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Testing the bosonic string with cosmology

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Abstract

A Kaluza–Klein cosmological model recently derived from the bosonic string theory is compared with the available data from the observations of cosmic microwave background anisotropies.

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Superstring, supergravity, and other theories with an extended gravitational sector share the idea of compactified spacelike extra dimensions. These modern theories motivate the study of Kaluza–Klein (hereafter "KK") multidimensional cosmology in which the four observable dimensions expand and the extra dimensions shrink, or stay small as the universe evolves. In recent years a large amount of literature on KK cosmology has been published ([1] and references therein). KK cosmological models are believed to mimic supergravity, superstring or M-theories, but they are severely constrained by the observations of the cosmic microwave background, although certain scenarios are not ruled out [2]. There is hope that a direct derivation of KK cosmologies from a more fundamental high energy theory will provide a rigorous framework for KK cosmology, and will modify the values of the KK parameters in such a way that the current observational constraints will be circumvented. This goal was not accomplished until the recent remarkable discovery that the multidimensional cosmologies studied in [1] can be obtained from the bosonic string theory in the limit of an infinite number of extra dimensions [3]. The starting point is the tree-level effective action[1] [4]

$$S_{eff} = -\frac{1}{2k_0^2}\int d^{D_0}x\sqrt{|g^{(0)}|}\exp(-2\Phi)\left[R[g^{(0)}] + 4g^{(0)\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + C\right] , \qquad (1)$$

where k_0^2 is the D_0–dimensional gravitational constant, Φ is the string dilaton, and C is the central charge deficit. After performing the compactification of d_1 extra dimensions and a conformal transformation to the Einstein frame, the following cosmological effective action is obtained [3]:

$$\begin{aligned} S &= -\frac{1}{2k_0^2}\int d^{D_0}x\sqrt{|\hat{g}^{(0)}|}\left[\hat{R}[\hat{g}^{(0)}] + \hat{g}^{(0)\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + R[g^{(1)}]\exp(-2\lambda_{(1)}\varphi)\right. \\ &\quad \left. +2\Lambda\exp(-2\lambda_{(2)}\varphi)\right] , \end{aligned} \qquad (2)$$

where φ is the Einstein frame string dilaton, Λ is the cosmological constant in $D_0 + d_1$ dimensions, and $R[g^{(1)}]$ is the Ricci curvature of the manifold of extra dimensions. The cosmological dilatonic coupling constants are given by

$$\lambda_{(1)}^2 = \frac{D_0 + d_1 - 2}{d_1(D_0 - 2)} , \qquad (3)$$

[1] The metric signature is $- + + +$. The speed of light assumes the value unity, and Newton's constant is related to the Planck mass by $m_{pl} = G^{-1/2}$. Quantities in the Einstein frame are denoted by a caret. The Ricci scalar in the present paper is the opposite of Ref. [3].

$$\lambda_{(2)}^2 = \frac{d_1}{(D_0 + d_1 - 2)(D_0 - 2)} , \tag{4}$$

as functions of the number of observable and extra dimensions (D_0 and d_1, respectively). The value of the string dilatonic coupling constant,

$$\lambda_s = \frac{1}{\sqrt{D_0 - 2}} , \tag{5}$$

is reproduced in the limit $d_1 \to \infty$. This limit is not unphysical: a large number of extra dimensions ($d_1 \geq 36$–40) has been invoked by many authors in order to generate a large amount of entropy in the observable 4–dimensional universe [5]. It was suggested in Ref. [3] that observational cosmology can provide a natural test for string theories – this is indeed the case for the KK cosmology presented in that paper. A typical feature of KK cosmology is the dominance of the dilaton field on the dynamics of the early universe – in the model under consideration, it corresponds to the identification of the string dilaton with the inflaton. It is currently believed that inflation is the only causal mechanism capable of solving the problems of the standard big bang model and which successfully generates density perturbations [6]. Hence a test for the string–inspired KK cosmology of [3] is to determine whether it provides a successful inflationary phase of the universe. To this end, it is necessary to rewrite the action (2) in the canonical form used in the literature on inflation[2] by means of the transformation

$$\varphi = k_0 \sigma , \qquad k_0 = \frac{\sqrt{8\pi}}{m_{pl}} , \tag{6}$$

which yields

$$S = - \int d^4x \sqrt{|\hat{g}^{(0)}|} \left[\hat{R}\left[\hat{g}^{(0)}\right] \frac{m_{pl}^2}{16\pi} + \frac{1}{2} \hat{g}^{(0)\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + V(\sigma) \right] , \tag{7}$$

where

$$V(\sigma) = \frac{R[g^{(1)}]}{2k_0^2} \exp(-\alpha\sigma/m_{pl}) + \frac{\Lambda}{k_0^2} \exp(-\beta\sigma/m_{pl}) , \tag{8}$$

$$\alpha = 4\sqrt{2\pi}\, \lambda_{(1)} , \qquad \beta = 4\sqrt{2\pi}\, \lambda_{(2)} , \tag{9}$$

[2]Unfortunately, the use of a incorrect form of the action seems to be common in the literature on KK theories ([2] – see also footnote 11 of Ref. [7]). However insignificant this error may appear to be, it profoundly affects the final results on the viability of the KK model under consideration [2].

and where the value $D_0 = 4$ has been used. In the limit $d_1 \to \infty$, in which $\lambda_{(1,2)}$ reproduce the string coupling constant (5), the potential (8) reduces to the single exponential

$$V(\sigma) = V_0 \exp(-\sqrt{16\pi}\,\sigma/m_{pl}) \,, \tag{10}$$

where $V_0 = k_0^{-2}\left(\Lambda + R[g^{(1)}]/2\right)$. This potential is well-known from the power-law inflationary scenario [8], and gives a coasting universe with scale factor $a(t) = a_0 t$, which is non-inflationary. The power-law inflationary potential $U(\sigma) = U_0 \exp(-\sqrt{16\pi/p}\,\sigma/m_{pl})$ gives a scale factor $a = a_0 t^p$ and a spectral index of density perturbations $n = 1 - 2/p$. The combined statistical analysis of the *COBE* and Tenerife observations of cosmic microwave background anisotropies [9] sets the constraint $p \geq 20$. We conclude that the KK cosmology derived from the bosonic string in [3] is not viable. The reason may be that the limit $d_1 \to \infty$ of the bosonic string action does not correspond to the real universe, or that the early universe was not dominated by the string dilaton, or perhaps the action (1) is not the correct one. The multidimensional cosmological models considered in [3] corresponding to finite values of the parameter d_1 (not derived from string theory) were compared to the observational data in Ref. [2], and the viable models still lack a rigorous derivation from a high energy theory.

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