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Domain Walls in Brans-Dicke Theory

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Abstract

It is shown that a thick domain wall with planar symmeby having $T_0^0 = T_2^2 = T_3^3 \neq 0$ and $T_1^1 = 0$ cannot remain in static equilibrium in Brans -Dicke theory of gravitation.

Introduction

The topological defects occur through a series of phase transitions [1] in the early Universe. One such defect is a domain wall, whcih is formed when a discrete symmetry is spontaneously broken. Recently the study of domain walls with finite thickness has gained renewed interest [2,3,4] because of the proposal for a new scenario of galaxy formation due to Hill, Schramm and Fry [5]. Since the domain walls associated with such a late time phase transition, are light compared with those associated with a GUT phase transition, they do not dominate the energy density of the Universe. The gravitational field of infinitely thin walls has been calculated by Vilenkin [6], Ipser and Sikivi [7], whereas the walls mentioned in [2-4] maybe said to be thick. In these thick walls the metric is time dependent, whereas the scalar field is static.

For the thick domain wall in curved spacetime we assume the wall to have planar symmetry with two commuting killing vectors describing transational invariance in the plane parallel to the wall and a third Killing vector related to a rotational rymmetry about the x-axis perpendicular to the wall. It is symmetric about $x = 0$ plane. The lagrangian for the scalar field is taken to be $L = \frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu - V(\phi)$, where ϕ is assumed to be a function of x alone. It has been shown by Widrow that for such a field purely static metric exhibits unphysical behaviour at large $|x|$, so long as $V(\phi)$ is positive.

One may wonder if it is possible to obtain such a static thick domain wall in the framework of Brans Dicke theory [8], which has gained renewed interest in the context of the socalled extended inflation [9]. In the first step are considered $T^0_0 = T^2_2 = T^3_3 \neq 0$ and $T^1_1 = 0$. This form of the energymomentum tensor has been used by Raychaudhuri and Mukherjee [10], who had shown in a general way that such walls in Einstein's theory cannot remain in static equilibrium.

In this note it shown from the field equations in a straight forward manner that such static domain wall cannot exist even in Brans Dicke theory as well.

Field Equations for a Static domain wall in Brans-Dicke Theory

We choose the general static metric with planar symmetry in the following form.

$$ds^2 = e^{2\alpha}dt^2 - e^{2\beta}dx^2 - e^{(\beta-\alpha)}(dy^2 + dz^2), \tag{1}$$

where α and β are functions of x alone. The energy momentum tensor for a static scalar field mentioned above can be obtained in a straightforward manner as given below

$$T_0^0 = T_2^2 = T_3^3 = \rho = \frac{1}{2}e^{-2\beta}\phi_1^2 + V(\phi) \tag{2}$$

and

$$T_1^1 = -\frac{1}{2}e^{-2\beta}\phi_1^2 + V(\phi) \tag{3}$$

Our field equations in Brans Dicke theory will be

$$\alpha_{11} = \frac{8\pi}{\psi}[T_0^0 - \frac{w+1}{2w+3}T]e^{2\beta} - \alpha_1\frac{\psi}{\psi} \tag{4}$$

$$\beta_{11} + \frac{1}{2}(3\alpha_1 + \beta_1)(\alpha_1 - \beta_1) = \frac{8\pi}{\psi}[T_1^1 - \frac{w+1}{2w+3}T]e^{2\beta} - \frac{w\psi_1^2}{\psi^2} + \beta_1\frac{\psi_1}{\psi} - \frac{\psi_{11}}{\psi} \tag{5}$$

$$\frac{1}{2}(\alpha_{11} - \beta_{11}) = -\frac{8\pi}{\psi}[T_2^2 - \frac{w+1}{2w+3}T]e^{2\beta} + \frac{1}{2}(\beta_1 - \alpha_1)\frac{\psi_1}{\psi} \tag{6}$$

Here ψ stands for the Brans Dicke scalar field and the subscript '1' indicates derivative with respect to x- cordinate. Considering the case for $T_1^1 = 0$, the Bianchi indentity $T_{1;\mu}^{\mu} = 0$ leads to $\beta_1 = 0$ for a nontrivial situation. The equation (3) can now be written as

$$\alpha_{11} + \frac{\alpha_1\psi_1}{\psi} = \frac{8\pi\rho}{\psi}[1 - \frac{3(w+1)}{(2w+3)}]^{e^{2\beta}} \tag{7}$$

and the equation (5) reduces to

$$\frac{1}{2}(\alpha_{11} + \frac{\alpha_1 \psi_1}{\psi}) = -\frac{8\pi\rho}{\psi}[1 - \frac{3(w+1)}{(2w+3)}]e^{2\rho} \tag{8}$$

The equations (6) and (7) are clearly inconsistent showing that a static domain wall with planar symmetry and $T_1^1 = 0$ does not exist in Brans Dicke theory. It is to be seen, however, if the result alters when one assumes $T_1^1 \neq 0$. The work in this direction is in progress.

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