IUCAA -29/99

July ' 99

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An Autonomous Institution of the University Grants Commission

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# POWER LAW INFLATIONARY SCENARIO WITH A SCALAR FIELD EXPONENTIAL POTENTIAL MODEL

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## ABSTRACT

For the purposes of making a viable cosmological scenario, it is known that the universe must evolve into the radiation dominated Friedmann stage after enough inflation and successful reheating. This can be achieved if the potential, $V(\phi)$, has a global minimum with respect to $\phi$. While the realistic potentials which appear in particle and supergravity theories are too complicated to be handled analytically, the forms of the exponential potentials used in the literature thus far also do not display these features at the same time. Here we suggest a simple exponential potential which has a global minimum (like the one dictated by the particle-theory potentials) and describes the inflationary scenario rather satisfactorily. Attempts are made to investigate the solutions of the corresponding classical equation of motion for the scalar field $\phi$ and accordingly the power law inflation scenario is discussed. The tunnelling probability and the density perturbations are also computed within this framework.

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## 1. Introduction

With a view to producing symmetry breaking one needs to introduce in the ambit of discussion a potential function $V(\phi)$ with a nontrivial classical minimum. While such features of the potential must show up in a natural manner on the basis of some dynamical arguments, Coleman[1], Guth[2] and Guth and Weinberg[3] have endeavoured to look for their origin in the renormalized version of "massless" scalar electrodynamics. However, the corresponding mathematical structure of $V(\phi)$, obtained in these cases for T = 0, viz.

$$V(\phi) = \frac{25}{16}\alpha^2 [\phi^4 \ln(\phi^2/\sigma^2) + \frac{1}{2}(\sigma^4 - \phi^4)] \tag{1}$$

turns out to be difficult to handle analytically. A form such as this will be termed as the realistic one. Here $\alpha^2(= 1/45)$ is the strong coupling constant and $\sigma(\simeq 1.2 \times 10^{15}$ GeV) is the broken-symmetry minimum value[4] of $V(\phi)$. As compared to several other known forms, the structure (1) has a smooth behaviour near $\phi = 0$ and also explains several temperature-dependent features.

Other competing mechanisms to understand the inflationary universe have found expression[5-11] in terms of various versions of inflation characterized, for instance, by chaotic inflation, extended inflation, stochastic inflation among others. In the case of chaotic inflation one considers a massive non-interacting scalar field $\phi$ with the Lagrangian density

$$\mathcal{L} = (M_P^2/4\pi)\ R + \frac{1}{2}\partial_\mu\phi\ \partial^\mu\phi - V(\phi), \tag{2}$$

with $G = M_P^{-2}$ as the gravitational constant; $M_P = 1.2 \times 10^{19}$ GeV the Planck mass, R the curvature scalar and m is the mass of the scalar field appearing in $V(\phi)$ such that $m \lll M_P$. If the scalar field $\phi$ is sufficiently homogeneous in some domain of the universe, then its behaviour inside this domain is governed by the equations

$$\ddot{\phi} + 3H\dot{\phi} = -(\partial V/\partial\phi), \tag{3}$$

$$H^2 + (k^2/a^2) = (\varkappa^2/3)[\frac{1}{2}\dot{\phi}^2 + V(\phi)]. \tag{4}$$

Here $V(\phi)$ is the effective potential of the field $\phi$; $H = (\mathring{a}/a)$, $a(t)$ the scale factor of the locally Friedmann universe (inside the domain under consideration); $\varkappa^2 = (8\pi/M_P^2)$, and $k = +1, -1, 0$ correspond to the closed, open or flat universe, respectively. In this work we confine ourselves to the case of the flat ($k=0$) universe.

It is now well known that the resolution of the appropriately posed cosmological problems of horizon and flatness are attributed to the inflation scenario or to the exponential expansion of the cosmic scale in the early universe[12). In particular, the cosmic scale factor $a(t)$ is found to grow as $a(t) = a_o t^p$, ($p > 1$). In fact such a power law expansion is realizable mainly through the scalar field $\phi$ with an exponential potential, in that the potential $V(\phi) = V_o \exp(-\lambda\phi)$ dominates[7-9) the energy density of the universe. For the exponential form of $V(\phi)$ the solution of the nonlinear classical equation of motion (3), no doubt, exhibits[7) an attractor but the desired features mentioned above are missing from this form of $V(\phi)$. Also, in order to achieve a viable cosmological scenario the universe is expected to evolve into the radiation-dominated Friedmann stage after enough inflation and successful reheating. For this purpose the potential must have a global minimum- a requirement not fulfilled by the above purely exponential potential. To find an answer to this daunting problem several variants of the inflation scenario are discussed[11,12) in the literature.

While Yokoyama and Maeda[8) accounted for the energy dissipation by introducing a term, $c\,\mathring{\phi}$, in eq.(3) (the origin of this term can also be attributed to the couplings of some other fields, if present, with $\phi$), they proposed a toy potential for inflation, viz.,

$$V(\phi) = 2V_o\,[\cosh(\lambda\phi) - 1] \tag{5}$$

for the 'inflaton' field $\phi$. In the limiting cases this potential reduces to an exponential or a harmonic form

$$V(\phi) \simeq V_o \exp(-\lambda\phi) \qquad \text{for } -\phi \gg (1/\lambda), \qquad (6a)$$

$$\simeq V_o \lambda^2\phi^2 \qquad \text{for } |\phi| \ll (1/\lambda). \qquad (6b)$$

depending essentially on the relative strength of the inflaton field $\phi$ vis-a-vis $\lambda^{-1}$. Interestingly, the forms (6a) and (6b) have been used in recent years[11] to describe the inflation scenario in different stages by what are known as exponential and new inflations, respectively. While form (5) does not accommodate the desired features, its use in (3) is no less challenging for the purpose of mathematical treatment. On the other hand, a generalized version of the limiting cases (6a) and (6b) is found[10] particularly useful from the point of view of investigating the inflation scenario. Therefore, in order to address several additional aspects pertaining to inflation we propose, in the present work, an effective potential of the form†

$$V(\phi) = V_o (\phi-\phi_o)^2 e^{\lambda(\phi-\phi_o)} , \qquad (7)$$

where $\phi=\phi_o = 1.2 \times 10^{15}$ GeV is the position of the global minimum in $V(\phi)$ of (7) and there also exists (cf. Fig.1) a maximum at $\phi = \phi_o - (2/\lambda)$. The quantities $V_o = 77.35 \times 10^{28}$ GeV$^2$ and $\lambda = 2.86 \times 10^{-15}$ GeV$^{-1}$ have been fixed in accordance with the realistic potential (1). The potential (7) exhibits a (mandatory) huge slow-roll regime as well as a minimum at $\phi = 0$ (cf. Fig.1) for these values of the parameters.

In Sect. 2, we discuss the dynamics of the system and show that there exists an attractor for the solution of eq.(3)

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† Mathematically, it is not difficult to visualize the origin of the harmonic factor in (7) if one confines only to the second order variations of $V(\phi)$ with respect to $\lambda$ through $(\partial^2 V(\phi)/\partial\lambda^2)$ and such a variation makes sense since $\lambda$ is also linked[7] with n in (4+n)-dimensional Kaluza-Klein type supergravity theories.

with $V(\phi)$ given by (7). In Sect.3, we demonstrate the possibility of existence of yet another attractor in the solution of (3) by introducing a friction term, $(\sum_m c_m \phi^m)\dot{\phi}$, in it. Necessity of such a term arise[13] in the study of inflation with thermal dissipation. Slow-roll scenario is also discussed here. The tunnelling probability and the density perturbations corresponding to potential (7) are estimated in Sect.4. Finally, concluding remarks are made in Sect. 5.

## 2. Dynamics of the System

Now we investigate the solutions of the classical equations of motion (3) for the potential (7). After defining $f = \phi_o - \phi$, eqs.(3) and (4) can be rewritten, respectively, as

$$\ddot{f} + 3H\,\dot{f} + V_o f(2-\lambda f)\, e^{-\lambda f} = 0 \tag{8}$$

$$3H^2 = \varkappa^2 \left(\frac{1}{2}\,\dot{f}^2 + V_o\, f^2\, e^{-\lambda f}\right). \tag{9}$$

Note that $V_o$, $\lambda$, $\varkappa$ and $H$ are true constants and obviously no variation in their values is possible while studying the stability of the solutions of (8). Exact integration of eq.(8) is possible[14] if one drops the second term in eq.(8) and sets the initial conditions as $f = 0$, $\dot{f} = 0$ at $t = 0$ (which correspond to the solution at the global minimum). However, such a solution of (8) cannot be fully justified and we therefore look for the solution of eq.(8) without imposing the above conditions. For this purpose, we transcribe eq.(8), for convenience, as

$$\ddot{x} + 3H\dot{x} + V_o x(2-\lambda x)\, e^{-\lambda x} = 0 \tag{10}$$

and prefer to write it equivalently in terms of a pair of first order differential equations in two different ways, namely

**Case A :**

$$\dot{x} = y \tag{11a}$$

$$\dot{y} = -3Hy - V_o x(2-\lambda x)\, e^{-\lambda x}. \tag{11b}$$

Case B : $\overset{\circ}{x} = y - 3Hx$ (12a)

$$\overset{\circ}{y} = - V_o x \; (2-\lambda x) \; e^{-\lambda x} . \qquad (12b)$$

With the reduction (12) note that eq.(10) can immediately be identified[15] with the Lienard equation

$$\overset{\circ\circ}{x} + g(x) \overset{\circ}{x} + h(x) = 0,$$

where $g(x) = 3H$ is a constant and $h(x) = V_o x(2-\lambda x) \; e^{-\lambda x}$ satisfies the required conditions[15] in order that a Liapunov function $F(x,y) = \mathscr{K}(x) + \frac{1}{2} y^2$, with $\mathscr{K}(x) = \int_0^x h(u)du$ can be constructed.

The Liapunov function finally turns out to be

$$F(x,y) = V_o x^2 \; e^{-\lambda x} + \frac{1}{2} y^2 , \qquad (13)$$

which is positive definite for all real x and y. Similarly, $\overset{\circ}{F}$ is obtained from

$$\overset{\circ}{F} = \frac{\partial F}{\partial x} \overset{\circ}{x} + \frac{\partial F}{\partial y} \overset{\circ}{y} \qquad (14)$$

after using (13), (12a) and (12b) as

$$\overset{\circ}{F} = -3HV_o \; x^2 (2-\lambda x) \; e^{-\lambda x}$$

which is negative definite as x varies over $0 < x < (2/\lambda)$ or $\phi$ varies over a region $\phi_o - 2/\lambda < \phi < \phi_o$ where the slow-roll of $\phi$ in the potential dominates. Thus, the origin (0,0) in the x-y plane is a **uniformly stable** critical point, i.e., there is a unique attractor at $\phi = \phi_o$ of the potential. However, for the region $0 \le x \le (2/\lambda)$ $\overset{\circ}{F}$ is negative semidefinite and therefore (0,0) is **asymptotically stable.**

Alternatively, one can as well carry out an analysis of the phase portraits on the x-y plane for the system (12): In this case one obtains three critical points $C_1$: (0,0), $C_2$: ($2/\lambda$, $6H/\lambda$), $C_3$: ($\infty,\infty$). It can be shown[15] that $C_1$ is a stable node, $C_2$ a saddle (unstable) and $C_3$ an unstable critical point (since one of the eigenvalues in this case is zero, implying a critical point which is not isolated). In the same way, if one carries out the phase plane analysis for the case A (cf. eqs.(11a) and (11b), then the nature of the critical points and subsequently the topology of the trajectories turns out to be the same except for the fact that the axes now stand rotated relative to the case B.

## 3. Slow-Roll Scenario and the Effect of Dissipation

It may be mentioned that in exponential inflation the dependence of $\phi$ on the time variable t is negligibly small during the inflationary stage whereas it is not so in the case of power law inflation. For the former case $\ddot{\phi}$ in eq.(3) can be dropped and consequently this equation implies that $\dot{\phi} = -\frac{1}{3H}(\frac{\partial V}{\partial \phi})$. Thus, the inflaton field $\phi$ rolls down the pure exponential potential more rapidly then it rolls down the potential (7) in the vicinity of the global minimum. To some extent such a phenomenon can be attributed to the couplings of the $\phi$-field with other fields, if present, which in turn implies that the inflation proceeds through some dissipation processes. While the role of such friction forces has been investigated[7,8] in the literature by inserting a term of the type c $\dot{\phi}$ in eq.(3), it is also argued that this term owes its existence to the particle creation[8]. This is perhaps the simplest way (as one retains only the first power of $\dot{\phi}$) of accounting for the dissipation in the system. If one resorts to account for the dissipation in a different (and possibly a little more complicated) way, then the corresponding dynamical system is going to exhibit several other interesting features as far as the nature of attractor (stability of solutions) is concerned. For example, for the friction term of the type $(\phi^2-1)\dot{\phi}$ or $c_o(\phi^2+\dot{\phi}^2-1)\dot{\phi}$ the existence of a limit cycle in the system can be readily demonstrated.

In order to account for the thermal dissipation in the inflation scenario, Lee and Fang[13] have recently suggested the use of a term of the type $(c_m\phi^m)\dot{\phi}$ with $c_m > 0$. One may as well use more generalized version of this form, namely $(\sum_m c_m\phi^m)\dot{\phi}$ in eq.(3). While the first form clearly leads to a limit cycle for certain values of $c_2$ and $m = 2$, we analyse the latter case here in detail. Eq.(10) can now be written in the form

$$\ddot{x} + 3H\dot{x} - (\sum_m c_m x^m)\dot{x} + V_o x(2-\lambda x)\, e^{-\lambda x} = 0. \tag{15}$$

The phase plane analysis of eq.(15) can be carried out in exactly the same way as is done for the system (12) (cf. Case B). We again obtain the same set of critical points $C_1, C_2, C_3$ out of which $C_1$: (0,0) is a stable node as before. For the critical point $C_2$: $(\frac{2}{\lambda}, \frac{6H}{\lambda})$ which was a saddle (unstable) earlier, the situation now is different however. In fact, the eigenvalues in the present case are given by

$$\Lambda_{1,2} = -\frac{1}{2}(3H-\sum_m c_m(2/\lambda)^m)$$
$$\pm \frac{1}{2}[\{3H-\sum_m c_m(2/\lambda)^m\}^2+8e^{-2}-12\sum_m c_m H(2/\lambda)^m]^{1/2}. \tag{16}$$

Clearly, for $m = 0$ and $c_o = 0$ one recovers the earlier result i.e. a saddle at $c_2$: $(\frac{2}{\lambda}, \frac{6H}{\lambda})$; otherwise note that for

$$[3H-\sum_m c_m(2/\lambda)^m]^2 + 8e^{-2} < 12H\sum_m c_m H(2/\lambda)^m$$

one can establish the existence of a stable spiral for certain values of m and $c_m$. Thus the introduction of dissipation of the type discussed here in the above system will definitely bring in the role of more attractors which otherwise was not possible in the approach of Yokoyama and Maeda[8]. Moreover, once the dissipation is present it will also accelerate the process of reheating[16].

## 4. Tunnelling Probability and Density Perturbations

In this section we demonstrate the computation of some results within the framework of the present exponential potential model. For this purpose we make use of the standard techniques available in the literature. In particular, we carry out an order-of-magnitude estimation of the quantities which have been of pivotal importance in recent years; namely (i) the tunnelling probability P through the barrier of potential (7) (cf. Fig.1), and (ii) the density perturbations $(\delta\rho/\rho)$.

In order to estimate the tunnelling probability P we assume, following Linde[17], the formation of bubbles of a new phase and make use of the Euclidean approach to the tunnelling theory. This approach enables us to compute the decay probability of the false vacuum and the expression for P in this case is given by[17]

$$P = A \exp(-S(\phi)), \tag{17}$$

where $S(\phi)$ is the Euclidean action corresponding to the solution of the equation

$$\Box \phi = \frac{\partial^2 \phi}{\partial t^2} + \nabla^2\phi = \frac{dV}{d\phi} \equiv V'(\phi) \tag{18}$$

and takes the form

$$S(\phi) = \int d^4x \left[ \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla\phi)^2 + V(\phi) \right]. \tag{19}$$

The factor A in eq.(17) has the dimensions of $(\text{mass})^4$ and related to the action S through the relation

$$A = \left(\frac{S}{2\pi}\right)^2 \left[ \frac{\det'[-\Box + V''(\phi)]}{\det[-\Box + V''(\phi)]} \right]^{-1/2}, \tag{20}$$

where "det′" represents the functional determinant of the operator $[-\Box+V''(\phi)]$ such that its vanishing eigenvalues, corresponding to the zero-modes of the operator, are ignored. Here $\phi$ is assumed to satisfy the boundary condition $\phi = 0$ as $x^2+t^2 \longrightarrow \infty$ and $V(\phi)$ satisfies $V(0) = 0$. The latter condition is achieved from (7) by redefining $V(\phi)$ as $V(\phi)-V(0)$. Recognizing the fact that the quantities $\phi(0)$, $\sqrt{V''(\phi)}$ and $r^{-1}$ (where r is the typical size of the bubble) lie within an order of magnitude of each other, the factor $[\det'[-\Box+V''(\phi))/\det(-\Box+V''(\phi))]^{-1/2}$ on the right hand side of eq.(20) can be estimated so as to be of the same order i.e., of the order of $(r^{-4}, \phi^4(0), (V'')^2)$. Thus, under the same assumption $S(\phi)$ can be estimated, after using (4) in (19), as

$$S(\phi) \sim r^4.(3H^2/\varkappa^2) \sim (V''(\phi))^{-2}.\ (3H^2/\varkappa^2)$$

and subsequently the factor A as $A \sim (S^2(\phi)/4\pi^2)\ [V''(\phi)]^{-1}$. Substituting the value of $V''(\phi)$ at $\phi \equiv \phi_1 = (\phi_0 - 2/\lambda)$ as $20.931 \times 10^{28}$ $GeV^2$ and the values of other constants, the tunneling probability P from (17) is estimated to be $P \simeq 2.55 \times 10^{-33}$ $GeV^2$. It may be mentioned that it is the factor A (and not the factor $\exp(-S(\phi))$ in (17) that dominates the values of P. Alternatively, one can also compute P using the theory of Hawking and Moss[18]. In this case one considers the tunneling from $\phi = 0$ through the barrier with a maximum at $\phi = \phi_1$ and as a result, P is given by

$$P \sim A \exp\left[ - \frac{3M_P^4}{8} \left( \frac{1}{V(0)} - \frac{1}{V(\phi_1)} \right)\right]. \tag{21}$$

On using the values of V(0) and $V(\phi_1)$ as obtained from (7) and the value of $A \sim 2.98 \times 10^{-33}$ $GeV^2$ from (20), the relation (21) yields the value of P as $P \simeq A \exp(-6.42 \times 10^{16})$. It may be mentioned that in the Hawking-Moss theory (cf. eq. (21)) the exponential factor dominates over the value of A whereas in the case of Euclidean approach (cf. eq.(17)) the factor A dominates over the exponential instead. In either case the value of P turns out to be enormously small. This value is even smaller than that obtained for the chaotic type inflation[17] where $P \sim \exp(-8\pi^2/3\bar{\lambda}) \sim 10^{-10^3}$ for the potential $V_1(\phi) = \frac{1}{2} M^2 \phi^2 - \frac{1}{4} \bar{\lambda} \phi^4$.

Following Linde[17], we also carry out an estimation of the quantum fluctuations in the present model under highly simplifying assumptions. Towards this end we use the expression

$$\frac{\delta\rho(k)}{\rho} = \frac{48}{5} \sqrt{\frac{2\pi}{3}}\ \frac{V^{3/2}(\phi)}{M_p^3\ V'(\phi)}, \tag{22}$$

where the quantity on the right hand side is computed at the value of the wave number $\bar{k}$ such that $\bar{k} \sim H(\phi)$. Using $V(\phi)$ from (7) an estimate of $(\delta\rho/\rho)$ from (22) at $\phi = 0$ furnishes a value of $6.39 \times 10^{-12}$ and that at $\phi = 1.0 \times 10^{15}$ GeV gives rise to $(\delta\rho/\rho) = 7.43 \times 10^{-11}$. While the variation in the value of $(\delta\rho/\rho)$ in the present model is just within one order of magnitude for the allowed range of the scalar field $\phi$, it is, however, much smaller than the value $10^{-4} - 10^{-5}$ expected for the spectral amplitude on a galactic scale and for a specially normalized spectrum.

## 5. Concluding Discussion

We have made an attempt to understand the inflation scenario within the framework of an exponential potential model for the scalar field inducing the power law inflation. While this scalar field potential accounts for almost all the desirable features as dictated by the particle physics theories, the solutions of the corresponding equation of motion are found to exhibit an attractor. It is shown that another attractor also comes into play if one accounts for the dissipation by introducing a term of the type $-\left(\sum_m c_m \phi^m\right) \dot{\phi}$ in the equation of motion. In fact the importance of a similar term is realized recently[13] to understand the inflation scenario with thermal dissipation.

Other features of this exponential potential model investigated in the present work pertain mainly to the estimation of the tunnelling probability P and the quantum fluctuations through the computation of $(\delta\rho/\rho)$. It is found that the use of either of the methods, namely the Euclidean approach[17] and the theory of Hawking and Moss[18], yields enormously small values for the tunnelling probability as compared to the one estimated (see, Ref. (17)) on the basis of a chaotic- type scalar field potential. Similar is the trend of the results obtained for $(\delta\rho/\rho)$ in the present model. As the maximum between the "false" and "true" vacua is not very pronounced in potential (7) (cf. Fig.1) we expect better understanding of the slow-roll phenomenon and subsequently that of the reheating mechanism in this model. Further studies are in progress.

**Acknowledgements**

We express our gratitude to Professor J.V. Narlikar, and Professor Ajit Kembhavi for the warm hospitality at IUCAA, Pune, where a good deal of this work was carried out. Thanks are also due to Professor Varun Sahni for many useful discussions and to Mr. Satyen Sinha and Dr. Shyamal K. Banerjee for their help with the computation. RSK acknowledges the award of a Research Scientist Scheme by UGC, New Delhi during the course of this work.

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## Figure Caption

Fig. 1. The effective scalar field potential $V(\phi)$ described by Eq.(7).

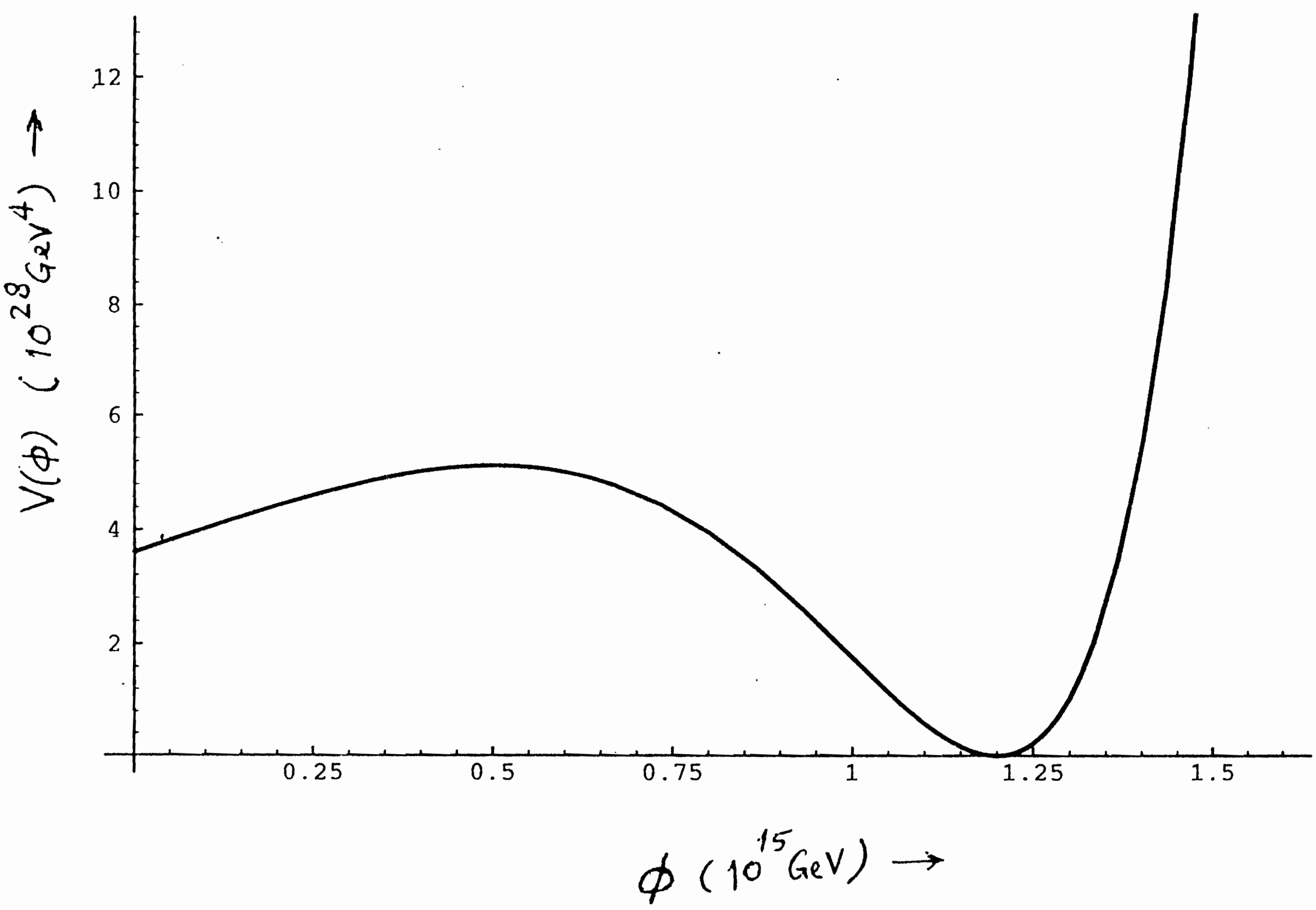


Fig. 1