$, thus satisfying the condition of statistical homogeneity in the expanding Universe.

(2) Among various solutions of the partial differential equation for $b(n,T)$ in the distribution, the only solution of the differential equation which satisfied all the boundary conditions of $b(n,T)$ is that given by eq. (44).

(3) The mean square fluctuations become negative for other solutions except for $b(n,T)$ given by eq. (44).

(4) The distribution function converges only for $b(n,T)$ given by eq.(44) under the

limiting conditins $b \longrightarrow 0$ which corresponds to Poisson limit. Thus the Poisson limit is an essential boundary condition which the distribution function must satisfy.

(5) The expression assumed earlier by Saslaw & Hamilton (1984) for $b(n, T)$ is now derived from more basic theory, thus removing it as an independent *ansatz*. This further strengthens the foundation of their theory.

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Table 1

Boundary Conditions for two-point correlation function.

b =	I	II	III
Boundary Conditions	$1+\beta nT^{-3}$	$\frac{\beta nT^{-3}-1}{\beta nT^{-3}+1}$	$\frac{\beta(nT^{-3})^{\alpha}}{1+\beta(nT^{-3})^{\alpha}}$
(1) $n \to \lambda^{-3}n$, $T \to \lambda^{-1}T$ then $b(\lambda^{-3}n, \lambda^{-1}T) = b(n,T)$	satisfied	satisfied	satisfied for all values of α
(2) As $nT^{-3} \to \infty$, then $b \to 1$	not satisfied	satisfied	satisfied for all values of α
(3) As $b \to 0$,then $S \sim ln\,(\frac{NT^{3/2}}{V}) + S_{\circ}$	not satisfied	not satisfied	satisfied for all values of α
(4) $\Phi(b) = 0$ for $b = 0$	not satisfied	not satisfied	satisfied for all values of α
(5) $b\Phi'(b) - \Phi(b) = 0$	not satisfied	not satisfied	satisfied only for $\alpha = 1$

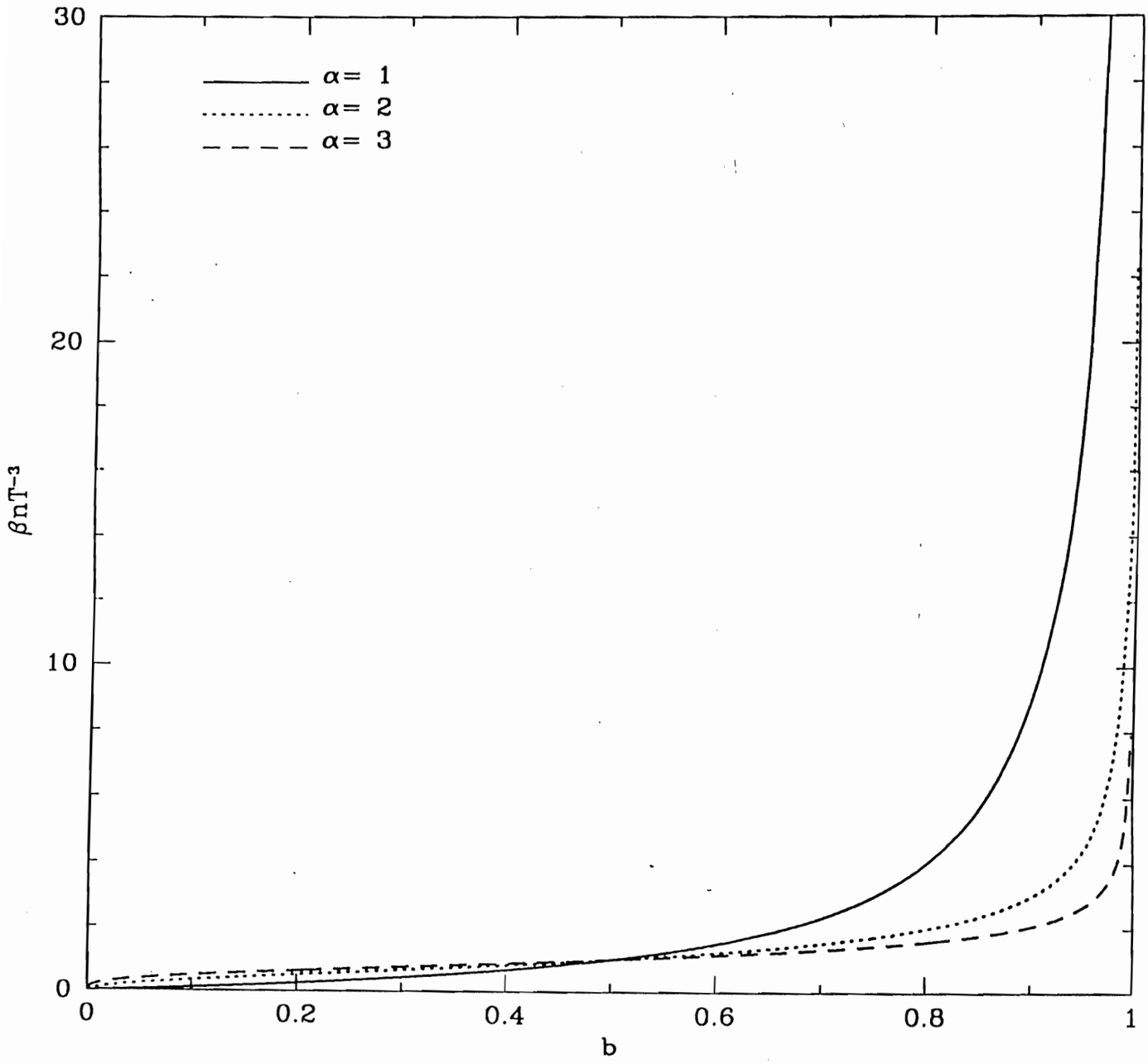
30
20
10
0
0
0.2
0.4
0.6
0.8
1
b
βnT-3
α= 1
α= 2
α= 3

Figure Caption:

Fig. 1. Variation of b with nT^{-3} for different values of α .