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A program for a problem free Cosmology within a framework of a rich class of scalar tensor theories

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Abstract

A search for a problem free cosmology within the framework of multicomponent non-minimally coupled scalar tensor theories is suggested. In an SO(2) invariant theory for example, we outline a program that can accommodate a coasting Milne universe with a distribution of non - topological soliton solutions [NTS's]. The interior of these solutions would be domains where effective gravitational effects would be indistinguishable from those expected in standard Einstein theory. For a large class of non - minimal coupling terms and the scalar effective potential, the effective cosmological constant identically vanishes in the region exterior to the NTS's and dynamically approaches a small value in the interior region. We describe features of a problem free cosmology that follows.

I. Introduction

The prospect of a dynamic geometric action forms the basis of Einstein's theory of gravitation. A geometric action however, is not uniquely fixed by the requirement of general co variance alone. Just for the sake of simplicity, and for the sake of getting the canonical Newtonian theory in the weak field limit, one requires the action to be of the second order in the metric. This gives the standard Einstein - Hilbert action of general relativity [GTR]:

$$S = \int d^4x\sqrt{-g}[\beta R + \Lambda] \tag{1.1}$$

with Λ the cosmological constant. This form for the gravitational action is virtually defended as a faith. The success of the three classical tests as well as the basic simplicity, economy and elegance of this theory at the classical level has indeed given a widespread feeling that this theory must be correct. Admittedly Einstein's theory is elegant, however one can not be sure whether nature has quite the predilection for elegance and economy that man possesses.

The last six decades have seen growth of empirical evidence of discrepancy between predictions of General Relativity and observations. Oort [1932] was perhaps the first to demonstrate that the acceleration caused by visible matter in our neighbourhood in the Milky Way is only half of what is experienced by stars in their motion perpendicular to the galactic disk. Zwicky [1933], and more recently Kent et al [1982], further discovered acceleration discrepancies ranging from factors of two to three to about 100 in clusters of galaxies. In the outskirts of most galaxies, a centrifugal acceleration upto a factor of 5 or more than that expected from visible matter is needed to account for the anomalous velocity curves (Albada[1985], Hernquist et al [1987]). Faith in Einsten's theory has led astronomers to make out a case for non luminous matter to explain the above descrepancies.

The requirement for non - luminous [dark] matter is also made out over larger cosmological distances. Over the last four decades it has been fashionable to maintain "cosmological book - keeping" within the framework of the standard Freidman - Robertson - Walker [FRW] solution. This framework, however, comes with its own peculiar problems. The FRW model read with GTR is plagued with an initial singularity. GTR is also sick on account of a lack of a consistent quantum mechanical framework. The latter problem arises on account of the fact that the coupling constant in the theory [the gravitational constant] is dimensional. A consistent account of structure formation is far from understood. The age of the universe predicted in the model is significantly smaller than age estimates of old stellar objects. The observed large scale homogeneity and isotropy of the universe can not be dynamically generated in a FRW cosmology on account of the so called "horizon problem". The stability of the FRW solution, moreover, requires a fine tuning of the density parameter in the theory unless it is exactly equal to the critical density. This is referred to as the flatness problem. These problems can be squarely resolved if the FRW universe is made to undergo inflation at some early epoch. Such a behaviour can be accounted for in the standard cosmological model by a dynamicaly varying equation of state of matter

in the universe and elaborate schemes have been proposed to make this possible, (see eg. Brandenberger [1997] for a recent review). The paradigm however, requires a scalar field with a very special profile for its effective potential. Standard convexity and triviality theorems in quantum field theory cast a doubt on whether such an effective potential can indeed be realised. Assuming that this formal problem would be resolved, inflationary scenarios ensure the favoured value for the density parameter of the FRW universe viz.: the closure density, in turn ensuring: the density parameter $\Omega = 1$. This defines what is now regarded as the so called Standard - Big - Bang model [SBB]. Over the last decade, the SBB has suffered a lot of stress.

What are heralded as major successes of SBB are: (1) The Big Bang Nucleosynthesis [BBN] prediction of light element abundances and the Cosmic Microwave Background [CMBR]; and (2)consistency of distance indicators and redshifts between theory and observations.

From the light element abundances one finds that the observed D and $_3He$ can be generated in BBN only if $\Omega_B \approx 2\%$. The game plan in SBB is to choose a suitable combination of hot and cold dark matter [HDM; CDM] content to account for structures at different scales. While the progress in this direction has been on for some time, there is now enough accumulated evidence that puts the whole programme in doubt.

There has been a steady growth of empirical evidence indicating that Ω over scales of a giga parsec is significantly less than one. Figures of .5 to .8 have been quoted in literature. The flatness problem [fine tuning] then stares SBB in the face. There is further evidence of excessive baryonic DM inconsistent with BBN from the intensity of x-ray from centres of clusters of galaxies. It has been proposed that a small cosmological constant be incorporated in the model. While this would not do away with the fine tuning problem, the resolution that it may achieve is seriously in doubt. The allowed range of the hubble parameter and the cosmological constant is severely constrained by gravitational lensing, structure functions, the BBN, xray from clusters and most of all from the age estimates of globular clusters [Bagla(1996),]. It was believed that the age of $\approx$ 12 gyrs could be accommodated in a very small parameter space in which we could have a concordant cosmological model. However this space is shrinking. Two recent observations that have suffered a heavy blow on Std. model are: (1) the Lyman alpha observations of deuterium absoption lines - differing in intensity from the expected value by a whole order of magnitude [Steigman 1997] and [2] the observation, at redshift of around two, of a galaxy with some 3.5 billion yr. old clusters [Kennicut (1996)]. In the first instance one finds it difficult to have any dynamical evolution model that can account for such a wide variation of deuterium in Std cosmology. On the other hand. if one believes the higher abundance of deuterium to be the correct primordial result, any realistic galactic chemical evolution would yield very high residual $3He + D_2$. Such high values are not consistent with observations. The combined data ($D, 3He, 4He$ and $Li7$) with the usually assumed uncertainties are inconsistent with standard ($N_\nu = 3$) BBN for a conservative choice of 3He survival factor unless we assume that some of the uncertainties have been underestimated. With the baryon entropy ratio η determined by the combined $D - 3He$ and $7Li$ constraints, BBN predicts $Y_p = 0.246 \pm 0.002(1\sigma)$, where the error includes the uncertainties from the $D - 3He$ and $7Li$ constraints and from the BBN calculations. This

value for Y_p required for BBN consistency is 0.014 above the observed value. We thus find ourselves on the verge of several possible interesting inconsistencies : 1) If the primordial $4He$ fraction is established to be less than 23.9% either $D + 3He$ estimates will have to be revised or some more dramatic cosmological or particle physics based alteration in BBN predictions are required. 2)Recent estimates of the baryon fraction in rich clusters suggest that this fraction can be rather large. If the Universe is flat, then the baryon fraction derived, based on the rich cluster estimates is at least a factor of 2 larger than that allowed by BBN estimates, even with systematic uncertainties allowed for. Whether this is an indication that the universe is not flat, or an indication that the cluster estimates themselves suffer from possible large systematic uncertainties remains to be seen.

The second observation implies an age of around 18 gyrs for the present universe. For such an age there is no concordant parameter space for a viable model. There is no concordance in a model with zero cosmological constant while a small non - vanishing cosmological constant would need a very fine tuning for a dynamic realisation.

To conclude, over the last more than fifteen years, the state of art in cosmology has been reduced to designing and incorporating a suitable inflationary paradigm within the framework of the standard big - bang model [SBB]. Though inflation can resolve the horizon, flatness and the monopole problems and has a promise to give an ansatz for primordial density fluctuations, it is still too early and naive to defend this paradigm as a faith. For one, the inflaton field requires a fine tuning of its dynamical parameters leaving hardly any "naturalness" in the model. Further, a transition from an almost empty inflated patch to a matter filled universe (the reheating and the graceful exit problems) can not be dynamically realised in most versions of the inflationary scenario. Most versions of inflation find the smallness of the cosmological constant an embarrassment. The "natural" value of the cosmological constant in most unification schemes, the inverse square of the Planck length, differs from observational bounds by some 120 orders of magnitude ! On the other hand, with better instrumentation and observations, the parameter space for a viable cosmology within SBB (inflation included) is shrinking [Bagla, 1996]. Perhaps the worst constraints being imposed by age estimates of old clusters and parameters for structure formation. The SBB may well be on the verge of a crisis.

In the next section we outline a cosmological model in which the universe explodes as a non - empty Milne model from an initial arbitrarily high density state. This necessarily requires the effective gravitational constant to vanish identically. It is proposed to dynamically realize such an expanding [coasting] model as a solution to a generic class of scalar tensor theories in which the scalar field is arrested at a value where its non - minimal coupling to the scalar curvature diverges. The trace of matter stress energy tensor is a dynamic parameter that determines the evolution of the scalar field. One can expect variations of some 8 to 10 orders of magnitude in this parameter as the temperature of the universe [kT] transits through the mass of any particle species. One can ensure that such a universe witnesses successive first order phase transitions. For a multicomponent scalar field, one can have non - trivial, non - topological soliton solutions in the theory. These can be expected to be pinched off from the ϕ meson plasma as regions of true ground state percolate. What is desired is to start off with a highly correlated matter distribution that explodes with a "Big - Bang" as a cloud bounded by a light front in a flat [Minkowski]

space time, make it go through a succession of phase transitions and achieve the same expanding cloud with a distribution of NTS's distributed near surfaces where true ground state regions collide. We show that each NTS is a domain that separates the exterior $\mid \phi \mid = 0$ region from the interior where the scalar field could be arrested to an arbitrary value. The interior is thus a region with an effective gravitational constant and an effective cosmological constant determined by the interior values of the non minimal coupling term $U(\mid \phi \mid)$, and the scalar effective potential $V(\mid \phi \mid)$ respectively. The essential point is that we expect each NTS domain to expand while conserving the charge and the energy of the NTS. This necessarily implies that the expansion would be accompanied by a drift of $\mid \phi \mid$ to a value that ensures smaller and smaller values of $V(\mid \phi \mid)$. In the limit, the effective gravitational constant would approach a universal value inside all NTS's - determined by a value $\mid \phi_0 \mid$ and the bulk paremeters describing the matter stress energy tensor. This would be an effective solution to the cosmological constant problem.

The program essentially requires non - minimal coupling [NMC]. There is no compelling principle to constrain the coupling of a functional of a scalar field with the scalar curvature. Coleman and others [1970] have used a conformal coupling for a single component scalar field: $U(\phi) = \phi^2/6$ to get decent renormalisable properties of the stress energy tensor. Zee[1979], and earlier, Deser [1970] have considered NMCS to generate effective Brans - Dicke like theories virtually indistinguishable from GTR at low energies. While Dolgov [1982] has demonstrated that a consistent description of Goldstone bosons requires the absence of NMCS, he has also explored [though not successfully] a rising NMC as a mechanism to dynamically reduce the effective cosmological constant [Dolgov et al 1982]. Madsen [1987] has extensively reviewed properties of a large class of NMCS. There seems to be no consensus on any particular principle that may be used to determine the NMC. For our purpose we propose an ansatz that leaves the form for the NMC unspecified as an otherwise arbitrary function $U(\phi)$. We require the model to support non - topological - soliton solutions [NTS's] and determine the form for the NMC by minimizing the energy of the NTS. This is what is outlined in the next section.

Section II describes requirements on a two component scalar field theory in order that it generate an effective theory of gravitation indistinguishable from the GTR. In the class of ST theories considered, one can solve for the interior metric exactly. However, on account of the intrinsic non - linearity and complexity of the field equations, we have not been able to develop rigourous arguments to establish stability of the soliton solutions. The description may well be taken as a suggestive and heuristic argument and is the best we have been able to manage at this stage.

Section III, in its corresponding titled subsections, translates aspects of standard model in the model described here. We report on the status of nucleosynthesis, structure formation and related aspects in our model. Section IV summarises the predictions of the model and highlights further essential work that need be done on the model presented here. In the appendix we describe the essential properties of a two component scalar tensor theory. Expressions for a conserved energy - momentum pseudo - tensor are derived.

II. A Milne model with a difference:

We take the impasse described in the introduction as reason enough to revisit the favourite model of Milne [1935], who considered the evolution of an exploding universe from a highly correlated state [a vanishingly small ball of particles in thermodynamic equillibrium] localized near the neighbourhood of the point x = y = z = 0 at t = 0, in a Minkowski spacetime. At any later time, the universe consists of an isotropically expanding swarm of particles of all speeds bounded by the speed of light. Special relativity tells us that the universe looks the same (isotropically expanding ball bounded by a light front) in all Lorentz frames that coincide at the origin x = y = z = 0 at t = 0. In this sense the universe is homogeneous and isotropic about every such Lorentz observer - strictly obeying the so called cosmological principle. In co - moving coordinates the Minkowski metric describing the expanding ball reduces to

$$ds^2 = dt^2 - t^2[\frac{dr^2}{1+r^2} + r^2 d\Omega^2] \tag{2.1}$$

This is nothing but the standard open FRW metric with the scale factor a(t) = t. The most appealing feature of this model is that at any time, every observer can see the entire universe : there is no horizon in the model.

$$\int_0^t \frac{dt}{a(t)} = \infty \forall t > 0 \tag{2.2}$$

There is no flatness problem either as the rate of expansion of the universe is not constrained by any "critical density" parameter. This however, may be regarded as a trivial solution to the flatness problem. Eqn[2.1] is a solution to Einstein's equations only if the product of the gravitational constant and the density $G\rho$ vanishes. In canonical Einstein theory, the Milne metric can thus only be a solution for an empty ($\rho = 0$) universe. The model is thus put away without any further ado. However, one can try to make out a case for a search for models in which the universe coasts freely over large distances and has gravitating domains localised in pockets. In this article we report on our study of requirements on classes of scalar - tensor [ST] theories in which the large scale dynamics of a non - empty universe is described by eqn[2.1] on account of the vanishing of the effective long - distance gravitational constant.

We consider a ST theory characterised by an arbitrary non - minimal coupling of a scalar field with the scalar curvature in an effective action:

$$S = \int \sqrt{-g} d^4x [U(\phi_1, \phi_2)R + \frac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - V(\phi_1, \phi_2) + L_m] \tag{2.3}$$

Here $\phi_i, (i = 1, 2)$ are the two components of a scalar field, $U(\phi_i)R$ is a non - minimal coupling of the scalar field with the scalar curvature R, $V(\phi_i)$ is the scalar effective potential and L_m is the contribution from the rest of the [matter] fields. Throughout our analysis we treat $\phi_{1,2}$ as components of a classical field. The essential features of such a theory

are described in the Appendix A. As shown, the stress energy tensor for the rest of the matter fields has a vanishing co - variant divergence thereby confirming to the equivalence principle.

We now consider a particular two component ST theory invariant under SO(2) rotations in the internal ϕ_1, ϕ_2 space. The action being described by eqn[3] with $U(\phi_1, \phi_2)$ and $V(\phi_1, \phi_2)$ being functions of ϕ_1, ϕ_2 through the SO(2) invariant $\mid \phi \mid \equiv \sqrt{\phi_1^2 + \phi_2^2}$. Let $V(\phi)$ be inclusive of an additive constant whose source could be the characteristic cut - off mass scale that appears when we renormalise any quantum [matter] field. It could even be an arbitrary integration constant. It is this constant that manifests itself as an arbitrary cosmological constant in the theory.

If the model can consistently support a divergent $U(\phi) \longrightarrow \infty$ at $\mid \phi \mid = 0$, flat space is a solution for $\phi = 0$ for an arbitrary $V(\phi = 0)$. This gets rid of the cosmological constant problem. However, this is a trivial cure as the blowing up of $U(\phi)$ implies a vanishing of the effective gravitational constant, and we all know that, but for gravitation, the additive constant in the effective potential has no dynamical role in physics. What we want is a cure to the problem in the presence of gravitation.

The invariance of the theory under SO(2) rotations in $[\phi_1, \phi_2]$ space implies the conservation of any non - topological charge for a configuration having a compact support on any spacelike hypersurface Σ. The conserved current and the consequent conserved charge are given by:

$$J_\mu = \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 \tag{2.4}$$

$$Q = \int_\Sigma d\Sigma J^o \tag{2.5}$$

In flat spacetime, charge conservation plays a key role in the existence and stability of non - trivial, non - topological soliton solutions [NTS's]. These are the "Q - balls" of Coleman [1985]. For these solutions we have $\phi = \phi(r) = \phi_o$ for radial coordinate r less than some radius R_o, and ϕ quickly going to zero outside this radius. The two regions are separated by a transition zone of thickness independent of the total charge Q. The total charge and the internal energy of the soliton are degenerate for a given volume. The surface energy is proportional to the surface area of the solution. Thus the total energy for a given charge of the solution is minimum for a sphere. There is no limit to the size of these solutions. The size is determined, however, by the total charge of the solution.

We outline properties of "gravitating Q - balls" in the theory described by the action eqn[2.3]. Holding the action stationary under canonical variations of the metric and the scalar field components gives the field eqns[A.1, A.2] in the appendix. We desire a solution in which the non - minimal coupling term $U(\phi)$ blows up at $\phi = 0$. This is not an undesirable feature. Potential solutions to the cosmological constant problem using a dynamically diverging non - minimal coupling has been explored extensively in literature [Dolgov 1982, see also Weinberg 1989]. In any case the standard Brans Dicke theory comes with a linear coupling of the Ricci scalar with ϕ. The theory can be recast in terms of $\psi = \phi^{-1}$ and would then meet the requirement of blowing up at $\psi = 0$. Such a behaviour of $U(\phi)$ ensures that a Ricci flat - and even a Minkowski - spacetime would be a solution to the resulting Einstein equations - irrespective of the magnitude of the contribution from other terms in the stress energy tensor. As stated before, this is what is termed

as a "trivial" solution to the cosmological constant problem. However, if in addition, we have appropriate conditions for the existence of non - topological soliton solutions with $\phi = \phi_o$ in the interior of a large domain, configurations with canonical attractive domains inside the soliton and with zero gravitational and cosmological constants outside would be ensured. We shall not describe solutions to the above equations by assigning any particular functional dependence of $U(\phi)$ and $V(\phi)$. For the present, it is satisfying to demonstrate the possibility of existence of soliton solutions. Taking the cue from the flat space solutions we look for time dependent solutions in which the scalar field rotates in the internal ϕ_1, ϕ_2 space with angular frequency ω

$$\phi_1 = \phi(r)sin(\omega t); \phi_2 = \phi(r)cos(\omega t) \tag{2.6}$$

We look for a spherically symmetric, static metric solution described by the metric:

$$ds^2 = g_{oo}(r)dt^2 - g_{rr}(r)dr^2 - r^2[d\theta^2 + sin^2\theta d\phi^2] \tag{2.7}$$

The configuration that is sought would have $\phi(r)$ locked to an almost constant value inside a spherical domain and transiting to the exterior region across a thin surface. Taking the trace of [A.1] enables us to eliminate the scalar curvature in [A.2]. The expressions for the scalar curvature and the equation for $\phi(r)$ reduce to:

$$R[U - 3U'^2] =$$

$$\frac{1}{2}[T^{\alpha}_{m\alpha} - \omega^2 g^{oo}\phi^2 + g^{rr}(\phi_{,r})^2 - 6g^{rr}\phi_{,r}\phi_{,r}U'' + 6g^{oo}\omega^2\phi U' + 4V(\phi) - 6U'V'] \tag{2.8}$$

$$[1 - \frac{3U'^2}{U}]\nabla^2\phi - \omega^2\phi g^{oo}[1 - \frac{U'\phi}{2U}] + V'$$

$$-\frac{U'}{2U}T^{\alpha}_{m\alpha} - (\phi_{,r})^2 g^{rr}\frac{U'}{U}[\frac{1}{2} - 3U''] - \frac{2U'V}{U} = 0 \tag{2.9}$$

Here $\nabla^2\phi = -(g^{rr}\sqrt{-g}\phi_{,r})_{,r}/\sqrt{-g}$. We can demonstrate the existence of NTS's by considering the following special case for which the effective potential V may have a general convex profile and U' quickly vanishes as one moves away from $\phi = 0$ where it diverges. A simple example that could serve as a prototype for the NMC, that one could use is $U(\phi) = [\alpha + \beta/\phi^2]$. Such an assumption is however not necessary and one could establish existence for more general NMC's. The "overshooting / undershooting" argument described by Coleman [1985] goes through with some alterations. Consider W defined by

$$W'(1 - \frac{3U'^2}{U}) \equiv V' - \frac{2U'}{U}V \tag{2.11a}$$

W has a minimum at $\phi = 0$. We shall consider W to have a profile as outlined in Fig. 1. We shall further impose

$$F(U) \equiv (1 - \frac{3U'^2}{U})^{-1}(\frac{1}{2} - 3U'')\frac{U'}{U} < 0 \tag{2.11b}$$

It is easy to demonstrate that for such a W and U constrained by eq(2.11) NTS's would exist. Indeed, in the flat - space limit, the wave eqn(2.9) for the scalar field reduces to:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + F(U)\frac{d\phi}{dr}^2 = \hat{W}' \tag{2.12}$$

where

$$\hat{W}' \equiv W' + (1 - \frac{3U'^2}{U})^{-1}[-\omega^2\phi(1 - \frac{U'}{2U}) - \frac{U'}{2U}T_m] \tag{2.13}$$

has the same profile as that of W for a sufficiently small ω and a sufficiently small trace of the matter stress energy tensor T_m. To ensure overshooting consider a configuration which has the scalar field fixed in the interior for a large enough radius near $\phi = \phi_a$ so that the second term in eqn(2.12) is small and thereafter shooting off towards $\phi = 0$. The field would surely overshoot $\phi = 0$. This is just the overshoot argument for 'Q - balls' outlined by Coleman [1985] - the third term merely enhances the overshooting on account of the constraint (2.11b) on $F(U)$. (This constraint is sufficient for overshooting. It is not a necessary condition though we have not found an argument that could ensure overshooting in general). The "undershoot" point is simpler to demonstrate. For $\hat{W}$ with the profile of Fig. 1, we have the trivial undershoot point which is the maximum of $\hat{W}$. This establishes the existence of NTS's.

With the scalar field $\mid \phi \mid$ trapped to a uniform value in the interior of a spherical domain, the exact interior solution can be written down immediately. We plug the metric (2.7) into the field equation (A.1) for a uniform scalar field in the interior to give the solution regular in the interior:

$$ds^2 = \frac{\omega^2\phi_o^2}{V(\phi_o)}dt^2 - \frac{dr^2}{1 - Cr^2} - r^2[d\theta^2 + sin^2\theta d\phi^2] \tag{2.14}$$

with $C \equiv V(\phi_o)/4U(\phi_o)$. Expressions for the conserved charge and the internal energy of such a solution follow from (2.5) and (A.16):

$$Q = \omega\phi_o^2 g^{oo} v = vV/\omega \tag{2.15}$$

$$E = 2vV \tag{2.16}$$

Here v is the invariant three volume inside coordinate r. To get the total energy one would also add the surface term which would be proportional to the area of the sphere. In order that the NTS be spherical, we require the surface energy to be positive definite. The total energy for a given charge, without the surface term, is degenerate in volume. With the surface term, the energy would be minimised for the sphere.

Once a NTS is formed with the scalar field arrested at a value ϕ_a in the interior, the configuration would evolve to a state of lower energy by changing ϕ_a. The energy could be transmitted to a changing energy of rotation of the scalar field in internal space and also to the expansion of the wall. A drift of the scalar effective potential $V(\phi)$ to a vanishingly small value while conserving the total charge must be accompanied by the volume v blowing up to infinity. This is the only way one could preserve the metric signature as $V(\phi) \longrightarrow 0$.

The larger the NTS becomes, the longer would be the time expected to synchronise the rotation of the field throughout the interior and the slower would be any further drift to $V(\phi) \longrightarrow 0$. Thus a NTS would approach $V(\phi) \longrightarrow 0$ in infinite time.

Thus in the quasi - static approximation described above, the solutions would approach a flat configuration in the interior. The effective gravitational constant would approach a value $[U(\phi_o)]^{-1}$ irrespective of the initial value that the NTS may be borne with. This would dynamically generate the universality of the induced gravitational constant.

The above Q - ball solution is thus going to be a region with an effective gravitational constant $\approx [U(\phi_o)]^{-1}$ in its interior, flat space with zero cosmological constant in its exterior and an interior effective cosmological constant completely determined by the magnitude of $V(\phi_o)$ and the conserved charge of the soliton. The charge of the soliton is an arbitrary but conserved dynamical parameter. The cosmological constant in the exterior of these domains is strictly zero and the effective cosmological constant approaches zero dynamically. We propose calling such domains : gravity [g] - balls. The interior of these solutions would be regions endowed with canonical gravity.

Section III

The program that we have embarked upon is to look for an ansatz that would allow non - trivial, non - topological soliton [NTS] configurations in which we can have canonical gravitational constant G_{eff} inside compact domains varying in size from a few kpc to say 1Mpc and having the effective gravitation vanishing outside. Material particles would freely stream outside and gravitationally attract each other canonically inside such domains. There is no limit to the size of a NTS. In order to ensure its stability against decay, all one may do is to forbid coupling of the scalar field with matter [Kusenko 1997].

The cosmology that our group is exploring follows an assumption that the universe has a beginning at some time that we may define as t = 0. The wave function of the universe at creation is a highly correlated superposition of all possible states in a flat spacetime. The subsequent evolution is described as an explosion of that state. Any co moving observer would consider itself as a centre of the explosion. This universe would be just an exploding Milne model - but unlike the Milne model, it is non - empty. The metric is just the FRW metric with k = -1 and the scale factor $a(t) = t$. Such a metric has no horizon problem. Further, as the expansion is not constrained by a critical density, there is no flatness problem. The cosmological constant is 'predicted' to vanish identically. The Hubble parameter is precisely the inverse of the age t. Thus the age of the universe inferred from a measurement of the Hubble parameter is 1.5 times the age inferred by the same measurement in SBB. The deceleration parameter is predicted to vanish.

The standard classical cosmological tests, viz.: the number count, angular diameter and the luminosity distance variation with redshift are comfortably consistent in such a cosmology. These results are reported in [Sethi and Lohiya 1996] where particle trajectories inside a G - ball have been studied in detail. A typical G - ball would be a region where a galaxy could form. It is easy to obtain an exact solution to the particle trajectories that follow from the action S (eqn[2.3]) in the thin wall approximation. We have shown that velocity of a typical test particle increases linearly with the distance fron the centre of the

ball. A light ray travels undeflected outside a G - ball but deflects from the boundary of the G - ball. The ball thus acts as a gravitational lens. We are studying constraints on the effective refractive index of a G - ball from observational data on gravitational lensing.

We have explored an account of structure formation in the universe by the formation and successive fragmentation of G - balls in an expanding universe. The non - minimally coupled scalar field is treated as an effective classical field and the universe, at t = 0, is assumed to be trapped at $\phi = 0$. The profile of $\hat{W}$ is expected to change for large variations of T_m. Assuming the universe to be arbitrarily hot with all particles relativistic near $t = 0$, $T_m \approx 0$ initially. The profile of $\hat{W}$ would be as described in Fig. 1. Cooling of the universe with expansion would be accompanied by a peculiar variation of T_m everytime the temperature of the universe transits across the rest mass of any particle species. T_m would start with a small value at ultra - relativistic temperatures, with the particle plus antiparticle density comparable to the photon number, peak near temperature near the rest mass of the species and finally drop drastically at annihilation by a factor n_p/n_γ [the present particle entropy ratio]. One may conjecture that at some such transition the profile of $\hat{W}$ alters to that described in Fig 2. A large condensate - a coherent state of scalar particles would form as the universe would roll away from $\phi = 0$. This is similar in nature to the coherent states of a scalar condensate often encountered in ["new"] inflationary cosmology. As $\hat{W}$ returns back to its original profile, the condensate would become unstable with respect to the formation of true ground state bubbles. Domains comprising of different charge density would develop. As the theory admits NTS's, the lowest energy state in a given charge sector would be a "Q - ball" - which, with the non - minimal coupling that we have considered, would be the gravity balls in our theory. The variation of T_m is not necessary for the above scenario to work. The transition of the profile of $\hat{W}$ from Fig. 2 to Fig. 1 at some early epoch is all that is essential.

The initial distribution of G - balls may be specified by simulating collisions of true ground state bubbles in a first order phase transition. It is easy to visualise that NTS's would be formed by pinching off charged configurations just before the intersection of colliding bubbles of true ground state [Freeman et al 1988]. This gives a distribution of NTS's that lie along surfaces weaving around voids. We find the basic picture quite encouraging and worth developing. To simulate such a distribution, we divide a given spatial region into an $N \times N \times N$ grid. We find a value N = 50 to 100 easy to play with on a 180 MHz computer. With M nucleation sites chosen at random in the region, we explore the grids that are equidistant from a pair of nucleation sites with there being no other site at a smaller distance. The locus of such a grid network is just a 3 - D Voronoi tessellation. The network is just where all the energy of the scalar field would get swept into. Results of typical simulation are presented in Figs. 3. We have made a user friendly software in Visual C(++) that generates the grid - boundary of a Voronoi tessellation using the above ansatz. The program further attributes weights to every point on the grid proportional to the distance from the nearest nucleation site provided its distance from the grid point is the same as that of another nucleation site from the grid point. The weights would be a measure of the surface energy density swept to the wall of an expanding true ground state bubble. The program can further evaluate the autocorrelation of the weight distributions in comparison to a random distribution. We have found the emerging

pictures quite encouraging and we plan to contrast them with known deep supercluster and void surveys.

Assuming that at some (pre - recombination) epoch, a distribution of G - balls materializes in the ϕ meson plasma, a random distribution would keep coasting as the universe expands and give a vanishing two point autocorrelation at large scales. To get a feel for the autocorrelation over smaller scales, we have considered a single ball that successively fragments in an expanding universe into say 1000 fragments - at each fragmentation, conserving energy and momentum. We have written a code that simulates such fragmentation and evaluates two point autocorrelation ψ for the final distribution. We repeatedly find $\psi \approx [r_o/r]^n$ with $n \approx 1.7$ to 1.9 and with r_o being the distance over which the autocorrelation vanishes [Lohiya et al 1997].

We finally announce some very encouraging results in nucleosynthesis in a coasting cosmology. Energy momentum conservation implies that the effective temperature T scales as $a(t)T = tT =$ constant. Taking $T_o = 2.7K$ and $t_o \approx 10^{10}$ years gives the age of the universe of the order of years at $T \approx 10^{10}K$, i.e $T_9 = 10$. The universe takes some 500 years to cool to 10^8K. It can be easily inferred that weak interactions would decouple at $\approx 1.2 \times 10^8K$. This means that the neutron proton ratio keeps falling as $\approx exp[-15/T_9]$. At $T_9 \approx .9$, deuterium burning into other light nuclei becomes more efficient than neutron decay. This would direct any neutrons to the nucleosynthesis channel. Unfortunately there being hardly any neutrons left, the standard analysis would imply virtually no nucleosynthesis. However, as weak interactions are still not decoupled, inverse beta decay would lead to more neutrons getting formed and getting into the nucleosynthesis channel once the above 'deuterium bottleneck' is cleared. We have made elaborate modifications of the standard codes written by Kawano to suit the much stiffer rate equations and find that for a baryon - entropy ratio $\eta \approx 10^{-8}$ we get 23.9% He^4 and metallicity some 10^9 times that obtained in SBB [Batra et al 1996]. This is quite close to the actual metallicity seen in low metallicity interstellar clouds and globular clusters. One of the predictions of the scenario follows from the fact that weak interactions are in equilibrium in the e^+e^- annihilation epoch. This implies the equality of the relic effective neutrino and the microwave background temperatures. The bad news is that we get inadequate D. It is possible to get around this problem in an inhomogenous model with pockets of high neutrino degeneracy $\xi_\nu = -5$ and $\eta \approx 10^{-10}$. Such mixing may arise as a natural fall out from the electroweak phase transition epoch. Another simple idea that we are exploring is the possibility that the interior of the G - balls at this epoch would sustain a higher temperature plasma than the exterior. Once D burning freezes out in the exterior of the G - ball, any D diffusing out from the interior of the ball would survive the external environment. The background Milne model with zero effective gravitational constant can sustain large inhomogeneities both in the temperature as well as baryon density unthinkable in the SBB. Large inhomogeneities that can sustain regions where deuterium burning has frozen out while more neutrons are still forming and converting to D_2, would account for larger D_2 than what we have managed in a homogeneous model.

We are continuing to study aspects of this model. The recombination epoch occurs at $t \approx 10^7$ years. As G - balls support a higher interior temperature, the number of G - balls within a beam width would determine the deviation of CMBR temperature from the

average. Consider a random distribution of G - balls in a large volume. If one samples a small portion of the volume, the rms fluctuation of the number of these balls from the average would be proportional to the inverse square root of the number of balls in the sample. The number of balls being proportional to the sampled volume, we expect such CMBR anisotropy to increase linearly with decreasing beam width. The acoustic peak is expected at six minutes and the Silk angle is 1/60th of the corresponding angle in SBB [Gahlaut and Lohiya 1996]. The Silk mass is some ten times its corresponding value in the standard model. Taking the G - ball size to be less than the Silk length, one would have a typical G - ball emerging out of the last scattering surface at recombination as a gravitating region with a very smooth distribution of mass in its interior. We intend exploring the collapse of this cloud to form galactic structures within the ball.

The most bothersome issues that one has to address are constraints on $U(\phi)$ and $V(\phi)$ that would support such large NTS's. At present we are satisfied by our belief that the fine tuning of $U(\phi)$ and $V(\phi)$ in our model is far milder than constraints that are normally imposed on the scalar effective potential in most realizations of a suitable inflationary model in SBB. In any case a lot of the work that is reported here would hold for any coasting cosmology and is not a unique feature of the model described by eqn[2.3].

III. Conclusion

There are two distinct aspects of the work presented here. First is the viability, advantages and consequences of a coasting cosmology. The second aspect is the possible realisation of such a coasting. The absence of the horizon, flatness and age problems and the possibility of concordance of nucleosynthesis with observations distinguish a coasting cosmology. This is enough motivation for a search of a paradigm that could hold the effective G to a constant value in bounded domains. Such domains can be realised in higher order theories. For example, in a fourth order theory with the action:

$$S = \int d^4x\sqrt{-g}[\alpha C^2 - \beta R]$$

the dynamics of a conformally flat FRW metric is completely determined by the repulsive βR term. Domains of attractive effective gravitation can occur as non - conformally flat perturbations of the FRW metric. Thus the cosmological consequences discussed are not unique features of the model discussed here.

We have demonstrated that in a whole class of two - component scalar tensor theories in which the non - minimal coupling diverges and for which the classical effective potential vanishes at some point, classical scalar field condensates can occur as NTS's. The effective gravitational constant inside all domains would approach a universal value and the effective cosmological constant would drift to zero.

The cosmology described in this article essentially requires the scale factor to coast linearly with time. The particular model outlined here is deficient in several respects. Firstly we have treated the scalar field as purely classical. The stability of the NTS against decay has not been studied. We feel that the peculiar behaviour of the NMC at $\phi = 0$ would prevent any stable ϕ particle states to be defined near $\phi = 0$. Further we have assumed that the expansion of an arbitrary NTS could be followed in a quasi - static approximation along a one parameter family of stable NTS's with a smaller and smaller effective potential V in their interior. This is merely a conjecture and requires to be proved.

We feel that the classical analysis outlined in this article is worth developing. The dynamical tuning of the effective cosmological constant to a small value and the effective gravitational constant to a universal value are compelling features that would keep us on our effort to raise the toy model described here to the status of a viable cosmology.

The idea of exploring NMC for the purpose of getting stress energy of the saclar field condensate to compensate the cosmological constant was also suggested by Dolgov. It was shown however that the NMC itself diverged. It was indeed suggested that a spatial variation of the scalar field - and hence a gravitational constant be explored for a non - trivial model. What we have shown is that in a model where the NMC diverges through most of the space and is finite over compact domains, the compact domains can be expected to inflate. Conservation of charge and energy would then ensure that the cosmological constant approaches zero inside these domains and identically vanishes outside.

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Appendix A

We describe properties of a generalised two component scalar tensor theory and derive expressions for a conserved pseudo energy tensor for the theory. The theory is described by the action:

$$S = \int \sqrt{-g} d^4x [U(\phi_1, \phi_2)R + \frac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - V(\phi_1, \phi_2) + L_m] \tag{2.3}$$

Here $\phi_i, (i = 1, 2)$ are the two components of the scalar field, $U(\phi_i)R$ is a non - minimal coupling of the scalar field with the scalar curvature, and $V(\phi_i)$ is the scalar effective potential. Requiring the action to be stationary under variations of the metric tensor and the fields ϕ_i, gives the equations of motion:

$$U(\phi_i)[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R] = -\frac{1}{2}[T_m^{\mu\nu} + T_\phi^{\mu\nu} + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu}U(\phi)]^{;\lambda}_{;\lambda}] \tag{A.1}$$

$$g^{\mu\nu}\phi^i_{;\mu;\nu} + \frac{\partial V}{\partial \phi^i} - R\frac{\partial U}{\partial \phi^i} = 0 \tag{A.2}$$

Here $T_m^{\mu\nu}$ is the energy momentum tensor of matter constructed from L_m, and

$$T_\phi^{\mu\nu} = \partial^\mu \phi^i \partial^\nu \phi^i - g^{\mu\nu}[\frac{1}{2}\partial^\lambda \phi^i \partial_\lambda \phi^i - V(\phi)] \tag{A.3}$$

We have considered, for the present, a ϕ - independent L_m. It would be reasonable to demand that this theory confirms to the equivalence principle. To demonstrate that this

indeed holds i.e. $T^{\mu\nu}_{m;\nu} = 0$, we note that the contracted Bianchi identity satisfied by the Einstein tensor gives:

$$[U(\phi_i)^{-1}[T^{\mu\nu}_m + t^{\mu\nu}])_{;\nu} = 0 \tag{A.4}$$

with

$$t^{\mu\nu} \equiv T^{\mu\nu}_\phi + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu}U(\phi)^{;\lambda}_{;\lambda} \quad . \tag{A.5}$$

From the equation of motion (A.1) and the Bianchi identity it follows that

$$U(\phi)_{,\nu}[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R] = -\frac{1}{2}[T^{\mu\nu}_{m;\nu} + t^{\mu\nu}_{;\nu}] \tag{A.6}$$

The identity: $U(\phi)^{;\rho}R_{\rho\alpha} = U(\phi)^{;\lambda}_{;\lambda;\alpha} - U(\phi)^{;\lambda}_{;\alpha;\lambda}$ and the equation of motion (A.2) for the fields ϕ_i, together give

$$T^{\mu\nu}_{\phi;\nu} = U(\phi)^{;\mu}R \tag{A.7}$$

which in turn gives the desirable vanishing covariant divergence of the matter stress energy tensor.

One can find the expression for a conserved pseudo energy momentum tensor that would be conserved. To achieve this we proceed to express the vanishing covariant divergence of the matter stress energy tensor as:

$$[\sqrt{-g}T^\nu_{m\mu}]_{,\nu} - \frac{1}{2}g_{\tau\beta,\mu}\sqrt{-g}T^{\tau\beta}_m = 0 \tag{A.8}$$

To cast the lhs of the above equation into a total ordinary divergence one has to seek a representation of the second quantity in terms of an ordinary total divergence. This can be done as follows. First we make use of the equation of motion (A.6) to express the matter stress energy tensor in terms of the scalar field and the metric -dependent quantities: $T^{\tau\beta}_m = -t^{\tau\beta} - 2U(\phi)G^{\tau\beta}$. Second, note that the right hand side of this expression is merely the variational derivative of

$$J \equiv 2\int \sqrt{-g}d^4x[U(\phi)R + L_\phi] \tag{A.9}$$

under variations of the metric tensor, with boundary conditions that require the vanishing of metric and its first derivative variations on the boundary of a (3+1) - dimensional mainfold over which this integral has been taken. For more general variations, one would get contributions from the surface integrals as well. We consider the standard decomposition of $\sqrt{-g}R$ into a pure divergence term and a simple expression involving only the metric and its first derivatives:

$$\sqrt{-g}R = \mathrm{A} + [\sqrt{-g}g^{\sigma\rho}\Gamma^\alpha_{\sigma\alpha}]_{,\rho} - [\sqrt{-g}g^{\sigma\rho}\Gamma^\alpha_{\sigma\rho}]_{,\alpha} \tag{A.10}$$

with

$$\mathrm{A} \equiv \sqrt{-g}g^{\sigma\rho}[\Gamma^\alpha_{\sigma\rho}]\Gamma^\beta_{\alpha\beta} - \Gamma^\alpha_{\beta\rho}]\Gamma^\beta_{\alpha\sigma}] \tag{A.11}$$

It follows that the functional derivative of J with respect to the metric tensor is the same as that of

$$H \equiv \int d^4x[\mathrm{B} + \sqrt{-g}L_\phi] \tag{A.12}$$

where

$$\mathrm{B} \equiv [U\mathrm{A} - \sqrt{-g}g^{\sigma\rho}\Gamma^\alpha_{\sigma\alpha}U_{,\rho} + \sqrt{-g}g^{\sigma\rho}\Gamma^\alpha_{\sigma\rho}U_{,\alpha}] \tag{A.13}$$

In other words

$$\sqrt{-g}UG_{\mu\nu} + \sqrt{-g}[U_{;\mu;\nu} - g_{\mu\nu}U^{;\alpha}_{;\alpha}] + \frac{1}{2}\sqrt{-g}T_{(\phi)\mu\nu}$$

$$= \frac{\partial}{\partial g^{\mu\nu}}[\mathrm{B} + \sqrt{-g}L_\phi] - [\frac{\partial(\mathrm{B} + \sqrt{-g}L_\phi)}{\partial g^{\mu\nu}_{,\lambda}}]_{,\lambda} \tag{A.14}$$

We define $\hat{\mathrm{B}} \equiv \mathrm{B} + \sqrt{-g}L_\phi$. The expression for the ordinary derivative of $\hat{\mathrm{B}}$ and the field equation for the scalar field ϕ then easily enable us to express the second term in eqn(A.8) as a total divergence. This gives:

$$[\sqrt{-g}T^\nu_{m\mu} - \hat{\mathrm{B}}\delta^\nu_\mu - \frac{\partial\hat{\mathrm{B}}}{\partial g^{\tau\beta}_{,\nu}}g^{\tau\beta}_{,\mu} - \frac{\partial\hat{\mathrm{B}}}{\partial\phi_{,\nu}}\phi_{,\mu}]_{,\nu} = 0 \tag{A.15}$$

For $\nu = o$ the expression within the brackets integrated over a spacelike hypersurface is thus invariant under time translations for a distribution of matter and the rest of the terms in (A.15) having a compact support over the surface. This is the expression for the pseudo energy momentum tensor that we seek. The quantity

$$P_\mu \equiv \int_\Sigma d\Sigma[\sqrt{-g}T^o_{m\mu} - \hat{\mathrm{B}}\delta^o_\mu - \frac{\partial(\hat{\mathrm{B}}}{\partial g^{\tau\beta}_{,o}}g^{\tau\beta}_{,\mu} - \frac{\partial\hat{\mathrm{B}}}{\partial\phi_{,o}}\phi_{,\mu}] \tag{A.16}$$

evaluated on a constant spacelike hypersurface Σ, is thus conserved. This may be viewed as the generalisation of the energy momentum four vector for the scalar - tensor theory described by eqn[2.3]. The formalism presented here is general and can be used to determine the energy momentum four vector for any Brans - Dicke theory in particular. As in standard general relativity, P_μ is not a generally covariant four vector as A and B are not scalar densities. The intrinsic non - covariance of the energy momentum density of the gravitational field has its origin in the intimate connection between geometry and the gravitational field. Had the expression been covariant, one could always have gone into a preferred [freely - falling] frame to ensure vanishing of an arbitrary localised gravitational field.

The above form for the energy momentum pseudo-tensor for the generalised Brans - Dicke theory can also be obtained by considering a variation of the coordinate system instead of the metric field. The analysis enables us to express the gravitational energy - momentum in a very compact form. To demonstrate this, we consider

$$H = \int \hat{\mathrm{B}}d^4x \tag{A.17}$$

$\hat{\mathrm{B}}$ is a function of the metric, the scalar field and their first derivatives. Its variation is:

$$\delta\hat{\mathrm{B}} = \frac{\partial\hat{\mathrm{B}}}{\partial g^{\mu\nu}}\delta g^{\mu\nu} + \frac{\partial\hat{\mathrm{B}}}{\partial g^{\mu\nu}_{\lambda}}\delta g^{\mu\nu}_{\lambda} + \frac{\partial\hat{\mathrm{B}}}{\partial\phi}\delta\phi + \frac{\partial\hat{\mathrm{B}}}{\partial\phi_{,\lambda}}\delta\phi_{,\lambda} \tag{A.18}$$

Consider a change of coordinates of the form:

$$\hat{x}^{\alpha} = x^{\alpha} + \epsilon\xi^{\alpha} \tag{A.19}$$

Retaining terms to the first order in ϵ, we get the following variations:

$$\delta g^{\mu\nu} = \epsilon(\xi^{\mu}_{,\alpha}g^{\alpha\nu} + \xi^{\nu}_{,\alpha}g^{\alpha\mu}) \tag{A.20}$$

$$\frac{\partial x^{\alpha}}{\partial\hat{x}^{\lambda}} = \delta^{\alpha}_{\lambda} - \epsilon\frac{\partial\xi^{\alpha}}{\partial x^{\lambda}} + O(\epsilon^2) \tag{A.21}$$

$$\delta g^{\mu\nu}_{,\lambda} = \epsilon[g^{\tau\nu}_{,\lambda}\xi^{\mu}_{,\tau} + g^{\mu\beta}_{,\lambda}\xi^{\nu}_{,\beta} - g^{\mu\nu}_{,\alpha}\xi^{\alpha}_{,\lambda} + g^{\tau\nu}\xi^{\mu}_{,\tau,\lambda} + g^{\tau\mu}\xi^{\nu}_{,\tau,\lambda} \tag{A.22}$$

$$\delta\sqrt{-g} = -\epsilon\sqrt{-g}\xi^{\alpha}_{\alpha} \tag{A.23}$$

$$\delta\phi = 0$$

$$\delta(\phi_{,\lambda}) = -\epsilon\phi_{,\alpha}\xi^{\alpha}_{,\lambda} \tag{A.24}$$

A restriction to linear transformations enables one to get an elegant form for $\delta\hat{\mathrm{B}}$. The Christoffel symbols transform as tensors under such transformations and hence $\hat{\mathrm{B}}$ as a scalar density. Thus

$$\delta\hat{\mathrm{B}} = \frac{\hat{\mathrm{B}}}{\sqrt{-g}}\delta\sqrt{-g} = -\epsilon\xi^{\alpha}_{,\alpha}\hat{\mathrm{B}} \tag{A.25}$$

Substituting the variations (A.20) to (A.24) for an arbitrary linear transformation into (A.18) and comparing the expression with (A.25) yields the identity:

$$\frac{\partial\hat{\mathrm{B}}}{\partial g^{\mu\nu}}g^{\alpha\nu} + \frac{\partial\hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,\lambda}}g^{\alpha\nu}_{,\lambda} - \frac{1}{2}\frac{\partial\hat{\mathrm{B}}}{\partial g^{\beta\nu}_{,\alpha}}g^{\beta\nu}_{,\mu} - \frac{\partial\hat{\mathrm{B}}}{\partial\phi_{,\alpha}}\phi_{,\mu} = -\frac{1}{2}\hat{\mathrm{B}}g^{\alpha}_{\mu} \tag{A.26}$$

The use of this identity yields a simple expression for the variation of $\hat{\mathrm{B}}$ under the general transformation (A.19):

$$\delta\hat{\mathrm{B}} = -\epsilon\hat{\mathrm{B}}\xi^{\alpha}_{,\alpha} + 2\epsilon\frac{\partial\hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,\lambda}}\xi^{\mu}_{,\tau,\lambda}g^{\tau\nu} \tag{A.27}$$

For transformations that have ξ together with its derivatives vanishing on the boundary, the variation of the metric tensor and its derivatives also vanish on the boundary of the manifold. H has a vanishing variation under such a transformation. The null variation of H:

$$\delta H = \int_{\Sigma}\delta(\frac{\hat{\mathrm{B}}}{\sqrt{-g}})\sqrt{-g}d^{(D+1)}x = 0 \tag{A.28}$$

reduces to

$$\delta H = 2\epsilon \int_{\Sigma} \frac{\partial \hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,\lambda}} \xi^{\mu}_{,\tau,\lambda} g^{\tau\nu} d^{(D+1)}x = 0 \qquad (A.29)$$

This may be integrated by parts twice. For the variation to vanish for arbitrary ξ^{μ} we get the following divergence law:

$$(\frac{\partial \hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,\lambda}} g^{\tau\nu})_{,\tau,\lambda} = 0 \qquad (A.30)$$

Thus

$$\sqrt{-g} F^{\tau}_{\mu} \equiv (\frac{\partial \hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,\lambda}} g^{\tau\nu})_{,\lambda} \qquad (A.31)$$

is a conserved quantity. Using the identity (A.26) and the field equation (A.1) gives:

$$\sqrt{-g} F^{\tau}_{\mu} = -\sqrt{-g} T^{(m)\tau}_{\mu} - \frac{1}{2}\hat{\mathrm{B}} g^{\tau}_{\mu} + \frac{1}{2}(\frac{\partial \hat{\mathrm{B}}}{\partial g^{\beta\nu}_{,\tau}} g^{\beta\nu})_{,\mu} - \frac{1}{2}\frac{\partial \hat{\mathrm{B}}}{\partial \phi_{,\tau}} \phi_{,\mu} \qquad (A.32)$$

This is just the expression that we had obtained for the stress energy pseudo - tensor by the variation of the metric tensor earlier. The expression in terms of a vanishing ordinary divergence (A.30) implies that

$$P_{\mu} \equiv -\int_{V} (\frac{\partial \hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,\lambda}} g^{o\nu})_{,\lambda} dV \qquad (A.33)$$

is a conserved quantity if V is the entire space at a given time. In the special case of a time independent metric, Gauss's theorem in 4 dimensions gives the energy momentum as surface integral over a 2 dimensional surface:

$$P_{\mu} = -\int_{\Sigma} (\frac{\partial \hat{\mathrm{B}}}{\partial g^{\mu\nu}_{,j}} g^{o\nu}) d\Sigma_j \qquad (A.34)$$

This gives a curious result that in the generalised Brans-Dicke theory, the generalised energy momentum in a D dimensional volume can be determined by the metric tensor field and its derivatives on the (D-1) - dimensional surface, the details of the field inside the volume being irrelevant.

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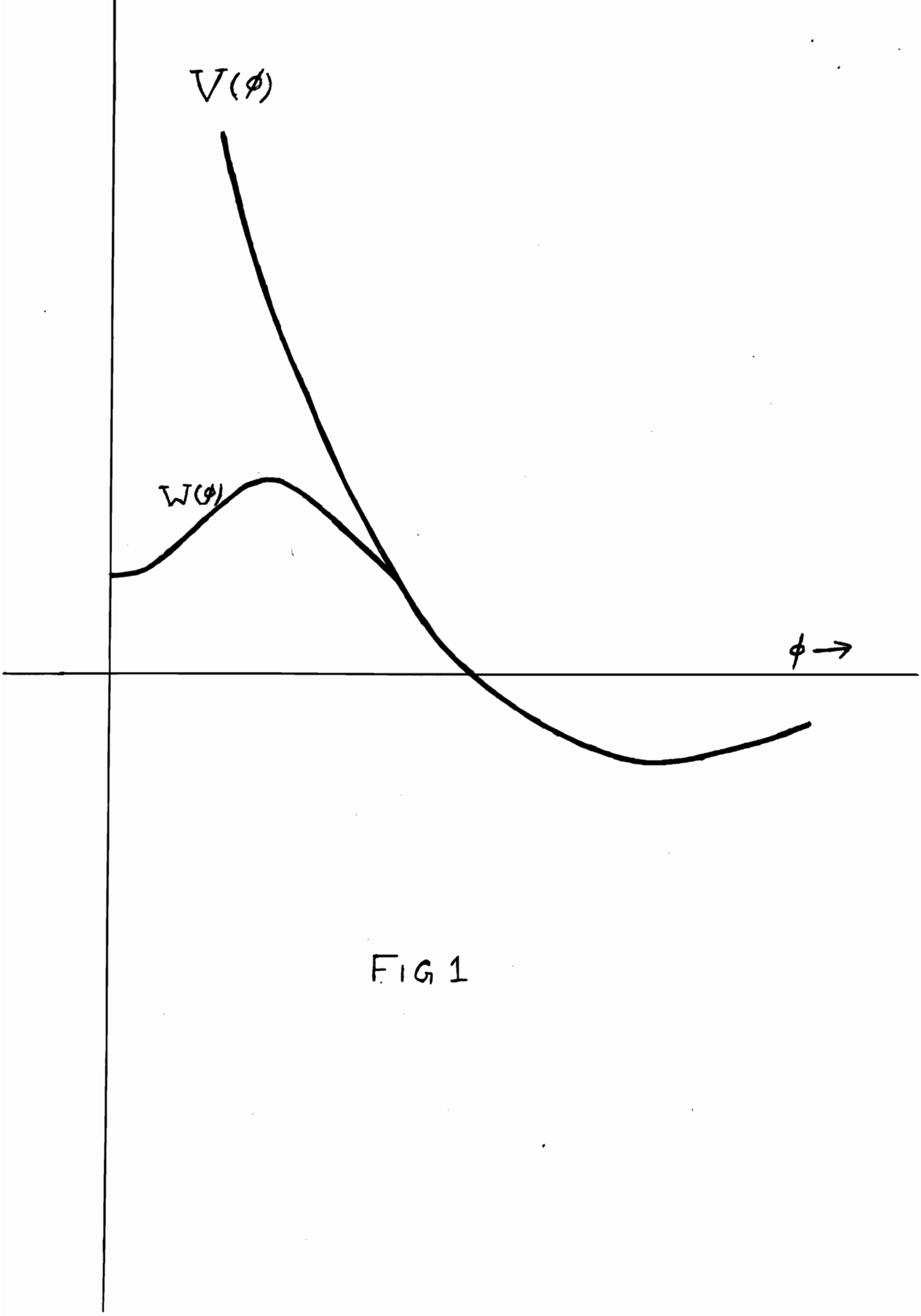

FIG 1

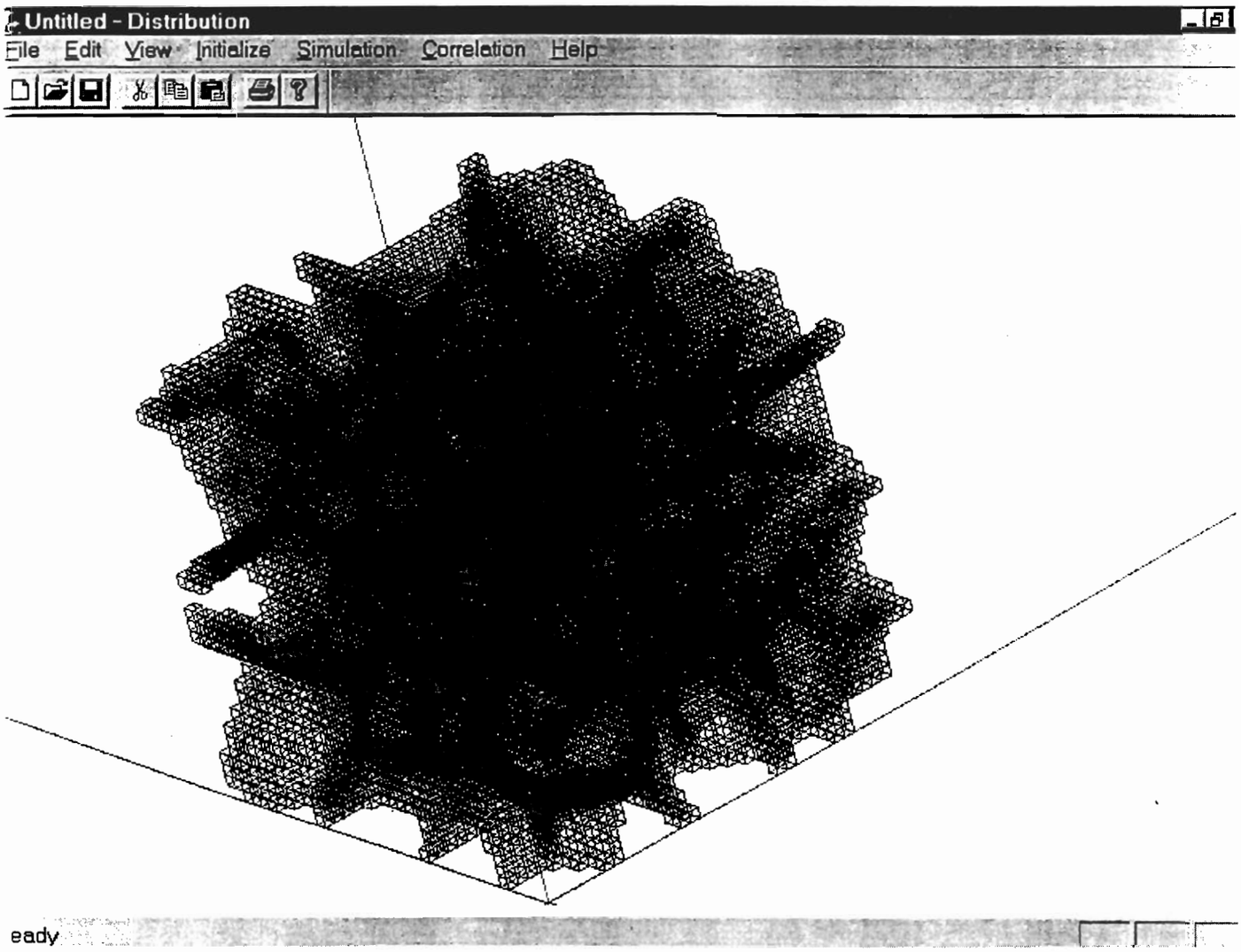
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FIG 2

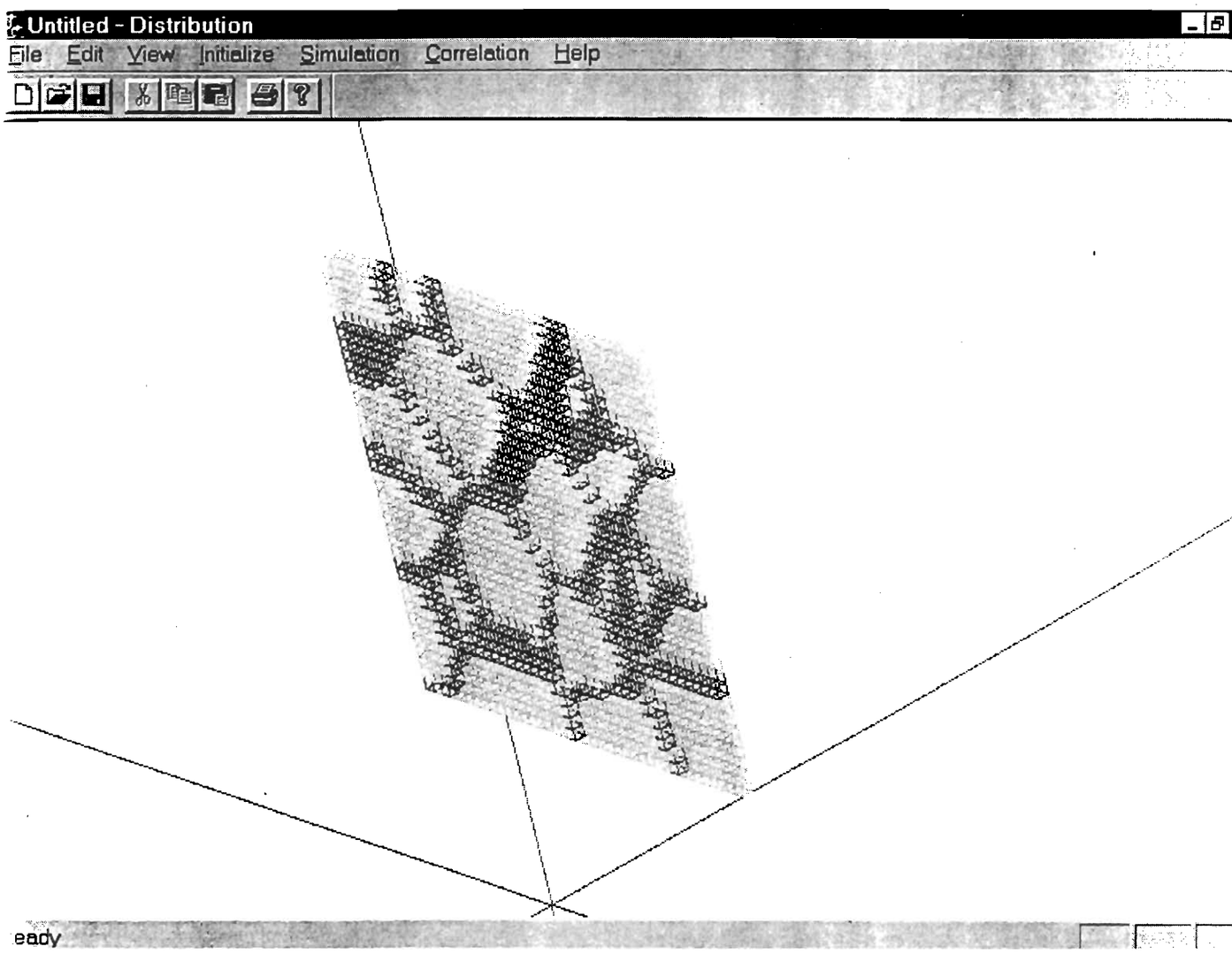
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FIG 3