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EVOLUTION OF INTRA-CAVITY FIELDS AT NON-STEADY STATE IN DUAL RECYCLED INTERFEROMETER

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Abstract

We describe how exactly the intra-cavity fields in a dual recycling cavity builds up its power before achieving a steady state value. We present the complete series representation of the intra-cavity light at any stage of evolution in its non-steady state.

Laser interferometric gravitational wave observatories[1] are currently being designed or constructed in several countries (LIGO, VIRGO, GEO, AIGO). The interferometer, in essence, is a rather elaborate transducer from optical path difference to output power. So, monitoring the change in output power, it would be possible to detect the changing curvature of spacetime induced by the passage of a gravitational wave. In order to reduce the shot noise level, an integral feature of these detectors will be the use of high power laser, in conjunction with variants of optical technique known as light recycling, an idea first proposed by Drever[2]. The first one of these techniques is called *power recycling*, in which, at a dark fringe operation of the interferometer, the outgoing laser light is recycled back into the interferometer by putting a mirror in front of the source, thus enhancing the laser power. When a gravitational wave passes through an interferometer, it modulates the phase of the laser light, thus producing sidebands which travel towards the photodetector [3]. So, another variant of optical technique called *dual recycling* incorporates a second recycling

mirror placed in front of the photodetector as shown in Fig.1. This arrangement can store signal sidebands for sufficiently long times to allow optimum photon noise sensitivity within a restricted bandwidth [4].

In this letter, we describe, how with the help of *Mathematica*, one can do calculation for the evolution of intra-cavity fields in the two recycling cavities of a dual recycled interferometer and finally achieve the expressions for the intra-cavity fields at the steady state condition. We present an exact series representation of the intra-cavity fields at any stage of their evolution before achieving the steady state. Hopefully, the following *Mathematica* exercise would be able to provide an alternative way of understanding the operation of the dual recycled cavity. Also, as pointed out at the end of this letter, this way of calculating transient power may find its application in the calculations of the power build-up in a dynamically tuned dual recycling interferometer, a technique suggested by Meers, Krolak and Lobo [5] for enhancing the sensitivity of observation of the *chirp* waveform of gravitational radiation from coalescing compact binary stars.

We first introduce the notations used to describe various components of the optical arrangement and finally describe the calculation.

Mirrors:

A mirror or beam-splitter (BS) is usually a layered dielectric medium on a substrate , which partially transmits and partially reflects incident light as shown in Fig.2. In all the following figures, rays of light labelled by a, b, x, y represent the complex amplitude of the beam's electromagnetic field. In this exercise, we make the following assumptions on the mirrors: (i) these are lossless, (ii) the substrate and the dielectric layers are linear media, (iii) mirrors are time-reversal invariant, (iv) any mirror has a reflection symmetry that exchanges inputs and outputs on the same side of the mirror, (v) there is no differential time delay between the transmitted and reflected lights and also the overall time delay across the mirror is negligible.

One can then arrive at the following simple input-output mirror relation:

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} t & -r \\ r & t \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}, \tag{1}$$

where t and r represent the transmission and reflection coefficients of the mirror. For a lossless mirror, $| t |^2 + | r |^2 = 1$. We consider all the beam-splitters to be of 50:50 type.

Fabry-Perot Cavity:

This can be simply described as consisting of two mirrors facing each other. The flowchart of the light paths has been shown in Fig.3. Light beams a_0 and b_0 enter the cavity from corner mirror (CM) and end mirror (EM) sides respectively. The beam x

represents the intra-cavity field that grows up in every bounce it gets from CM or EM, provided it adds up in phase with the incoming light there. The index j represents the number of round-trips of the x-field inside the cavity. The circle with the quantity L inside it represents the change in phase of the intra-cavity light due to the traversal of cavity of length L.

After a sufficient number of bounces, the intra-cavity field, x, achieves a steady state when it stops growing any more (i.e., the change after any bounce would be negligibly small). Such a state is described in Fig.4. At this condition the cavity 'reflection coefficient' can be easily calculated [6] by applying appropriate mirror relations and taking care of phase changes in the light fields as shown in Fig.4.

Dual Recycling Cavity:

The corresponding flowchart for a single delay-line dual-recycled cavity has been shown in Fig.5. To understand this figure one may also refer to Fig.1. The circle with the quantity l_1 or l_2 inside it represents the change in phase of the intra-cavity light due to traversal of power recycling cavity (PRC) of length l_1 between the beam-splitter (BS) and the power recycling mirror (PRM) or Signal Recycling Cavity (SRC) of length l_2 between BS and signal recycling mirror (SRM) respectively. The electromagnetic field a_0 represents the coherent beam of laser light entering through PRM. The field b_0 is in vacuum state in ordinary interferometers, but as suggested by Caves[7], in advanced interferometers, this may be made to be in squeezed vacuum state to reduce the shot noise. The fields x and y represent intra-cavity light beams in the Power Recycling Cavity (PRC) and Signal Recycling Cavity (SRC) respectively. The box with the label 'Propagation in Arms' represents the matrix relation:

$$\begin{pmatrix} x_3[j] \\ y_3[j] \end{pmatrix} = \exp\left[\frac{i2\omega L_a}{c}\right] \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{+i\theta} \end{pmatrix} \begin{pmatrix} x_2[j] \\ y_2[j] \end{pmatrix}, \tag{2}$$

where $k_0 = \omega_0/c$ and ω_0, L_a, c are the circular frequency of light beam, the arm length of the interferometer and the velocity of light respectively. The quantity θ represents the differential phase shift of light in the two arms of the interferometer. The end mirrors of dual recycling cavity are assumed to be perfectly reflecting ones.

A small programme in *Mathematica* can be easily written to implement this recursive procedure. After a sufficient number of bounces, j (say, 10), one can look at the expressions for $x_4[j]$ and $y_4[j]$. We have set $x_0[0] = y_0[0] = 0$ in the final expressions of $x_4[j]$ and $y_4[j]$ after finishing the recursive calculation because the initial intra-cavity fields were zero. Now, grouping the related terms together, we can guess the formation of a series in each case:

$$
\begin{aligned}
x_4[j] =& b_0 \Bigg[i t_2 Q_0 \sin\theta \sum_{k=0}^{K+1} (-r_1 r_2 Q_0^2 \sin^2\theta)^k \\
& \times \sum_{n=1}^{N} \{{}^{n+k-1}C_k\} (r_1 Q_1 \cos\theta)^{n-1} \sum_{m=1}^{M} \{{}^{m+k-1}C_k\} (r_2 Q_2 \cos\theta)^{m-1} \Bigg] \\
& + a_0 \Bigg[t_1 Q_1 \cos\theta \sum_{p=1}^{j} (r_1 Q_1 \cos\theta)^{p-1} - t_1 r_2 Q_0^2 \sin^2\theta \sum_{k=0}^{K} (-r_1 r_2 Q_0^2 \sin^2\theta)^k \\
& \times \sum_{n=1}^{N} \{{}^{n+k}C_{k+1}\} (r_1 Q_1 \cos\theta)^{n-1} \sum_{m=1}^{M} \{{}^{m+k-1}C_k\} (r_2 Q_2 \cos\theta)^{m-1} \Bigg], \qquad (3a)
\end{aligned}
$$

$$
\begin{aligned}
y_4[j] =& b_0 \Bigg[t_2 Q_2 \cos\theta \sum_{p=1}^{j} (r_2 Q_2 \cos\theta)^{p-1} - t_2 r_1 Q_0^2 \sin^2\theta \sum_{k=0}^{K} (-r_1 r_2 Q_0^2 \sin^2\theta)^k \\
& \times \sum_{n=1}^{N} \{{}^{n+k-1}C_k\} (r_1 Q_1 \cos\theta)^{n-1} \sum_{m=1}^{M} \{{}^{m+k}C_{k+1}\} (r_2 Q_2 \cos\theta)^{m-1} \Bigg] \\
& + a_0 \Bigg[i t_1 Q_1 \sin\theta \sum_{k=0}^{K+1} (-r_1 r_2 Q_0^2 \sin^2\theta)^k \\
& \times \sum_{n=1}^{N} \{{}^{n+k-1}C_k\} (r_1 Q_1 \cos\theta)^{n-1} \sum_{m=1}^{M} \{{}^{m+k-1}C_k\} (r_2 Q_2 \cos\theta)^{m-1} \Bigg], \qquad (3b)
\end{aligned}
$$

where

$$
\begin{aligned}
Q_0 &= \exp[i k_0 (2L_a + l_1 + l_2)] \\
Q_1 &= \exp[i k_0 (2L_a + 2l_1)] \qquad (3c) \\
Q_2 &= \exp[i k_0 (2L_a + 2l_2)]
\end{aligned}
$$

are the phase factors which satisfy $Q_0^2 = Q_1 Q_2$. The upper limits of the integers k, n and m are represented by K, N, and M respectively. For any value of j, we obtain all the terms in the series that are characterised by sets of values of these limits satisfying the following relationship:

$$
j = 2K + M + N. \qquad (3d)
$$

Interested readers may like to know the physical interpretation of all the terms appearing in series (3a) and (3b). One may note that any light gets multiplied by either $(i \sin\theta)$ or $(\cos\theta)$ depending on whether it changes its cavity (from PRC to SRC and *vice versa*) or not after passing through the BS and one arm.

For example, we may consider the series in the second term of the coefficient of a_0 in the intra-cavity field x_4[steady]. Any term in this series represent the contribution of a_0 light in x_4 after it completed n number of round trips in PRC and $(k+m)$ number of round trips in SRC. The number of possible ways of completing such a trip would be

$$\{{}^{n+k}C_{k+1}\}\{{}^{m+k-1}C_k\}.$$

As an easy example, let us take the contribution corresponding to $k=0$, $n=2$, $m=1$:

$$-2r_1r_2t_1\cos\theta\sin^2\theta,$$

which may be expanded as

$$(r_1\cos\theta)(i\sin\theta)r_2(i\sin\theta)t_1+(i\sin\theta)r_2(i\sin\theta)(r_1\cos\theta)t_1.$$

This means that this term has been contributed by two parts of a_0 light which made two round trips of PRC and one of SRC in the following two ways:

(i) →PRM; transmitted inside →BS, arm →SRC →SRM; reflected →BS, arms →PRC →PRM; reflected →BS, arm →PRC.

(ii) →PRM; transmitted inside →BS, arm →PRC →PRM; reflected →BS, arms →SRC →SRM; reflected →BS, arm →PRC.

The intra-cavity fields, x_4 and y_4 achieve their steady state value as j tends to infinity. Now, identifying

$$\sum_{n=1}^{\infty}\{{}^{n+k-1}C_k\}(r_1Q_1\cos\theta)^{n-1}=\frac{1}{(1-r_1Q_1\cos\theta)^{k+1}}, \tag{4}$$

etc. and after doing some algebraic manipulations, we finally arrive at the steady state expressions:

$$x_4[\text{steady}]=x_4[j\to\infty]=\frac{1}{M}[a_0t_1Q_1(\cos\theta-r_2Q_2)+ib_0t_2Q_0\sin\theta], \tag{5a}$$

$$y_4[\text{steady}]=y_4[j\to\infty]=\frac{1}{M}[ia_0t_1Q_0\sin\theta+b_0t_2Q_2(\cos\theta-r_1Q_1)], \tag{5b}$$

where

$$M=1-(r_1Q_1+r_2Q_2)\cos\theta+r_1r_2Q_1Q_2. \tag{6}$$

The steady-state expressions (5) can, of course, be calculated very easily by assuming that, under steady-state condition, the change in the intra-cavity fields after every round-trip of

the cavities is negligibly small [8]. One can then write the following two equations which can be solved to obtain the above-mentioned expressions.

$$\begin{aligned} x_4[\text{steady}] =& a_0 t_1 Q_1 \cos\theta + x_4[\text{steady}] r_1 Q_1 \cos\theta \\ &+ i b_0 t_2 Q_0 \sin\theta + i y_4[\text{steady}] r_2 Q_0 \sin\theta, \end{aligned} \tag{6a}$$

$$\begin{aligned} y_4[\text{steady}] =& b_0 t_2 Q_2 \cos\theta + y_4[\text{steady}] r_2 Q_2 \cos\theta \\ &+ i a_0 t_1 Q_0 \sin\theta + i x_4[\text{steady}] r_1 Q_0 \sin\theta. \end{aligned} \tag{6b}$$

The intra-cavity fields can achieve their maximum values at the resonance condition of the dual recycling cavity, i.e., when $l_1 = l_2$ and $Q_0 = Q_1 = Q_2 = 1$. This implies that the intra-cavity fields x_4 and y_4 add up in phase with the incoming light. For a sufficiently low value of r_2, this condition also represents *the broadband mode of operation* of a dual recycled cavity [4]. The expressions for the case of power recycling (when no SRM is used) can be easily obtained by setting $r_2 = 0$. Similar exercise can of course be carried out for interferometers incorporating Fabry-Perot cavities in the arms.

If a monochromatic gravitational wave hits the interferometer, the laser light gets phase modulated [3] and new modes of frequencies $\omega_0 \pm n\omega_g$ (n being any nonzero integer) are created. Of these, only the first two sidebands of frequencies $\omega_0 \pm \omega_g$ are of importance, since the sideband amplitudes are of order $\sim h^n$; h being the amplitude of the gravitational wave which is quite small. For exactly equal armlengths of the interferometer ($\theta = 0$), the dual recycling cavity, at any frequency, gets decoupled into two equivalent three mirror cavities. If $Q_1 = 1$, the laser light, a_0 becomes resonant with the three mirror power recycling cavity (PRM and two EMs), whereas the sidebands of laser light travel down the SRC. One can make one of these sidebands (say, $\omega_0 + \omega_g$) resonant with the three mirror signal recycling cavity (SRM and two EMs) by adjusting l_2, such that $Q_2 \exp[+i\omega_g(2L_a + 2l_2)] = 1$. This is called *the narrowband mode of operation.* Obviously, under this mode of operation, neither the b_0 light nor its sidebands (which, after getting created, travel down the PRC) can become resonant with any of the cavities.

Gravitational radiation from a coalescing compact binary system has a *chirp* waveform which continually increases in frequency and amplitude as the two stars spiral in towards each other. To observe this broadband waveform, dynamical dual recycling technique[5] was proposed, which incorporated two complementary techniques to improve the sensitivity of observation: (i) narrow band observation to achieve high signal to noise ratio at a given frequency, and (ii) tracking the chirp frequency by adjusting the tuning frequency by changing l_2 as long as the signal lies within the detector bandwidth. To implement this technique, it is very important to understand how exactly the signal (i.e., the sideband

that is tracked down) builds up its power in the presence of errors in tuning the SRC. A *Mathematica* programme simulating this technique in the way described above, therefore, may come to be handy and useful.

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Figure Caption

FIG.1 : Optical arrangement for dual recycling. (BS) - Beam Splitter, (EM) - End Mirror, (PRM) - Power Recycling Mirror, (SRM) - Signal Recycling Mirror, (PD) - Photodetector

FIG.2 : Mirror

FIG.3 : Flowchart of a Fabry-Perot cavity at non-steady state. CM - Corner Mirror, EM - End Mirror. The circle with L inside represents phase change due to traversal of cavity length, L.

FIG.4 : Steady-state of a Fabry-Perot cavity.

FIG.5 : Flowchart of a single delay-line dual recycled cavity at non-steady state. PRM - Power Recycling Cavity, SRM - Signal Recycling Cavity, BS - Beam Splitter. Circle with l_1 or l_2 inside represents phase changes due to traversal of length, l_1 or l_2 between BS and PRM or SRM respectively.

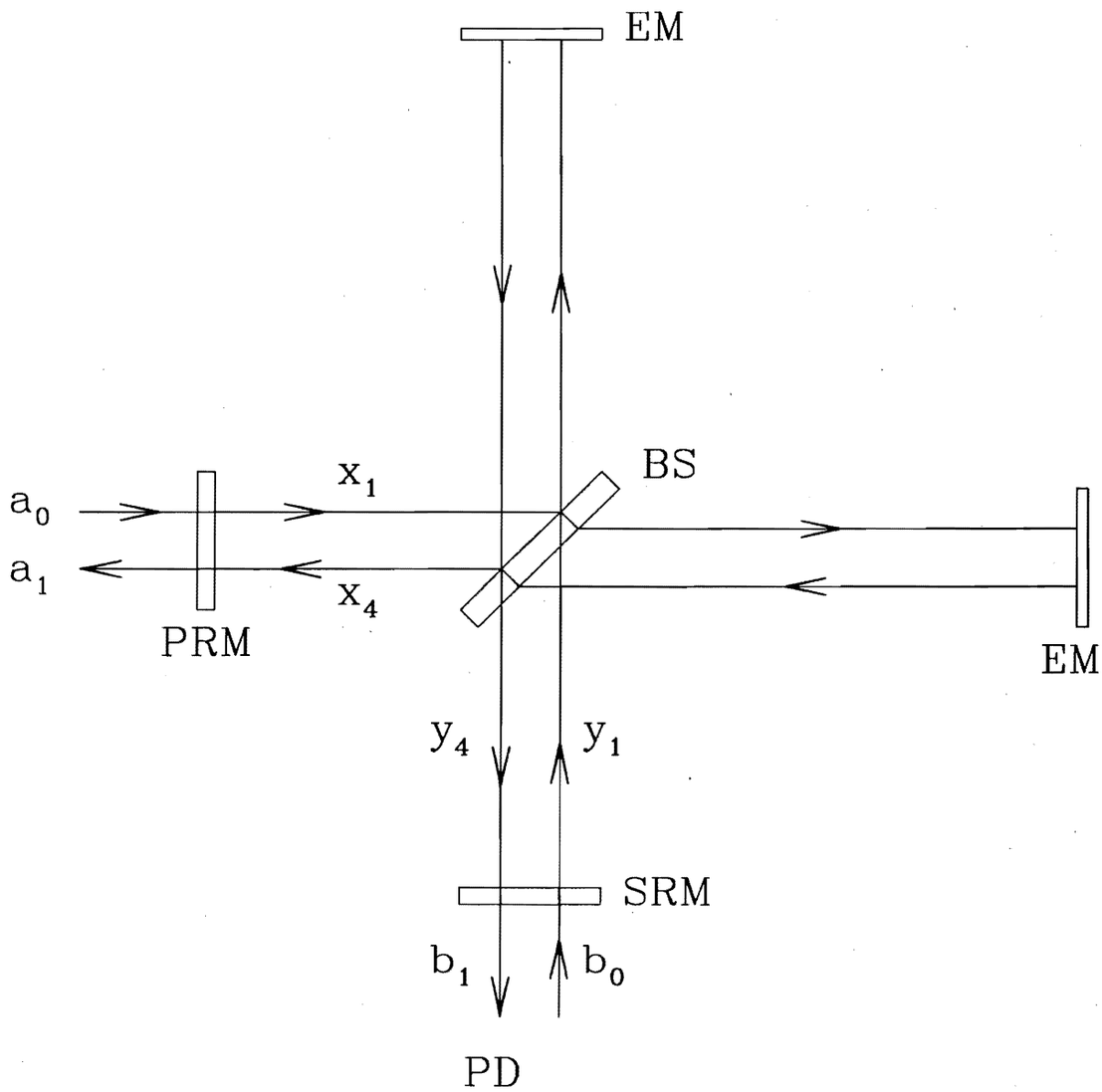

FIG. 1

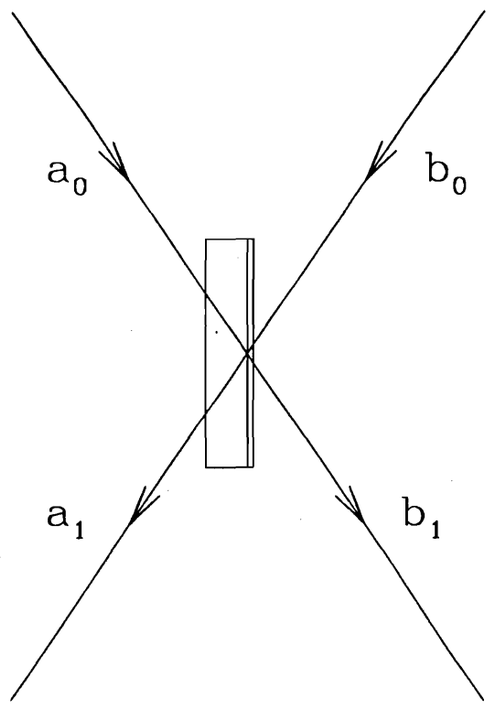

FIG. 2

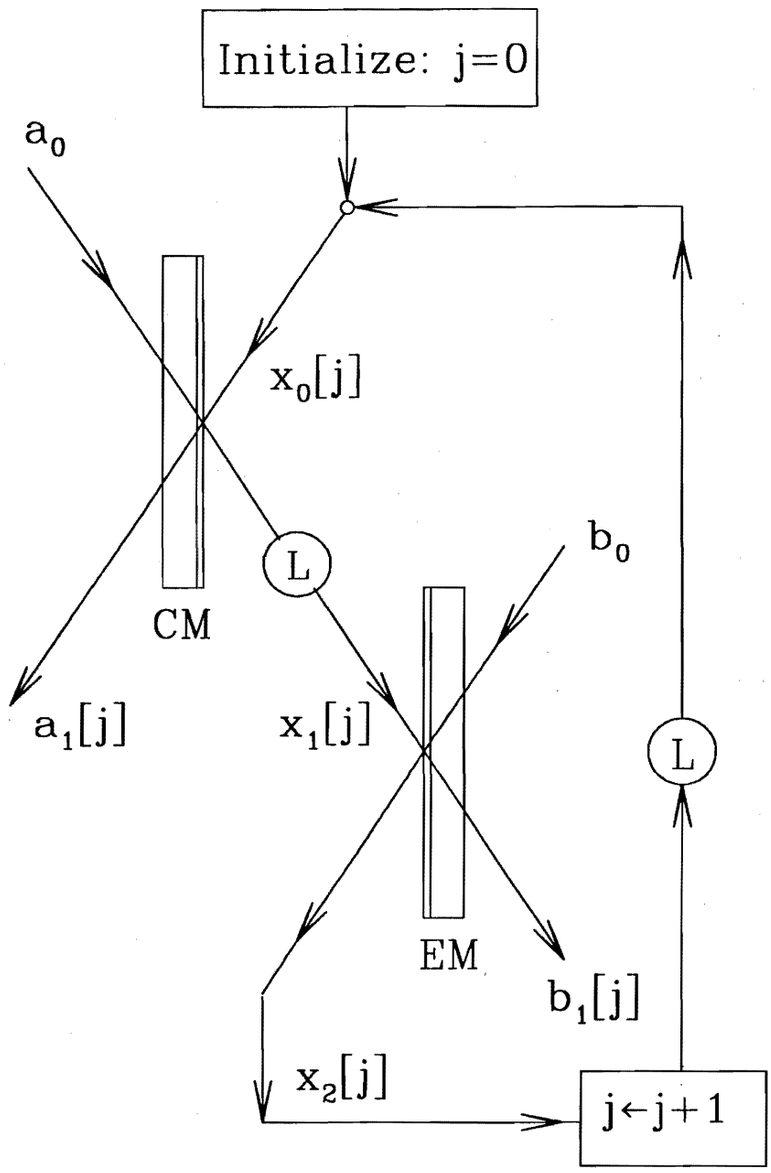

FIG. 3

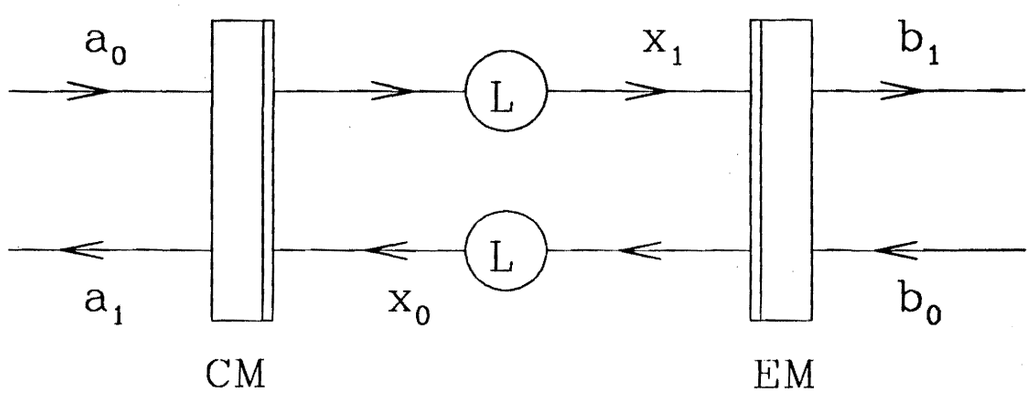

FIG. 4

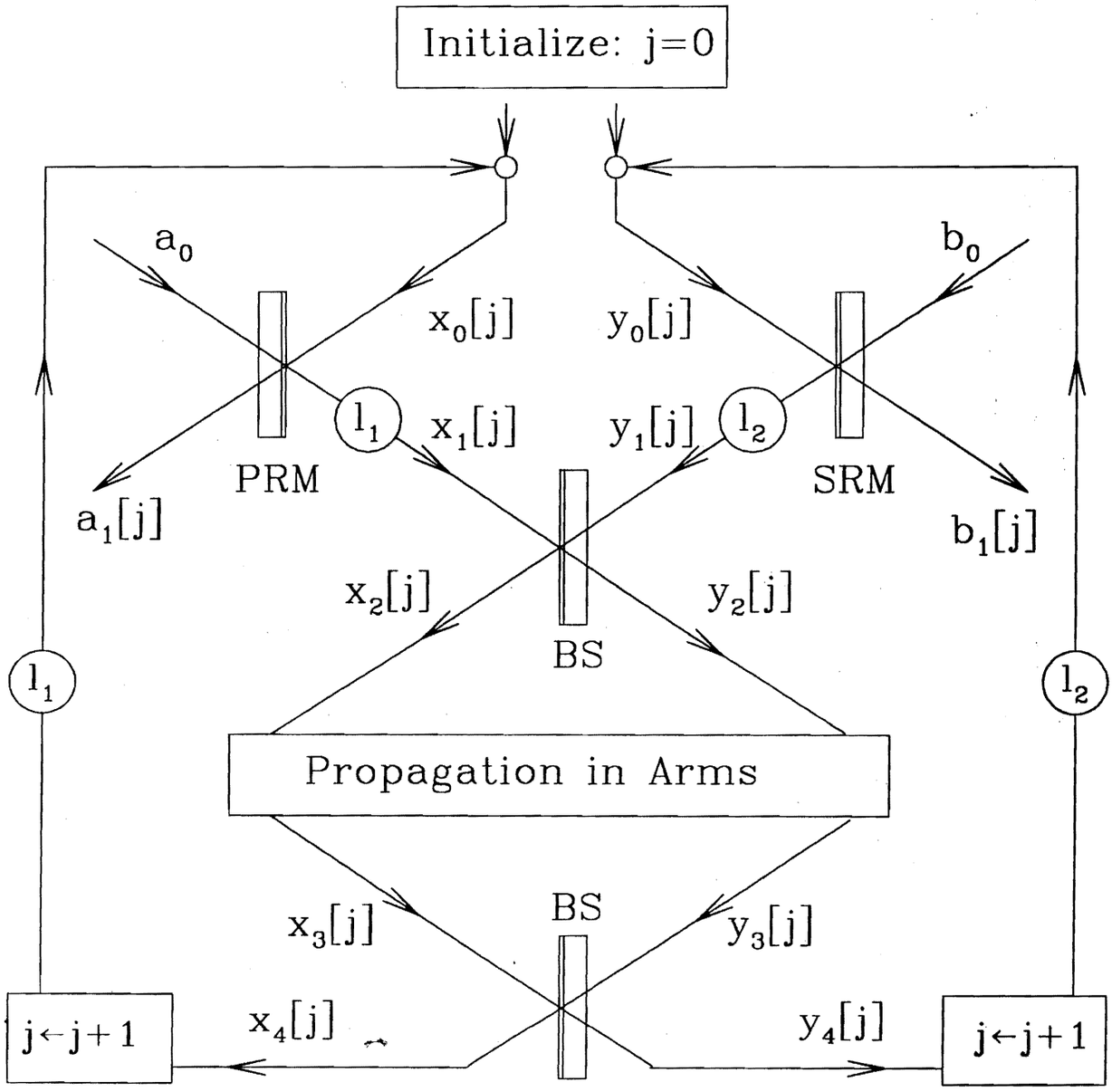
Initialize: j=0
a_0
b_0
$x_0[j]$
$y_0[j]$
l_1
$x_1[j]$
$y_1[j]$
l_2
PRM
SRM
$a_1[j]$
$b_1[j]$
$x_2[j]$
$y_2[j]$
BS
l_1
l_2
Propagation in Arms
$x_3[j]$
BS
$y_3[j]$
$j \leftarrow j+1$
$x_4[j]$
$y_4[j]$
$j \leftarrow j+1$

FIG. 5